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Resonant Frequencies and Quality Factors of LIGO "4-inch" Fused Silica Test Masses

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1 Introduction

This is a summary of studies of vibration modes of the LIGO 4-inch test masses, done in [1]. Similar measurements were also done in [3] with a different test mass.

We calculated the frequencies of the vibration modes using a numerical model. These frequencies were also found experimentally. For the modes predicted by the calculations we obtained a good agreement with the experiment. We also observed a mode not predicted by the calculations. This was a doubly degenerate mode at 48.9 kHz. The splitting was well within the resolution of the experiment. We also measured the quality factors of all the modes.

The frequencies of the test mass vibration modes, $(\omega = 2\pi f)$, were found by applying a force to the mass and sweeping its frequency. After the frequency was found we excited the mode and observed its decay. By measuring the mode decay time, τ, we can obtain the quality factor, Q. However, in the experiment we measured not the decay time, but the half-amplitude folding time t. The quality factor was found from the formula

$$Q = \frac{\omega\tau}{2} = \frac{\pi}{\ln 2}\, f\ t.$$

2 Test Masses Properties

The test masses are super-polished high grade fused silica cylinders. All test mass substrates are from the material 7940 (Silica Tetra-Chloride) grade OAA with homogeneity of less than 5×10^{-7}.

The dimensions of the test masses are shown in Table 1. There is a small wedge angle, θ, between the two flat sides of the cylinder. Therefore, the cylinder has a thin and a thick side. In the following table heights of the thick and thin side are denoted by H and h respectively. The diameter of the test mass is d. The wedge angles were not measured but calculated from $\theta \approx (H - h)/d$. The table shows the test mass dimensions in mm and the wedge angles in degrees. The accuracy for all length measurements is ± 0.05 mm. Therefore, the wedge angles are found with accuracy of ± 0.01 degrees.

Table 1: Test Mass Dimensions

dimension	COAA - 01 - 01 - 4.0	COAA - 01 - 04 - 4.0	COAA - 01 - 05 - 4.0
d	100.98	100.97	100.94
H	87.33	87.44	87.11
h	86.44	86.56	86.29
θ	0.51	0.50	0.47

3 Calculation of the Frequencies

We calculated the frequencies of the test mass vibration modes numerically. Table 2 shows the eight lowest frequencies. Since the test masses have very close dimensions we used averaged dimensions in the calculations. The model does not account for the wedge. Note, that the modes # 5 and # 7 were not found in the experiment. However, the experiment revealed a mode which was not predicted by the calculations.

Table 2: Frequencies Obtained from Simulation

# mode	frequency in kHz
1	30.36077
2	30.77705
3	34.85266
4	39.32173
5	43.37877
6	44.13600
7	56.94476
8	58.24740

4 Experimental Setup

The following description of the experimental setup is due to J. Carri [2]. The measurements were done using Michelson interferometer with intensity locking scheme. The test mass was suspended in vacuum and formed one arm of the Michelson interferometer. A second mirror rigidly mounted on the table served as the reference arm of the Michelson interferometer.

The high-voltage amplifier drove the board, which applied electric force to the entire area of the test mass. The amplifier obtained its input from a Phillips high-precision function generator set to the exact frequency of the test mass mode. The fringes of Michelson interferometer were observed on the photodiode. The photodiode output current was fed into a dc-coupled preamplifier, SR-560, with a low-pass cutoff frequency of 100 kHz. The output of the preamplifier was split into two parts. The first part went to the pole-zero shaping network, and then to the locking servo amplifier, which drove the shaker on the back of the reference mirror. The interferometer was locked halfway up a fringe. The second part of the photodiode output went to a Stanford Research lock-in amplifier. A function generator supplied the reference frequency to the lock-in amplifier. The reference frequency was offset from frequency of the test mass mode by 1-2 Hz to produce beats. The beats were observed on the chart recorder.

5 Measurement Results

In this section we present the experimental results. The results are shown in the following three tables, which correspond to different test masses. The caption for each table contains the substrate number for the test mass. The frequency range for the first test mass was not choisen carefully. This is why the first table contains only the modes within 30 - 44 kHz frequency range. The range of frequencies was extended to 60 kHz for the other two test masses. This was the limit of the spectrum analyzer used in the experiment. The asterisk (*) indicates the doublet which corresponds to the degenerate

Table 3: COAA - 01 - 05 - 4.0

f (kHz)	t (sec)	$Q(10^6)$
30.21483	8.4	1.1
30.79963	28.8	4.0
34.81730	2.0	0.31
39.01158	4.8	0.85
44.10573	8.0	1.6

Table 4: COAA - 01 - 01 - 4.0

f (kHz)	t (sec)	$Q(10^6)$
30.19615	8.8	1.2
30.72658	18.0	2.5
34.70159	5.2	0.82
38.95401	8.0	1.4
44.00773	9.6	1.9
(*) 48.97662	12.0	2.7
(*) 48.97821	12.0	2.7
58.05533	–	0.16

mode. The components of the dublet are only few hertz apart. The physical mechanism for lifting the degeneracy is not known. It is possible that the degeneracy was removed by the wedge on the mass. The other possibility is due the attachments on the mass at places where the suspension wire leaves the mass. The doublet is likely correspond to a non-axisymmetric mode of vibration of a cylinder.

Table 5: COAA - 01 - 04 - 4.0

f (kHz)	t (sec)	$Q(10^6)$
30.20406	11.2	1.5
30.71421	29.8	4.1
34.68894	6.8	1.1
38.94922	3.2	0.56
43.99494	20.0	4.0
(*) 48.95491	5.2	1.1
(*) 48.95799	4.2	0.93
58.04098	1.3	0.34

6 Conclusion

The frequencies of the test mass vibration modes agree with the prediction. The quality factors for those modes observed in the experiment fall into the expected range of $10^5 - 10^6$. The doublet presents an interesting case of a mode with lifted degeneracy. It requires further studies.

References

[1] J. Carri, J. Logan and M. Rahman, LIGO research, August 8, 1996.

[2] J. Carri, private communication

[3] S. Kawamura, et al, Specifications of the 40m Test Mass Suspension Prototype, LIGO Technical Document T-960162-D, October 18, 1996