LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
-LIGO-
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Note LIGO-T970174- 00- D 10/15/97

Interferometric Vernier Technique for Measuring the Lengths of LIGO Fabry-Perot Resonators

Matt Evans, Malik Rakhmanov
and Hiro Yamamoto

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

file /home/malik/documents/T970174.ps- printed November 14, 1997

Abstract

A technique for measuring a length of long baseline LIGO Fabry-Perot cavities is proposed. It is based on optical vernier naturally occuring when light resonating in the Fabry-Perot cavity has an RF sideband. By observing the resonance pattern and measuring the separation between successful resonances of the carrier and the sideband it is possible to determine the length of the resonator with high precision. The technique was applied to finding the length of the Fabry-Perot resonator of the LIGO "40m" Prototype. The precision obtained was 0.01%.

Keywords

Length measurement, Fabry-Perot cavity, resonance conditions, carrier, sidebands, modulation length.

1 Introduction

Historically Fabry-Perot interferometers were used for studies of fine structure of spectral lines and comparison of close wavelengths [1]. In those experiments the Fabry-Perot (FP) resonators were at most a few centimeters long. The length of such resonators was almost always known to a high degree of accuracy. Recently Fabry-Perot resonators were proposed to be used in the long baseline interferometric gravitational wave detectors [2]. The length of such Fabry-Perot resonators is chosen to provide sufficient sensitivity to the gravitational wave strain and can be as large as 4 km. It is one of the main parameters of the gravitational wave detectors. Thus knowing the length of the resonators with high accuracy becomes important.

Here we propose a technique for measurement of the length of the Fabry-Perot resonator based on its property to resolve close spectral lines.

The technique does not require any special equipment. The measuring device is the Fabry-Perot resonator itself. The input light for the FP resonator should consists of the at least two close frequencies. This condition can be easily satisfied. All interferometric gravitational wave detectors, which are currently under construction, have optical sidebands as an essential part of their signal extraction schemes.

The idea is to use two spectral lines with slightly different wavelengths, λ_c and λ_s, as an input for the FP resonator and observe a set of several resonances produced by the lines. To observe the set of resonances we need to make the length of the resonator vary over several wavelengths. This can be achieved by moving one of the mirrors of the resonator over a distance of several wavelengths.

It is interesting to note that neither λ_c nor λ_s need to be known accurately. Only the beat wavelength should be known with sufficiently high precision.

In LIGO interferometers the sidebands on the light are generated by applying phase modulation on a single frequency laser. The phase modulation with frequency f_{mod} creates an infinite array of the sidebands with frequencies separated by f_{mod}. The modulation wavelength is defined by the modulation frequency $\Lambda = c/f_{mod}$.

For the discussion here we choose the carrier and the first lower sideband as the two close spectral lines. If the carrier wavelength is λ_c the sideband has a wavelength of

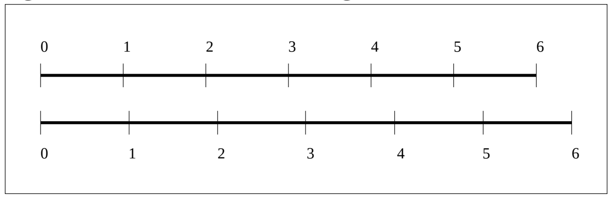
$$\lambda_s = \frac{\Lambda\lambda_c}{\Lambda - \lambda_c}. \tag{1}$$

If the carrier and the sideband were of equal amplitude the sum of the two waves would produce beats inside the FP resonator. One can see that the wavelength of the beat would be the same as the modulation wavelength.

2 Theory

The vernier technique is not new. It has been extensively used in various precision mechanical devices, such as micrometers. The technique is based on the fact that a much greater precision can be obtained if two slightly different length scales are used simultaneously. A picture of the vernier is shown on Fig.2.

Figure 1: Two different length scales as a vernier



For a given frequency of input light FP resonator creates an array of resonances in the space between the mirrors with the separation being equal to the half wavelength of light. If the two slightly different frequencies of light are used the resonator creates two spatial arrays of resonances with a slightly different spacings, which constitute an optical vernier.

