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Thermal-Noise Requirements for HAM Seismic Isolation

F. Raab

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California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

Abstract

Considerations for the required thermal noise performance of the HAM seismic isolation are given here. The requirement is derived based upon an analysis of the frequency noise on the light due to the combined influences of the motion of the mode-cleaner mirrors and the effect of feedback derived from the main interferometer. Given reasonable assumptions for operating parameters in the interferometer, it is found that the Science Requirements and the expected noise from suspended mode cleaner mirrors can be used to derive a bound on the frequency noise on the light in the absence of feedback. This allows a limit to be set for thermal noise in the final stage of the HAM seismic isolation and constrains the rsonant frequencies, Qs and effective masses for internal modes of the optical platform.

Keywords: HAM, seismic isolation, thermal noise, mode cleaner, feedback

1 INTRODUCTION

The thermal-noise requirements for HAM seismic isolation are set to ensure that the resulting frequency noise on the light, due to thermal-noise displacements of the mode-cleaner mirrors, does not significantly degrade performance of the initial LIGO interferometers. The connection between mode-cleaner-mirror displacements, the servo feedback which relates laser frequency to common-mode lengths in the main interferometer are explored in Sections 2 and 3. The thermal noise requirements placed upon the final stage of the HAM seismic isolation is then derived in Section 4.

2 MODE-CLEANER LENGTH STABILITY REQUIREMENT

2.1. Length Fluctuations in the Mode Cleaner and Frequency Noise

In the absence of feedback from the main interferometer, changes in mode-cleaner length Δl produce frequency fluctuations $\Delta\nu$ on the light transmitted by the mode cleaner. These are related by

$$\frac{\Delta\nu}{\nu} = \frac{\Delta l}{l} \tag{1}$$

where ν is the average carrier frequency for the light and l is the average mode-cleaner length. Generally, feedback from the main interferometer is used to suppress frequency fluctuations. We will assume that the quantities in equation (1) represent the frequency and length fluctuations in the absence of this feedback. In the presence of feedback, the frequency fluctuations are reduced to the level $\delta\nu$ given by

$$\frac{\delta\nu}{\nu} = \frac{\Delta\nu}{\nu} \cdot \left[\frac{1}{1 + A_{CM}}\right] \tag{2}$$

where A_{CM} is (to within a sign) the open-loop gain of the common-mode servo that provides this feedback. For definiteness the basic representation of this feedback loop is given in Figure (1).

The frequency noise obtainable in the presence of mode-cleaner length fluctuations and common-mode feedback is then

$$\frac{\delta\nu}{\nu} = \frac{\Delta l}{l}\cdot\left[\frac{1}{1+A_{CM}}\right]. \tag{3}$$

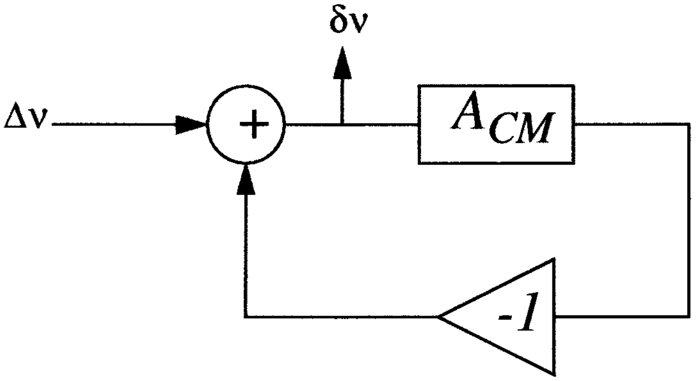

Figure (1) Schematic of feedback path from arm-common-mode servo.

2.2. Science Requirements on the Allowable Frequency Noise

The LIGO Science Requirement Document[1] (SRD) gives the overall strain-noise target h_{SRD} for a LIGO interferometer. This target is related to the allowable frequency noise on the light by the bound

$$\frac{\delta\nu}{\nu} \le \frac{1}{10}\cdot\beta\cdot h_{SRD} \tag{4}$$

where β is the common-mode-rejection ratio (CMRR) of the interferometer for frequency noise defined by the relation

$$\beta\frac{\partial h}{\partial\nu}\cdot\delta\nu = \frac{\delta\nu}{\nu} \tag{5}$$

and the factor of 1/10 is the prescribed safety factor.

2.3. Requirement on Mode-Cleaner-Length Fluctuations

Combining equations (3) and (4), we obtain

$$\frac{\delta l}{l} \le \frac{1}{10} \cdot \beta \cdot h_{SRD} \cdot [1 + A_{CM}] \qquad (6)$$

for the requirement on fluctuations in the mode-cleaner length.

