CALIFORNIA INSTITUTE OF TECHNOLOGY

Laser Interferometer Gravitational Wave Observatory (LIGO) Project

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Subject: Determination of the as-built LIGO Global Coordinate Axes for Hanford, WA: final analysis of the LIGO BT/VE interface survey monuments.

Notes: Revision B corrected a typographical error in paragraph A.2, page 3. Revision C corrects a calculational error involving the distortion in the Lambert Conformal Projection (easting & Northing grid coordinates) of a sphere. The effect changes slightly (by <1cm, but not negligible) axial displacements of benchmarks along the arms. Angular orientations are negligibly affected. The description of the correpsondence between x, y and Easting and Northing was also corrected.

Background

LIGO at Hanford has been surveyed by Rogers Surveying, Inc. (RSI) and IMTEC. Rogers performed a three-dimensional survey to identify the BT-VE interface points. These are labeled BTVEi, i={1 - 8}. BTVE1-4 are on the Y arm; BTVE5-8 are on the X arm. Ancillary (primary) benchmarks have also been laid out at the arm quarter points. Once the LVEA foundation is complete and available, the vertex will also be surveyed, and the constellation of NINE *Cardinal Points* {VERTEX, BTVE1,...BTVE8} will constitute the LIGO primary monuments for future alignment of BT, VE, and Detector subsystems.

RSI used a hybrid technique using GPS over long baselines and optical methods over short distances. They claimed sub-centimeter accuracy for horizontal, {x,y} data, but were not willing to certify accuracy in the vertical direction {z} due to their understanding of the limitations of GPS survey methods they employed. We are having Prof. Tom Herring (MIT Earth and Planetary Sciences) reanalyze the vertical GPS data to extract the ultimate accuracy from the RSI data. This analysis is still being performed.

IMTEC, which was familiar to the LIGO Facilities Group from SSC experience, was subsequently asked to survey the elevations of the as-built RSI monuments to an accuracy of +/- 0.005m. They performed this task using optical transit techniques. Their data analysis indicates accuracies of the order of a few millimeters were achieved. This, however, was after corrections (from look-up tables) were included for atmospheric effects on light propagation.

The RSI and IMTEC measurements of elevation compare favorably. This is shown in Figure 1.

The RMS deviation between the data sets is 4.6mm.

It has been the plan to augment the Cardinal Points with secondary benchmarks as required by both VE and Detector. PSI has recently requested such monuments to define the beam line axes within the LVEA and the mid- and end-station VEAs. The principal concern with regard to VE alignment is to ensure proper *angular* alignment of the VE to the beam tube centerlines. In this regard, Mike Zucker has raised the concern that +/- 5mm over short runs such as within the LVEA and VEAs correspond to potentially large angular uncertainties. For example, the distance between the (future) VERTEX and BTVE1 (Y arm) or BTVE5 (X arm) is 46m; thus a 1cm uncertainty in transverse coordinates would correspond to and angular uncertainty of 217 x 10^{-6} radians. The mid- and end-stations are comparable because the respective baselines between benchmarks are about 20m. At 4 km this (error with a 20m baseline) corresponds to a potential pointing error of 2m, which exceeds the BT clear aperture. It is required to maintain this pointing error below the diameter of a testmass (0.25m) to enable dead-reckoning alignment of the detector (and VE) components so that at first opportunity to transmit a beam between buildings, there is certainty of finding the target testmasses in the mid- and end-stations.

