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Requirements for Creep Testing of SEI Spring Elements

F. Raab

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California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

Abstract

Requirements for creep in spring elements for the inital LIGO seismic isolation are analyzed. An apparatus for evaluating spring elements against the requirements is discussed.

Keywords: mirror balancing, pitch adjustment, suspension

1 OVERVIEW

The requirements on possible creep in the spring elements of the seismic isolation for LIGO are analyzed here. Throughout this document the strain change accompanying the sudden release of stress in a structure under load is referred to as “creep”. Although both the 40-meter interferometer and the phase-noise interferometer use viton and metal stacks for seismic isolation, there was no testing for creep in the spring elements prior to installation. Any effects of creep on interferometer noise were assumed small in these elastomeric elements and measures were taken to mitigate the effects of drift in the spring elements. Damped metal springs are a candidate for the LIGO spring elements. Although the springs are loaded far below their yield stress, there is concern about the effects of creep in this new type of spring. In section 2 a model is described that relates the energy released in an impulsive relaxation due to creep to motion of the suspended test mass. From this model a bound is obtained on the energy that can be released impulsively without appearing above the noise floor of the interferometer. An apparatus is described in section 3, that can be used to set a bound on creep events. Unfortunately, it is found that the bound obtainable with simple apparatus is weak compared to the requirement.

2 REQUIREMENT ON CREEP IN SPRING ELEMENTS

2.1. Motion of Optics Platform from Creep in a Spring

It is reasonable to expect the probability of creep occurring in a particular spring to depend on the load on that spring. The design for the initial LIGO seismic isolation attempts to load all springs as equally using an integer number of springs. Therefore creep noise will be most severe in the final layer of the stack, because noise from the lower layers will be partially filtered by subsequent stack elements. To obtain a creep requirement, we therefore analyze a single layer stack with a mass M supported by N springs as sketched in Figure (1). All springs are assumed to have

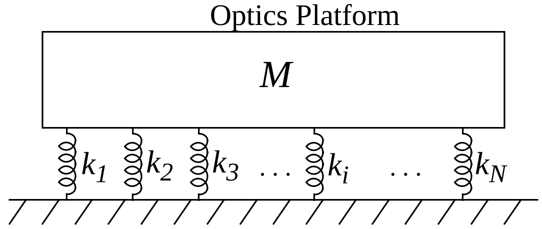

Figure (1) Sketch of a single-layer stack.

a uniform spring rate k_i and the static force exerted by each spring is given by

$$F_i = \frac{Mg}{N} \quad (1)$$

where g is the acceleration due to gravity. The spring rate is determined by the relation

$$k_i = \omega_0^2 \frac{M}{N} \quad (2)$$

where ω_0 is the angular resonance frequency of the single-layer stack. Suppose that at time t_0, creep occurs in spring 1, shortening the equilibrium length of spring 1. The force exerted by this spring changes according to

$$F_1 = k_1 x \Rightarrow k_1(x - \Delta x_1) = F_1 - \Delta F \quad (3)$$

where Δx_1 is the change in equilibrium length. This creep event releases a small amount of energy E in the spring given by

$$E \approx F_1 \Delta x_1 \quad (4)$$

where we have neglected the small changing force during the creep event. Combining equations (1) and (4) we obtain

$$\Delta x_1 \approx \frac{EN}{Mg} \quad (5)$$

for the change in equilibrium length of spring 1.

The other springs in the stack take up the change in force applied by spring 1 as the stack changes height. This is described by

$$\Delta F = \sum_{i=2}^{N} k_i \Delta x_i \quad (6)$$

where Δx_i is the change in height of the stack. An alternate relation for ΔF is obtained by combining equations (2), (3) and (5) to obtain

$$\Delta F = \frac{\omega_0^2 E}{g} \quad (7)$$

It is assumed that all spring rates are equal and that $\Delta x_i = \Delta x_P$ for all the springs. Combining equations (2), (3), (5) and (6) we obtain

$$Mg\Delta x_P \approx \frac{N}{N-1}E \tag{8}$$

for the change in height Δx_P due to the creep in spring 1 that released an amount of energy E. Note that the factor $\frac{N}{N-1}$ on the right-hand side of equation (8) is approximately unity to within the precision of the approximation made in deriving equation (4). Setting this factor to unity we have

$$Mg\Delta x_P = E \tag{9}$$

which obviously obeys conservation of energy. For the initial LIGO interferometer, the BSC downtube has a mass $M = 650$ kg. If $E = 10^{-9}$ J of energy is released by creep in a spring, the resulting motion of the optics platform will have an amplitude of $\Delta x_P = 1.6\times10^{-13}$ m.

2.2. Test Mass Response to Creep-Induced Optics-Platform Motion

Motion of the optics platform due to creep is filtered by the pendulum mode of the suspension. This can be expressed as

$$\Delta\tilde{x}_{TM}(f) = \eta \cdot \Delta\tilde{x}_P(f) \cdot \left(\frac{f_j}{f}\right)^2 \tag{10}$$

where η is a cross-coupling coefficient, f is a particular frequency component of the creep-induced motion and f_j is a resonant frequency of the suspension. For estimation, consider a creep event described by

$$\Delta x_P(t) = \Delta x_P e^{-t/\tau} \tag{11}$$

with duration τ. The spectral density of motion emitted in a time T of order τ is

$$S(f) \approx \frac{[\Delta\tilde{x}_P(f)]^2}{T} \cdot \frac{\tau^2}{1+(2\pi f\tau)^2} \tag{12}$$

which, as is typical of a burst, will decrease in magnitude as the measurement time T increases.