2.4. Assumed Open-Loop Gain for Common-Mode Servo

For purposes of deriving a limit for allowable fluctuations in mode-cleaner length for the SEI DRD[2], a gain profile for the CM servo was needed. Since the design of this servo was not yet completed in April 1996, a guess was made for the general properties of the gain profile[3]. It was assumed that the unity-gain frequency for this servo would be above 1 kHz and that the open-loop gain available at 100 Hz would be 60 dB.[1]

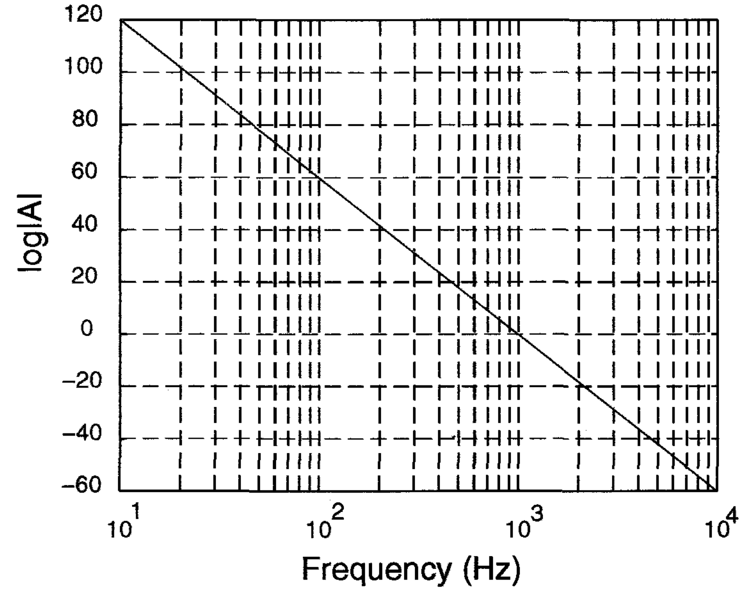

Figure (2) Gain profile assumed for frequency feedback from main interferometer.

For the purpose of estimation, the assumed gain profile was fixed at 60 dB at 100 Hz with an f^{-3} rolloff, as shown in Figure (2). This was considered a worst-case approximation to the gain profile that would result from the design process. Loop-stability considerations would require that the unity gain profile would be pushed to higher frequencies and/or the gain at 100 Hz would be increased to provide proper phase margin. Assuming that the quantity Δl was fixed according to

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[1] The gain at 100 Hz was chosen to suppress the expected mode-cleaner frequency fluctuations of $10^{-4}\,\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ resulting from thermal noise in the suspended mode-cleaner mirrors.

the gain profile of Figure (2), the actual gain profile adopted should give sufficient margin.

3 COMPARISON OF REQUIREMENTS FROM SRD AND DSR

The Detector Subsystems Requirement[4] (DSR) gives a requirement on the allowable frequency fluctuations in the absence of frequency feedback from the main interferometer. The requirement was based on accommodating the thermal noise associated with the mode-cleaner mirrors. In Figure (3) we compare the SRD and DSR requirements.

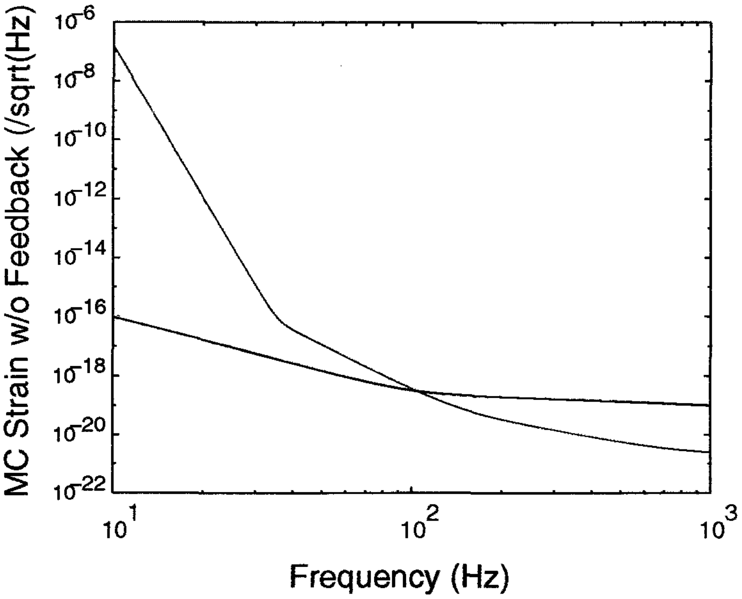

Figure (3) Limits on allowable fractional fluctuations in mode-cleaner length.

The frequency noise requirement is set equal to the upper curve in each frequency regime. This gives a more relaxed gain requirement at low frequencies where reaching the thermal-noise limit for the mode-cleaner mirrors is not required. At higher frequencies, where the gain profile of Figure (2) was unrealistically steep, this allows for a more gently sloped gain curve with better phase margin.