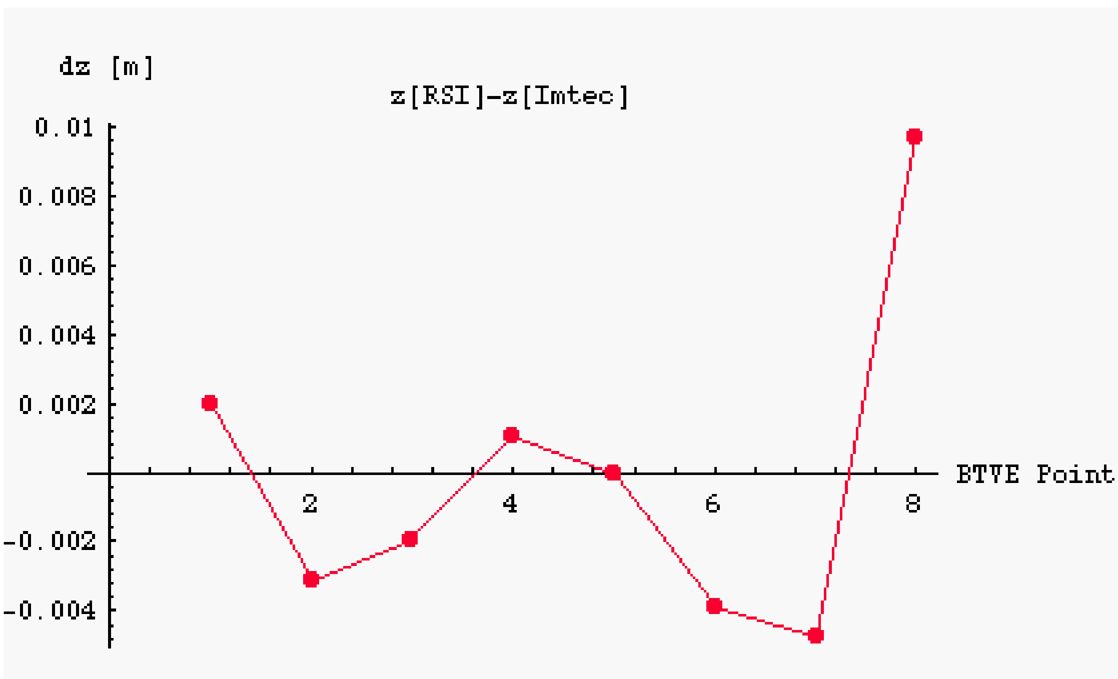

Figure 1 Elevation measurement differences between RSI and IMTEC

In order to achieve the needed pointing accuracies, it will be necessary to reduce the transverse uncertainty of the pointing references (benchmarks). One obvious way to do this is to use very long baselines (say, use VERTEX - quarter point or BTVE2 or BTVE6) to align within the LVEA and corresponding baselines for the other buildings. However, Allen Sibley has determined in discussions with RSI that the maximum optical range over which benchmarks may be used is approximately 200m. There do not now exist any markers at this distance near any of the station buildings. Resurveying to place these benchmarks is deemed prohibitively costly in both time and money. The discussion below outlines an approach based on optimally using *all* existing (and future VERTEX) BTVE benchmarks to define the *least squares* intersecting lines corresponding to the as-surveyed ensemble of markers, and then to derive offsets for each of the benchmarks such that alignment to the offset points would provide the additional pointing accuracy accrued by using 8 or nine reference points together.

Analysis and Results

A.The survey data of IMTEC and RSI were melded together as follows.

1. RSI data for the horizontal measurements were used: {x,y}. These appear on the map provided with the RSI Survey Report, LIGO C961787-O, "GPS Survey" File: 25195.DWG. The survey data appear in table "Coordinate Table-NAD83/91 Grid Coordinates (U.S. Survey Feet)" on that map. Columns "Northing" and "Easting" were used. Note that in order to obtain physical meters from the NAD83/91 "grid coordinates", it is necessary to divide grid displacements by the "Combined Factor" included in the RSI table. This accounts for the transformation from Lambert Conical Projection to the spherical geometry of the earth. This differential distortion (dilation) at the LIGO site for the NAD83/91 grid amounts to 0.125m per km, which is not negligible.
2. IMTEC data were used for the elevation measurements: {z}. IMTEC elevations are reported in Table 6, p. A-4, of the IMTEC Survey Report, LIFO C962208-00-O (a revised report has been issued correcting certain data reduction errors uncovered by LIGO; however I used the field "raw" data for this analysis). The arithmetic mean of RSI and IMTEC elevations were also tried. The RMS errors to planar fits were better for the IMTEC data alone: the RMS was 0.0019m for IMTEC alone (7 data points) and.0031m for the combined sets. The results presented below are for IMTEC elevation data only.
3. The survey was performed referring all GPS and optical measurements to BTVE5 as the ground truth. Thus there are actually 7 points (21 DOFs in 3D) in the fit: the Y arm is constrained to go through BTVE5.

B.The combined data set {x,y,z} was then used in a nonlinear regression analysis to two lines constrained to intersect at a common point (Mathematica Notebook /home/lazz/Models/Slab_Survey/Slab_Model_v5.ma). ***This analysis differs from the previous ones performed for Larry Jones (BT/CB&I) because the actual {x,y} coordinates for the benchmarks are included here***. Previously, it was assumed that the {x,y} were given by the $\{x_i,0\}$ for the X and $\{0,y_i\}$ for the Y arm, where the x_i and y_i are given by IMTEC's ranging measurements to the RSI benchmarks. The result of the fit to the Yarm beam centerline ***in NAD83/91 coordinates*** ($\hat{x}$ = –Easting; $\hat{y}$ = –Northing; $\hat{z}$ = Elevation) is:

$$n_{Yarm}' = \{-0.800806 \pm 1\times10^{-7}, -0.598923 \pm 1.4\times10^{-7}, -3.0\times10^{-6} \pm 0.\} \quad [1]$$

The resulting fit to the Xarm beam centerline is:

$$n_{Xarm}' = \{-0.598923 \pm 1.6\times10^{-7}, 0.800807 \pm 1.2\times10^{-7}, -6.284\times10^{-4}\pm0.\} \quad [2]$$

Errors in components are 1σ bounds.