Setting $T = \tau$ and conbining equations (10) and (12), we obtain

$$\Delta\tilde{x}_{TM}(f) = \eta \cdot \Delta\tilde{x}_P \cdot \left(\frac{f_j}{f}\right)^2 \sqrt{\frac{\tau}{1 + (2\pi f \tau)^2}}. \tag{13}$$

It is reasonable to expect that creep will tend to cause motion principally in the vertical direction along the major strain axis of the spring. For the pendulum mode we have $f_{pend} = 0.74$ Hz and cross-coupling of vertical to horizontal motion in the last seismic isolation stage could have an efficiency $\eta = 10^{-2}$. Alternatively, the cross coupling can occur as vertical motion of the suspension point is converted into horizontal motion of the pendulum. In this case, $f_{pend} = 11$ Hz and the efficiency is $\eta = 10^{-4}$. The noise generated by creep in the seismic isolation is most noticeable at lower frequencies. Table 1 lists the test mass motion that can be expected at 35 Hz, the lowest frequency before the seismic wall in the initial LIGO interferometer, due to release of a burst of energy $E = 10^{-9}$ J from creep in a spring in the top layer of seismic isolation..

Table 1.

Origin of Cross-Coupling	*Efficiency* η	*Resonant Frequency* f_j (Hz)	*Creak Duration* τ (ms)	*Test Mass Motion* Δx_{TM} $\mathrm{m}/(\sqrt{\mathrm{Hz}})$
Seismic Isolation	10^{-2}	0.74	10	2.9×10^{-20}
			5	3.5×10^{-20}
			2	2.9×10^{-20}
Pendulum	10^{-4}	11	10	6.7×10^{-20}
			5	7.5×10^{-20}
			2	6.4×10^{-20}

The initial LIGO noise target[1] at 35 Hz, obtained by extrapolating the anticipated spectrum above 35 Hz to lower frequencies, is 10^{-18} $\mathrm{m}/\sqrt{\mathrm{Hz}}$. Bursts arising from creep in the springs that release more than $E = 1.3\times10^{-8}$ J will cause excess noise in the interferometer.

3 TESTING FOR CREEP

A creep relaxation in the final layer of springs in the seismic isolation corresponding to $E = 1.3\times10^{-8}$ J would cause a displacement of the 650 kg downtube structure by 2×10^{-12} m. At typical frequencies for a burst duration of 5 ms, this corresponds to an acceleration of 8×10^{-9} g's. This is too small to be detected with conventional accelerometers. Alternatively one can test for the force released in the spring that underwent creep[2]. Using equation (7) the same energy release results in a change in force of approximately 5×10^{-7} N at the spring. Using a lead-zirconate-titanate piezoelectric transducer that is 5 cm in diameter and 1 mm thick, a change in force would generate an electrical response[3] of approximately 1 mV/N. A sketch of an apparatus that uses this effect is shown in Figure (2). Operating with an amplifier with $5\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ input-voltage

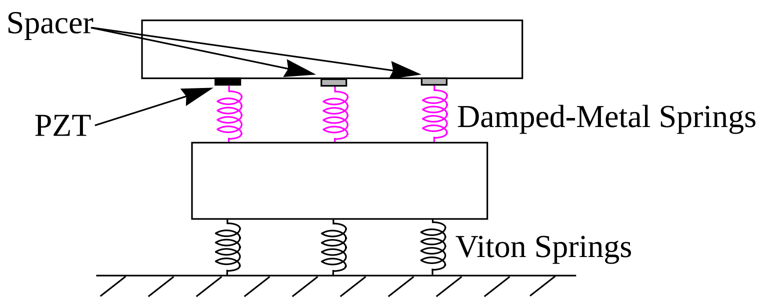

Figure (2) Sketch of apparatus for creep test in damped-metal springs.

noise, a voltage burst of 28 nV, corresponding to a creep relaxation of $E = 7\times10^{-7}$, would have a signal-to-noise ratio of unity. A signal-to-noise ratio of 5.3 would be required to identify a creep rate a factor of ten below the allowed limit for gaussian-noise pulses in a triple-coincidence burst search. Creep events with this signal-to-noise ratio would have a relaxation energy of $E = 3.7\times10^{-6}$ J, approximately three hundred times above the allowable level for the initial LIGO interferometers.

4 CONCLUSIONS

The requirements on creep arising in the final layer of springs in the LIGO seismic isolation has been verified. A test apparatus is described to measure burst created by creep relaxation using piezoelectric elements between the spring and the supported mass. Unfortunately the proposed test would be limited by amplifier noise to detecting events that are approximately 300 times larger than the LIGO requirement.

5 REFERENCES

[1] A. Lazzarini and R. Weiss, *LIGO Science Requirements Document*, LIGO-E950018-02-E.

[2] This method was suggested by D. Shoemaker.

[3] Data sheet from Piezo Kinetics Incorporated, P. O. Box 756, Bellefonte, PA 16823.