4 THERMAL NOISE REQUIREMENT ON OPTICS PLATFORM

4.1. Allowance for Thermal Noise from HAM Seismic Isolation

The strain requirements shown in Figure (3) are used to derive limits on the allowed motion of the HAM optics platform at the positions where the suspensions for the mode-cleaner mirrors are mounted. The mode-cleaner strain curve is multiplied by 12.1 m to convert strain to mirror displacement. This mirror displacement is then derated by a factor of 2 to allot only 25% of the mode-cleaner thermal noise to the seismic isolation and is divided by $\sqrt{2}$ to partition the requirement between the two HAM stacks that contribute. Dividing this corrected mirror displacement by the horizontal-to-horizontal transfer function of the suspension gives the required thermal noise allowance for the final stage of the HAM seismic isolation, which is shown in Figure (4).

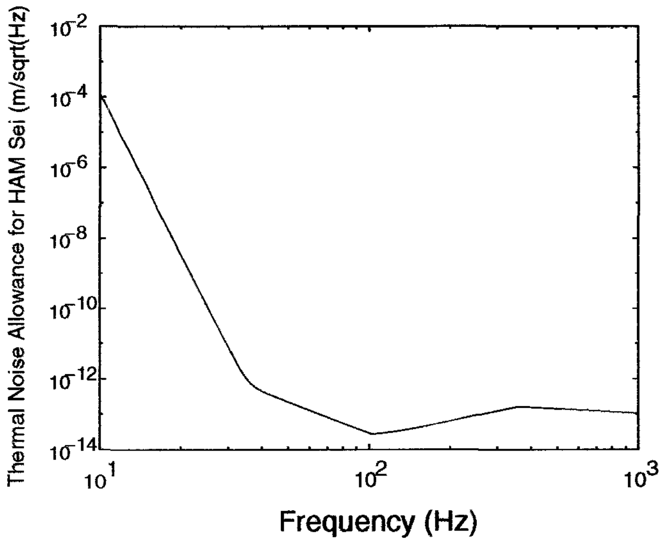

Figure (4) Allowed thermal noise from upper stage of HAM seismic isolation.

Contributions arising both from the springs and from internal resonances must remain below this level. It should be noted that the pendulum transfer function used to derive Figure (4) for frequencies above 350 Hz was set equal to a constant value (-107dB), rather than using a detailed transfer function including the violin resonances of the suspension wires. This approach is conservative, provided that care is taken to prevent accidental overlaps between resonances in the final stage of the seismic isolation and the violin resonances.

4.2. Limits on Frequencies and Qs of HAM Optics Platforms

The natural vibrations of the optical platform in thermal equilibrium with its surroundings (thermal noise) can excite the suspension near these resonances. The thermal component of optical-platform motion near these resonances is independent of the level of isolation of ground motion afforded by the seismic-isolation system and is given by

$$\tilde{x}_{platform}(f) = \left(\frac{4k_BTQ}{m\omega_r^{\ 3}}\right)^{1/2} \qquad (7)$$

where k_B is Boltzmann's constant, ω_r is the resonant angular frequency of the mode, Q is its quality factor and m is its effective mass. This quantity $\tilde{x}_{platform}(f)$ may be compared directly to the thermal noise allowance given in Figure (4). The table below gives representative values for the thermal noise assuming m=200 kg for the effective mass of an optical table mode at some particular frequency and Q.

In practice the effective mass must be found for each of the mode shapes and frequencies of the

Table 1: Evaluation of Equation (7) for *m* = 200 kg.

Frequency (Hz)	Q	$\frac{x(f)}{10^{-14}\mathrm{m}/\sqrt{\mathrm{Hz}}}$	*Acceptable?*
300	4000	0.7	yes
200	4000	1.3	yes
100	4000	3.6	no
100	1000	1.8	yes

optical table. Assuming that a finite-element method has solved for the motion $u(X, Y, Z)$ at each mass element $M(X, Y, Z)$, and that the transverse component of u averaged over the mounting area for the suspended optic is $x(X_0, Y_0, Z_0)$, the relation

$$\sum_{X, Y, Z} M(X, Y, Z) \cdot u^2(X, Y, Z) = m \cdot x^2(X_0, Y_0, Z_0) \tag{8}$$

may be used to solve for the effective mass.

5 CONCLUSIONS

Table 1 suggests that the thermal noise requirements for the HAM optical platforms can be met provided the gravest modes have effective masses of 200 kg or more, with Qs of less than a few thousand and frequencies above about 150 Hz. It should be remembered that this analysis is somewhat idealized, in the sense that the locations of the suspended mirrors on the optical tables cannot be specified for the various interferometer configurations that could be developed in future. Also, the analysis has not considered the response of the system to vertical or tilting motions of the optical platform where the optic is suspended. It would therefore be desirable to meet the requirements on the internal resonances with a considerable safety factor, if practical.

6 REFERENCES

[1] A. Lazzarini and R. Weiss, *Science Requirements Document,* LIGO-E950018-00.

[2] F. Raab and N. Solomonson, *Seismic Isolation Design Requirements Document,* LIGO-T960065-02-D.

[3] Based upon conversations with S. Kawamura and J. Camp.

[4] D. Shoemaker, *Detector Subsystems Requirements*, LIGO-E960112-05-D.