LIGO-T960131-00-D

FAX COVER PAGE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LIGO Project, Room 20B-145, 18 Vassar Street, Cambridge, MA 02139
(617) 253-4824, FAX (617) 253-7014

TO:	Fred Raab
ORGANIZATION:	LIGO at Caltech
FAX NUMBER:	(818) 304-9834
VOICE NUMBER:	
DATE:	July 15, 1996
TIME:	9:05 AM

FROM:	Rainer Weiss
ORGANIZATION:	LIGO at MIT
FAX NUMBER:	(617) 253-7014
VOICE NUMBER:	
REFER TO:	
SUBJECT:	

NUMBER OF PAGES FAXED INCLUDING THIS COVER SHEET:	20

NOTES:

Please see email

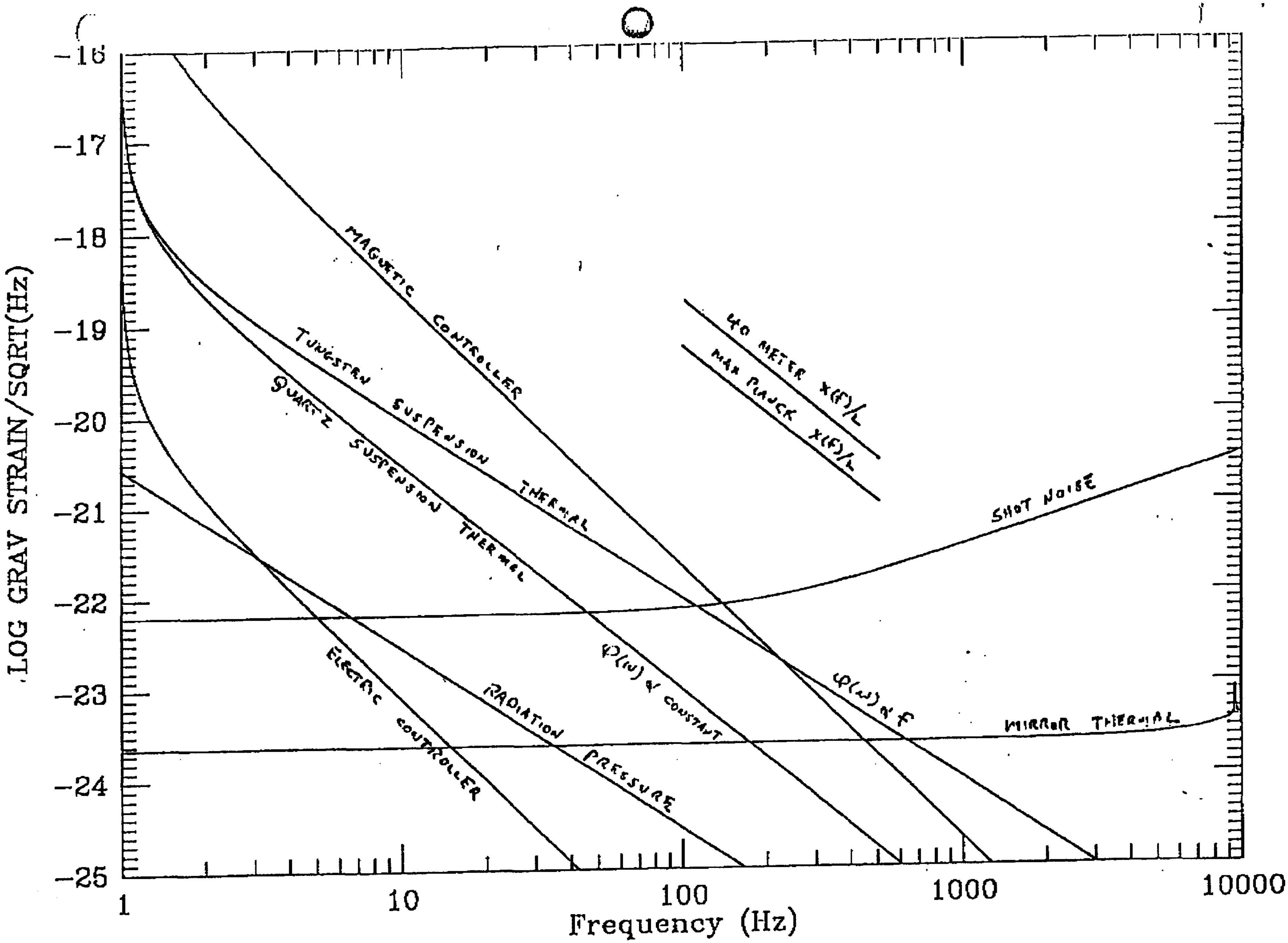
Laser Interferometer Gravitational-Wave Observatory