The light resonates in the FP cavity if the distance between the two mirrors equals to an integer number of half-wavelengths. Consider a situation when the front mirror is fixed and the end mirror moves away from the front mirror. Introduce a coordinate axis z along the optical axis of the resonator and choose the origin at the surface of the front mirror. As the end mirror moves it passes through successive locations were the resonant conditions are met. One can interpret this as if the resonances were located at

$$z_c = N_c \frac{\lambda_c}{2}, \tag{2}$$

where $N_c = 1, 2, \ldots$ is an integer number. Correspondingly the resonances for the sideband are located at

$$z_s = N_s \frac{\lambda_s}{2}. \tag{3}$$

with N_s being an integer number. Essential here is the fact that both sets of resonances begin from the same place, $z = 0$.

At some locations along z-axis the carrier and the sideband resonances coincide. The locations where this happens are given by

$$z = m \frac{\Lambda}{2}.$$

where m is an integer number

$$m = N_c - N_s. \tag{4}$$

The physical meaning of m is that it equals to the number of beats between the carrier and the sideband within the length z. The actual meaning of this condition is the following. If the end mirror is placed at these locations both the carrier and the sideband will resonate simultaneously. Thus the simultaneous resonances are located at the multiples of the modulation half-wavelength.

In general the length of the resonator can be represented as a sum

$$L = L_0 + \delta L.$$

Here L_0 is the closest multiple of the modulation half-wavelength

$$L_0 = m \frac{\Lambda}{2}, \tag{5}$$

and δL is the deviation of the resonator length from the last coincidence node. By definition δL is positive and less than $\Lambda/2$.

Imagine that you increase δL step by step from 0 to its maximum value. Each time you increment δL let the mirror swing through few successive resonances and observe the resonance pattern. The sideband resonance will be located somewhere in between the two carrier resonances. As you increment δL from 0 to its maximum value the sideband peak will move from one carrier to another.

Let δz be the separation between the sideband and the nearest carrier resonance from below. As δL changes from 0 to $\Lambda/2$ the sideband-carrier separation, δz, changes from 0 to $\lambda_c/2$. Since the two lengths, δL and δz, are linearly related they change proportionally

$$\frac{\delta L}{(\Lambda/2)} = \frac{\delta z}{(\lambda_c/2)}. \tag{6}$$

Mathematical derivation of this equation is given in Appendix.

The optical vernier method is based on finding the accurate value for the ratio given by eq.(6). Let μ be the ratio. If one knows the ratio one can find the deviation

$$\delta L = \mu \frac{\Lambda}{2}.$$

and therefore the total length

$$L = (m + \mu)\frac{\Lambda}{2}.$$

3 Applications

The method described above was applied to finding the length of the Fabry-Perot resonator of the South Arm of the "40m" prototype of LIGO at Caltech. The block diagram of the experimental setup is shown on Fig.3.

The beam from the Argon-Ion laser is phase modulated by the sinusoidal voltage at $f_{mod} = 32.7$ MHz produced by the oscillator. The Fabry-Perot resonator consists of the two mirrors suspended from wires. Two output signals were recorded in the experiment: transmitted light power from the photodiode (PD) and demodulated output from RF-tuned photodiode (RFPD). The modulation wavelength is Λ = 9.174 m.

The approximate length of the resonator known from previous measurements is

$$L = 38.5 \pm 0.2 \text{ m}.$$

Knowing the approximate length we can find the beat number exactly

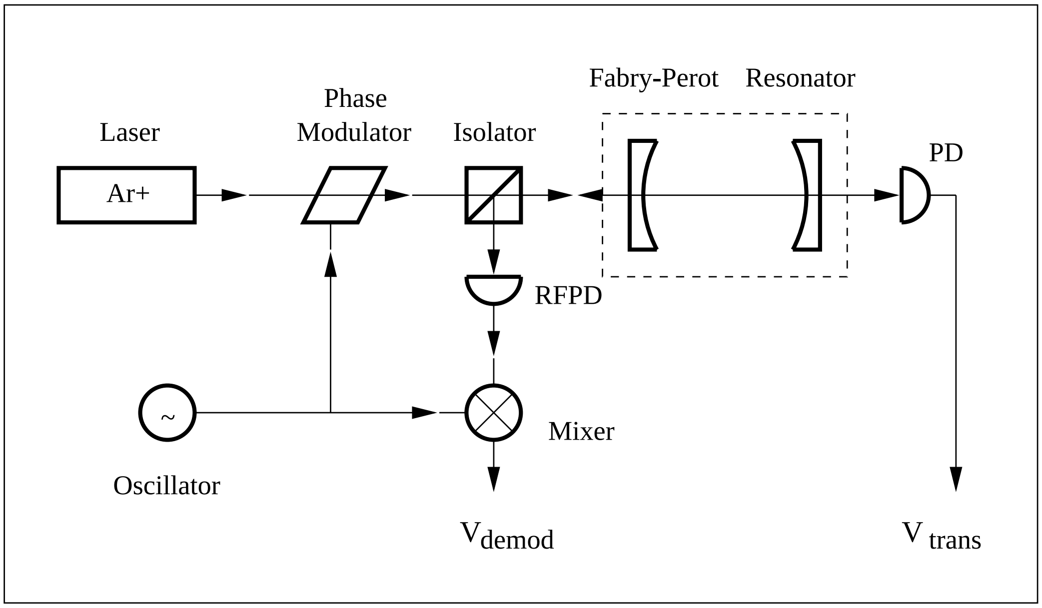
$$m = \left\lfloor \frac{L}{\Lambda} \right\rfloor = 8,$$

