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Motion of Optical Platforms Driven by Thermal Noise from Spring Elements

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Abstract

Thermal noise originating in the spring elements will drive motion of the BSC optical platform. This document estimates the thermal noise contribution that arises from the modes of the spring elements when the optical platform is treated as a rigid body. It is found that the thermal noise contributed by the simple-spring mode and by internal modes of the spring elements will not significantly affect the noise of the initial LIGO interferometers.

Keywords: microseism, vibration, seismic, isolation, range

1 OVERVIEW

At frequencies where seismic noise is sufficiently attenuated, thermal noise in the seismic isolation structure makes the largest contribution to the motion of the optical platform. In the initial LIGO interferometers seismic noise is expected to contribute strongly to test-mass motion at frequencies below 40 Hz. Above 40 Hz, it is expected that thermal noise from internal modes of the test mass and from pendulum and violin modes of the suspension should be the major background contribution. Thermal noise from the seismic-isolation components should be kept below these noise sources. In this document, the contribution of thermal noise from the spring modes of the final stage of the seismic isolation is estimated.

2 FORMULAE FOR ESTIMATION OF THERMAL NOISE

2.1. Simple Spring Mode

The equation for thermal noise from a lumped-element system of a mass M and a spring is given[1] as

$$\tilde{x}^2(f) = \frac{4k_BT}{M\omega}\frac{{\omega_0}^2\phi}{(\omega_0^2-\omega^2)^2+({\omega_0}^2\phi)^2} \quad (1)$$

where $\omega = 2\pi f$ and ω_0 is the resonant angular frequency of the system.

2.2. Internal Modes of the Spring

The spring has non-vanishing mass and will have self-resonant frequencies. Ordinarily a coupled-mode analysis is required but, because of the large mismatch in masses between the optical platform and the spring, a reliable estimate can be made by solving for the thermal noise in the self-resonant spring and then using the impedance mismatch between spring and optical platform to estimate the resulting recoil of the platform. The thermal noise in the spring is given by

$$\tilde{x}_s^2(f) = \frac{4k_BT}{m\omega}\frac{{\omega_s}^2\phi}{(\omega_s^2-\omega^2)^2+({\omega_s}^2\phi)^2} \quad (2)$$

where ω_s is the self-resonant frequency of the spring and m is the effective mass of the internal mode. The recoil produced in the optical platform is smaller in amplitude by a factor of m/M due to conservation of momentum. For the general case of the i^{th} spring resonance the contribution to the power spectral density for the optical platform is then

$$\tilde{x}_i^2(f) = \frac{4k_BT}{M\omega}\left(\frac{m_i}{M}\right)\frac{\omega_i^2\phi}{(\omega_i^2-\omega^2)^2+(\omega_i^2\phi)^2} \tag{3}$$

where ω_i is the angular resonant frequency of the i^{th} spring mode and m_i is its effective mass.

3 RESULTS

The contribution to motion of the optical platform due to thermal noise from the simple spring mode and one internal mode of the spring was estimated and compared to requirements for the initial LIGO interferometers. Since the final choice of spring materials is not yet known, an educated guess was made for the spring parameters. The parameters used are summarized in Table 1..

Table 1. Parameter Values used in Thermal Noise Estimate

Parameter	*Value*
M	1690 kg
ω_0	$2\pi \cdot 3\mathrm{Hz}$
ϕ_0	1/50
m_1	0.14 kg
ω_1	$2\pi \cdot 401.27\mathrm{Hz}$
ϕ_1	1/400

The resulting motion of the optical platform is given in Figure (1). The thermal noise is dominated by the simple spring mode at most frequencies, except in the neighborhood of the internal mode resonance near 400 Hz.

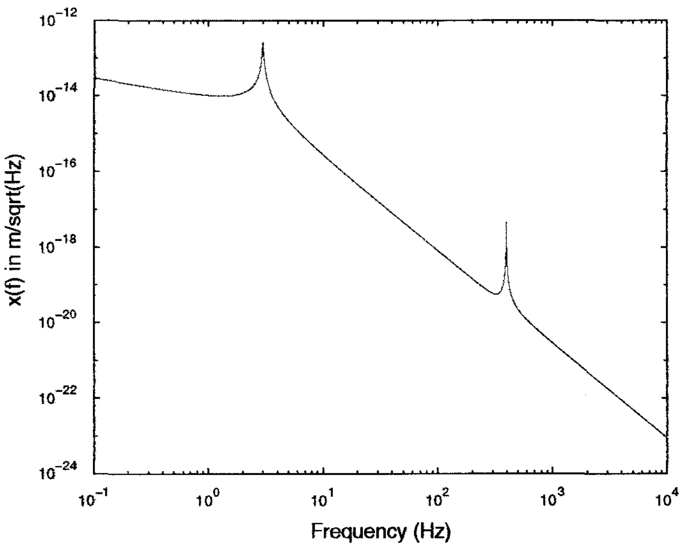

Figure (1) Motion of the optical platform due to thermal noise from modes of the spring elements

The optical platform motion is further filtered by the test mass suspension. The resultant motion of the test masses is compared to the LIGO Science Requirement[2] in Figure (2).

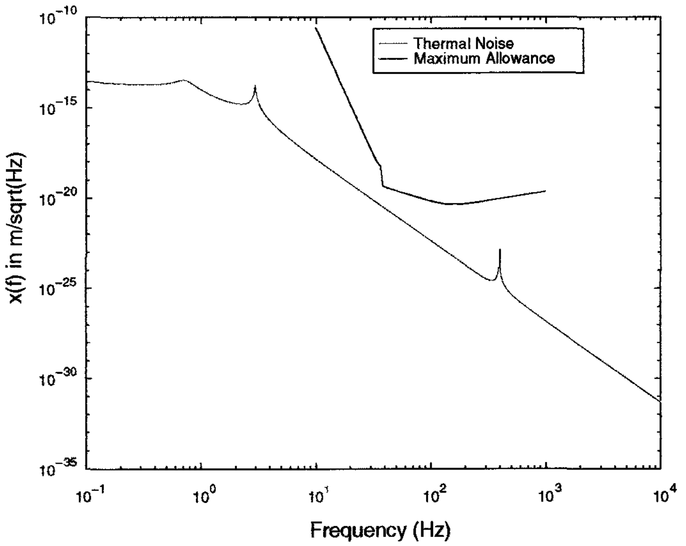

Figure (2) Thermal-noise contribution to test-mass motion from spring element modes (cyan) compared with the LIGO Science Requirement (red).

4 CONCLUSIONS

It appears from Figure (2) that the thermal noise in the optical platform due to modes of the spring elements is not problematical for the initial LIGO interferometers, provided the parameter values used are similar to those in Table 1. Note, however, that the violin resonances of the suspended test masses were not included in the transfer function of the pendulum used to obtain Figure (2). Violin resonances occur at frequencies of approximately 380 Hz and its harmonics. Clearly internal-mode resonances of the springs could be troublesome if they coincided with the violin-mode frequencies. The presence of violin modes also distorts the transfer function above the fundamental, so the gap between performance and requirement may be smaller at the higher frequencies than depicted in Figure (2). Another effect which was not taken into account is the amplification of the spring thermal noise due to internal resonances of the optical platform. These effects will be estimated by subsequent modeling.

5 REFERENCES

[1] P. R. Saulson, *Phys. Rev. D*, **42**, 2437, (1990).

[2] A. Lazzarini and R. Weiss, *LIGO Science Requirements Document*, LIGO-E950018-02-E, (1995).