1) Specifications for the isolation stage
   a) The estimates for the interferometer noise budget other than seismic and acoustic noise.
      i) Estimates of the thermal noise
      ii) Estimates of the controller noise
   b) The isolation transfer function desired
   c) The isotropy needed
   c) The rms motion of the test mass allowed
   d) The continuous and thermal drift permitted

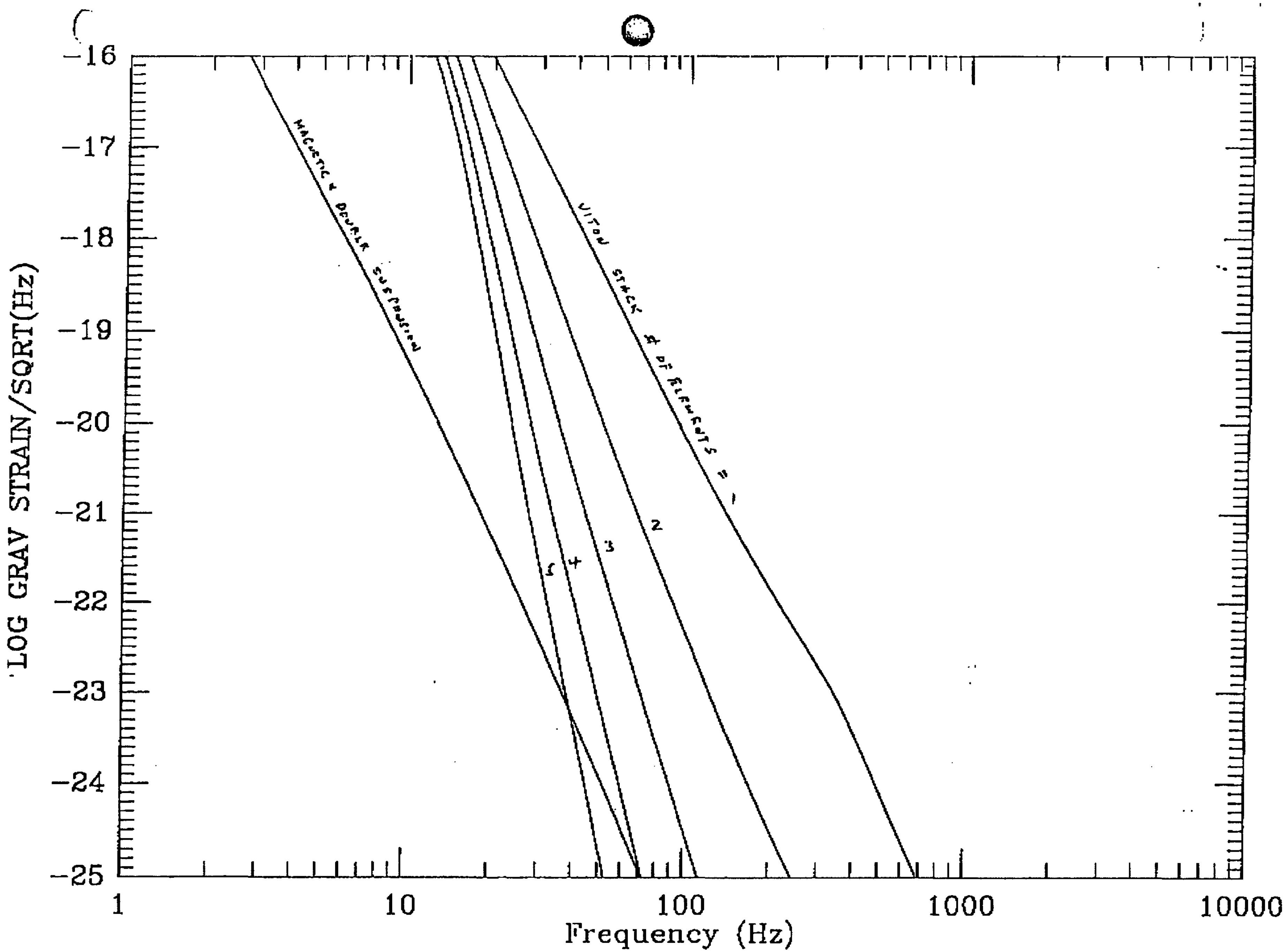
2) The spring design
   a) Vacuum compatible elastomers: VITON, SMRD, space program elastomers used in IUE and ST programs
   b) Encapsulated elastomers: bellows encapsulation, composite spring designs