where $\lfloor\rfloor$ stands for the greatest integer less than.

The ratio μ was found from the observation of the resonance pattern on the oscilloscope. The average ratio is

$$\mu = 0.4087 \pm 0.0004.$$

Figure 2: Setup of the experiment



The length of the resonator is

$$L = 38.5453 \pm 0.0036 \text{ m}.$$

One can see that the error in the length due to the error in the ratio is approximately 0.01%. Below we present the details of the measurement.

The set of resonances could be obtained either from the transmitted light power or from the demodulated output. The latter proved to be less noisy. Thus we used the resonances in the demodulated output. A trace used for the calculations of the ratio is shown on Fig.3. Five different measurements of the ratio corresponding to five consequitive free spectral ranges were analyzed.

The exact positions for resonances were found by setting the marker on the top of the highest peak within the fringe as shown on Fig.3. The marker positions of the carrier resonances provide a series of times $t_c(p)$, $p = 1, 2, \ldots, 5$. The time series should be converted into the series of mirror displacements $x_c(p)$ corresponding to these resonances. The same should be repeated for the sideband marker positions, $t_s(p)$. The mapping from time to displacement can be done if one knows the exact trajectory of the mirror $x(t)$. However this information was not available. The approximate trajectory $x(t)$ can be easily found from the same resonance set. Since the times $t_c(p)$ correspond to successive carrier resonances the displacements, $x_c(p)$, are separated by the carrier half-wavelength. Interpolating between the resonances we can find the the approximate trajectory of the mirror, $x(t)$. Once the trajectory is known we can find the sideband-carrier separations as

$$\delta z(p) = x(t_s(p)) - x(t_c(p)).$$

and therefore the ratios $\mu(p)$. The results are in the Table.1.