C.In this coordinate system, the normal to the plane is:

$$n_z' = \{-0.000379, 0.000501, 0.99999989\}. \quad [3]$$

In a second step the rotation required to bring the beam centerlines into the LIGO Global Coordinate System was determined. The transformation which takes $\{n_x', n_y', n_z'\}$ into axes aligned along the plane normal (*not the local vertical*) and the $\{X_{LIGO}, Y_{LIGO}\}$ is given as follows with $\phi = 126.79° \text{ [CW]}$:

$$R_\phi = \begin{bmatrix} \mathrm{Cos}\phi & \mathrm{Sin}\phi & 0 \\ -\mathrm{Sin}\phi & \mathrm{Cos}\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [4]$$

The corresponding unit vectors are:

$$n_{Xarm} = \{0.9999998, 0.0, -0.000628\}$$
$$n_{Yarm} = \{-1.23 \text{ x } 10^{-6}, 0.99999999, -3.0 \text{ x } 10^{-6}\}$$
$$n_z = \{0.0006284, 3.0 \text{ x } 10^{-6}, 0.9999998\} \quad [5]$$

These are the <u>*global coordinate axes*</u> for LIGO, Hanford Site. <u>***All vectors are referred to local vertical at the vertex: these global coordinate system vectors must be parallely transported from building to building***</u>.

<u>***Note that***</u> n_{Xarm} <u>***and***</u> n_{Yarm} <u>***are orthogonal to within 2 microradians:***</u>

$$n_{Xarm} \cdot n_{Yarm} = -1.22\times10^{-6} \quad [6]$$

Along n_x and n_y, the distances of the BTVE markers from the common vertex are as follows (in meters):

X arm: {BTVE5,BTVE6,BTVE7,BTVE8} = {45.999, 2007.498, 2026.999, 3988.494}
Y arm: {BTVE1,BTVE2,BTVE3,BTVE4}={45.999, 2007.494, 2026.996, 3988.493} [7]

D. With these results it is also possible to determine the residuals between the surveyed points, BTVEi, i={1,...,8}, and the best fit beam centerlines. The offset is defined as $\vec{\delta} = \vec{X}_{survey} - \vec{X}_{fit}$.

These deviations $\{\delta_i\}$, are (in *<u>milli</u>*meters, uncertainties are 3σ):

$$\begin{bmatrix} \delta_X^1 & \delta_Y^1 & \delta_Z^1 \\ \delta_X^2 & \delta_Y^2 & \delta_Z^2 \\ \delta_X^3 & \delta_Y^3 & \delta_Z^3 \\ \delta_X^4 & \delta_Y^4 & \delta_Z^4 \end{bmatrix} = \begin{bmatrix} 0 \pm 0.007 & 0 \pm 0.05 & 0.000015 \pm 0.001 \\ -1.21 \pm 0.29 & 0 \pm 1.96 & -0.87 \pm 0.001 \\ -0.74 \pm 0.29 & 0 \pm 1.98 & -0.98 \pm 0.001 \\ 0.91 \pm 0.57 & 0 \pm 3.9 & 0.93 \pm 0.001 \end{bmatrix}$$

$$\begin{bmatrix} \delta_X^5 & \delta_Y^5 & \delta_Z^5 \\ \delta_X^6 & \delta_Y^6 & \delta_Z^6 \\ \delta_X^7 & \delta_Y^7 & \delta_Z^7 \\ \delta_X^8 & \delta_Y^8 & \delta_Z^8 \end{bmatrix} = \begin{bmatrix} 0 \pm 0. & -0.064 \pm 0.03 & -1.23 \pm 0.001 \\ 0 \pm 0. & 0.211 \pm 1.18 & 2.7 \pm 0.001 \\ 0 \pm 0. & -0.054 \pm 1.19 & 2.8 \pm 0.001 \\ 0 \pm 0. & 0.03 \pm 2.35 & -2.5 \pm 0.002 \end{bmatrix} \quad [8]$$

E.This analysis has served to [i] verify the VERTEX location relative to the as-installed markers by requiring the two beam centerlines intersect at a common point and that they lie in the best-fit plane; [ii] confirmed the longitudinal locations of the as-installed benchmarks; [iii] determined the deviations from the best fit axes in the global coordinate system. The $\{\delta_i\}$ can be used to provide a more accurate alignment guide for the PSI and Detector. For example, the offsets for BTVE1 (Y arm) and BTVE5 (X arm) can be used to sight from the VERTEX along the best-fit line to the end station, etc. In essence, the offsets can provide the improved accuracy derived from a number of measurements at varying distances for each benchmark. <u>It should be noted that in spite of the very small errors quoted above, some of the associated pointing errors can still be as great as 100 microradians.</u>

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