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- LIGO -
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INITIAL LENGTH PRECISION OF LIGO SUSPENDED CAVITIES

JBC

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1 INTRODUCTION

The LIGO detector will employ a variety of suspended cavities whose initial DC lengths require specification within given tolerances. The cavity lengths are

1. mode cleaner
2. recycling cavity in-line length
3. recycling cavity perpendicular length
4. arm cavity

This note assumes specific lengths have been chosen for these cavities (TBD) and gives the precision to which they must be set.

2 MODE CLEANER

The mode cleaner length must be set to pass the RF sidebands: an RF frequency of 25 MHz (TBD, MEZ) gives a nominal length of 12 m. The RF frequency must be tuned to within ~100 Hz of the mode cleaner exact resonance to avoid RFAM from oscillator phase noise.[1] The RF oscillator has been speced to provide ~100 ppm tuning range, or 2500 Hz. This implies that the mode cleaner length should be set initially so that its 2nd resonance, f_2 (at twice the free spectral range), is within ~2000 Hz of the RF frequency:

$$\Delta L_{MC} = L_{MC} \cdot \frac{\Delta fsr}{fsr} = L \frac{\Delta f_2}{2fsr}$$

or $\Delta L_{MC} \sim 1$ mm

3 RECYCLING CAVITY

3.1. Average length

By "average length" we refer to one-half the sum of l_I and l_P, the recycling cavity in-line and perpendicular length. Once the RF frequency has been fixed so that the sidebands pass the mode cleaner, the recycling cavity average length must be set so that a lock of the carrier also causes the sidebands to resonate. We can regard an imprecision in length setting relative to the RF frequency as a detune of the RF frequency from the exact resonant value Ω_{res}. It has the following effects:

1. Oscillator phase noise coupling to detuning of the sidebands from recycling cavity resonance: This effect has been analyzed[2] and gives the requirement: $d\Omega / \Omega_{res} < 10^{-2}$
2. Loss of power buildup of sidebands in the recycling cavity: the allowed deviation from recyclling cavity resonance is $l_+ = 1 \times 10^{-10}$ m; this requirement was derived to allow a sufficient

1. Mode Cleaner Noise Sources, LIGO-T960164-00-D, sec. 4
2. Frequency, Intensity and Oscillator Noise in the LIGO, LIGO-T960019-00-D, app. 3

power buildup of both carrier and sidebands,[1] and imposes a round trip maximum sideband phase change of ~ 10^{-3} rad. A detune of the sidebands alone allows a higher phase offset of ~ 2 x 10^{-3} rad. The resonance condition $2 \Omega l_{rc} / c = \pi$ then gives:

$$\Delta l_{RC} / l_{RC} = 6 \times 10^{-4}$$

where Δl_{RC} is the length detuning from exact resonance. With l_{RC} ~ 10 m we have Δl_{RC} ~ 6 mm.

3.2. Asymmetry

The recycling cavity asymmetry δ_{RC} (= l_I - l_P) defines the sideband transmission to the dark port. With a nominal choice of δ_{RC} = 30 cm, a limit of $\Delta\delta_{RC}$ ~ 3 cm will be sufficient.

3.3. Individual lengths

We can meet the requirements for both Δl_{RC} and $\Delta\delta_{RC}$ by specifying a length tolerance of:

$\Delta l_I = \Delta l_P$ = 3 mm.

4 ARM CAVITY

The first requirement is that we wish to allow the possibility of an arm storage time match of < 1 : 10^3, so that $\Delta L_{arm} < 10^{-3} L_{arm}$, or ΔL_{arm} ~ 1 m.

The second requirement is to set the arm cavity length so that the 1st order sidebands are close to, but not at, exact antiresonance (so the 2nd order sidebands do not resonate in both the arms and recycling cavity.) Sideband antiresonance occurs at length intervals $\Delta L = 2 \pi c / 2 \Omega$ ~ 6 m; the length shift from this position must be at least $\lambda_{carrier} / 2$. Displacing the arm length from antiresonance will cause a phase shift in the reflected sidebands $\Delta\phi$; this phase shift can be accomodated by a corresponding change in the recycling cavity average length of $\Delta l_{RC} = l_{RC} \Delta\phi / \pi$. It is also important that the arm cavity length is kept from getting too close to sideband resonance, where the phase shift starts becoming large, as is seen in Table I.

Table 1: Recycling cavity length change to accomodate arm cavity length

Arm cavity displacement from sideband exact antiresonance (m)	$\Delta\phi$ (rad)	Δl_{RC} (cm)
1	0.01	3
2	0.02	7
2.5	0.06	15
2.9	0.3	75

1. Length Control RMS Deviations from Resonance, LIGO-T960067-00-D, sec 2.1

We assume here that the arm cavity length is less than 2 m away from sideband antiresonance so that real estate in the recycling mirror chamber is not taxed unduly. Then the above derived requirement on the precision of the recycling cavity average length (6 mm) implies an arm cavity length precision of ~ 20 cm. We use this number as the requirement.

5 SUMMARY

The following table lists the nominal cavity lengths and their specified precisions.

Table 2: Cavity Lengths and Precisions

Cavity	Nominal Length	Precision
Mode Cleaner	12 m (MEZ)	1 mm
Recycling cavity l_I	9.4 m (MEZ) + Δl_{RC}[a] + 0.15 m (JBC)	6 mm
Recycling cavity l_P	9.4 m (MEZ) + Δl_{RC} - 0.15 m (JBC)	6 mm
Arm Cavity	4000 + TBD (MEZ, JW, B^2)	20 cm[b]

a. Δl_{RC} is specified once L_{arm}, l_{RC} and Ω are known.

b. Assumes arm cavity length is no more than 2 m from sideband antiresonance