Figure 3: Demodulated output as a function of time

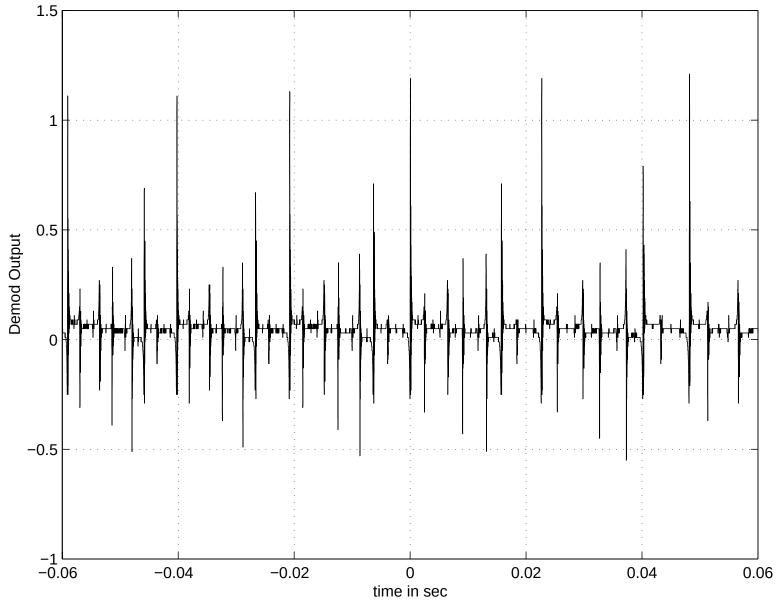


Figure 4: Magnified carrier fringe with a marker

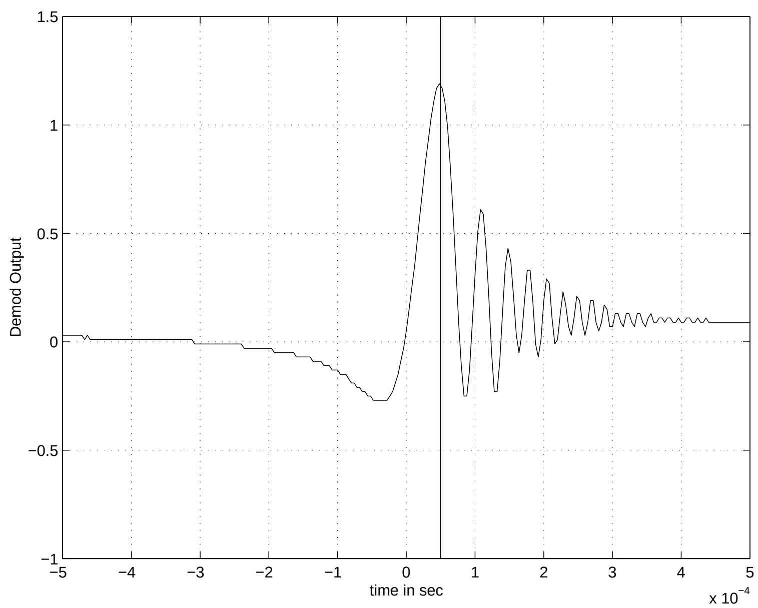


Table 1: Results of the experiment

# FSR p	Carrier t_c (sec)	Sideband t_s (sec)	Ratio μ
1	-5.9060e-2	-5.1405e-2	0.4072
2	-4.0230e-2	-3.2390e-2	0.4092
3	-2.0765e-2	-1.2450e-2	0.4087
4	0.0050e-2	0.9060e-2	0.4088
5	2.2705e-2	3.2685e-2	0.4097
6	4.8200e-2		

4 Uncertainty

The error in the determination of the length comes from

- uncertainty in the resonance due to the finite finesse of the FP resonator,
- uncertainty in the marker position due to a finite resolution of the trace,
- uncertainty in the mirror acceleration,
- uncertainty in the modulation wavelength.

In our measurement the dominant contribution to the error was the uncertainty in the mirror acceleration. This is due to the fact that we used only 5 points for polynomial interpolation between the resonances. Even with so few points the resultant error turned out to be 0.01 %, which indicates to the power of the method.

A careful study of possible systematic error in resonance location measurement was not performed. However, random errors in $x(t)$ dominate the largest concievable systematic error. In other words, the uncertainty in the trajectory, $x(t)$, in between the carrier resonances can be comparable to the FWHM of the resonance peaks. By simply increasing the number of points and doing the measurements over many free spectral ranges one can reduce the error in the determination of the trajectory. Other ways of improving the accuracy of the method, for example, to use several modulation wavelengths instead of one can also be studied.

5 Conclusion

The method of optical vernier is a simple and accurate way of obtaining the length of the Fabry-Perot resonators.

References

[1] Born M., Wolf E., *Principles of Optics*, p.338, Pergamon Press, 1970

[2] Abramovici A., et al, *LIGO: The Laser Interferometer Gravitational-Wave Observatory*, Science, Vol.256, p.281, 1992.

Appendix

Here we derive the proportionality condition, eq.(6). Using the definitions for the carrier and the sideband resonances we can write separation between the resonances as

$$z_s - z_c = N_s \frac{\lambda_s}{2} - N_c \frac{\lambda_c}{2}.$$

Then the calculation is

$$\begin{aligned} z_s - z_c &= N_s \left(\frac{\lambda_s}{2} - \frac{\lambda_c}{2} \right) - (N_c - N_s) \frac{\lambda_c}{2} \\ &= N_s \, \frac{\lambda_s \lambda_c}{2\Lambda} - m \, \frac{\lambda_c}{2} \\ &= \frac{\lambda_c}{2} \left(N_s \frac{\lambda_s}{\Lambda} - m \right). \end{aligned}$$

Dividing both sides of the equation by $(\lambda_c/2)$ we obtain

$$\frac{z_s - z_c}{(\lambda_c/2)} = \frac{z_s - L_0}{(\Lambda/2)}.$$