3) The 3D finite element modeling

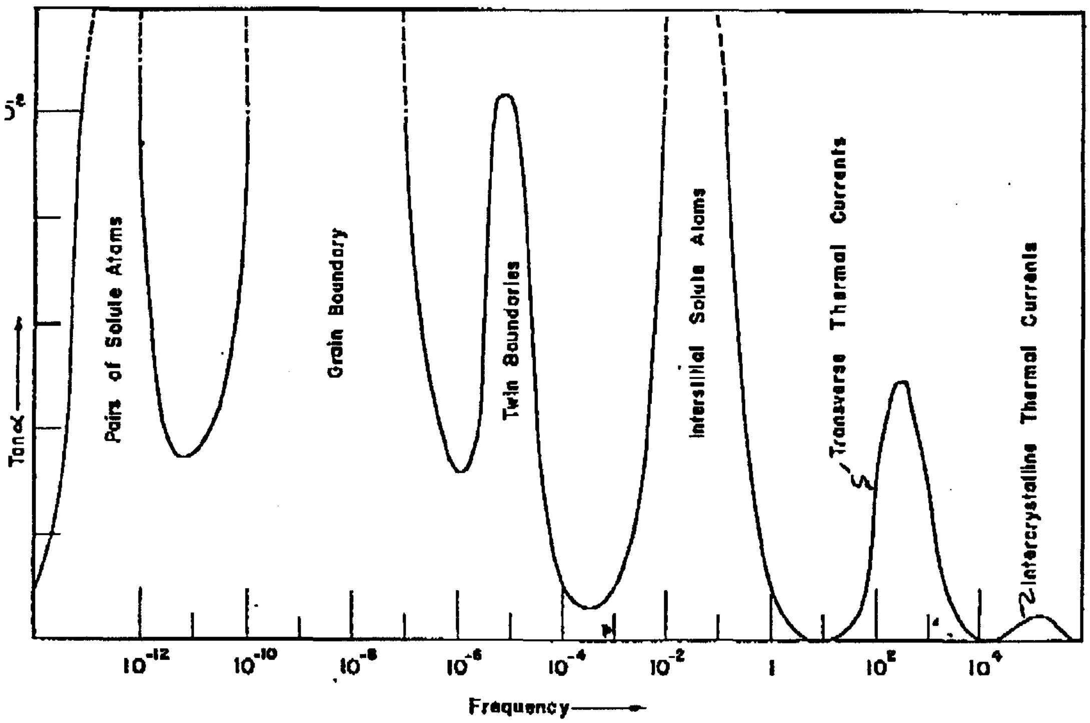
4) The test program
   a) Shaker drive amplitude vs frequency
   b) Accelerometer sensitivities : PZT, laser cavity
   c) Cross coupling measurements
   d) Drift and thermal dependence



OUTPUT OF GNPLOT WITH MODEL PARAMETERS FOR LIGO BROADBAND RECYCLED EXTERNAL MODULATION
REFER TO GNPLOT FOR MODEL PARAMETERS

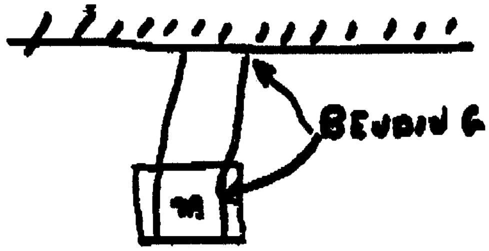


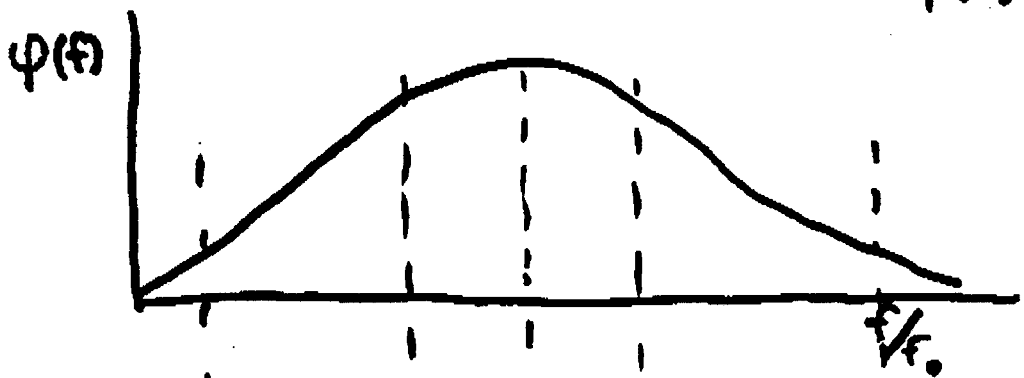
LOG GRAV STRAIN/SQRT(Hz)
−16
−17
−18
−19
−20
−21
−22
−23
−24
−25
1
10
100
1000
10000
Frequency (Hz)
MAGNETIC & DOUBLE SUSPENSION
VITON STACK # OF ELEMENTS = 1
2
3
4
5



A TYPICAL RELAXATION SPECTRUM

SUSPENSION THERMAL NOISE





$$\varphi(f) = \frac{\Delta\,(f/f_0)}{1+(f/f_0)^2}$$

$$f_0 = \frac{K_{TH}}{C_v\, a^2}$$

$$\Delta = \frac{Y \alpha^2 T}{C_v}$$

$$x_p(f) \propto \left| \frac{m^{1/4}}{n^{3/4} f^2} \right| \frac{1}{(mn)^{1/4} f^{5/2}} \left| \frac{n^{1/4}}{m^{3/4} f^3} \right|$$

EXAMPLE: $m = 10\,kg$, $n = 4$ WIRES, $f_p(0) = 1\,Hz$, $l = 25\,cm$

	a (mils)	f_0 (Hz)	Q (1 Hz)	$\varphi(f)$	$x_{TN}(f)$
SAPPHIRE	7	630	1.8×10^6	$\propto f$	$1/f^2$
TUNGSTEN	5	8700	8.0×10^6	$\propto f$	$1/f^2$
FUSED QUARTZ	31	3	5.0×10^6	CONSTANT	$1/f^{5/2}$

PUZZLE: HOW TO INCREASE a TO MAKE $f_0 \downarrow$
BUT ALSO KEEP $\dfrac{E_{ELASTICITY}}{E_{GRAVITY}} \ll 1$

MATERIAL PROPERTIES

	g/cm^3	dyne/cm²	dyne/cm²	dyne/cm²	watt/K cm	EXP 1/K	joule/gm °K	Poisson	300K	300K
	ρ	Y	Y	S_{max}	K_{TH}	α	C_{gm}	σ	Δ	D
TUNGSTEN	19	3.5×10^{12}		2×10^{10}	1.7	4.5×10^{-6}	0.143	.33)	7.83×10^{-4}	0.6
FUSED QUARTZ	2.2	7×10^{11}		5×10^{8}	1.4×10^{-2}	4×10^{-7}	0.74	.17	2.06×10^{-6}	0.00
SAPPHIRE	3.97	4.66×10^{12}		vs2 1×10 or (2.5×10^{9}, $7\times10^{[illegible]}$)	3.0×10^{-1}	5×10^{-6}	(Al_2O_3 0.83)	.17	1.061×10^{-3}	0.0
SILICON	2.42	1.7×10^{12}			1.5	2.5×10^{-6}	0.234	.22	5.6×10^{-4}	2.65

ZENER THERMOELASTIC DAMPING

$$\phi = \frac{\Delta\,\omega\tau}{1+\omega^2\tau^2} \qquad \Delta = \frac{Y\alpha^2 T}{C_{vol}} = \frac{Y\alpha^2 T}{C_{gm}\,\rho}$$

$$= \frac{\Delta\, f/f_0}{1+(f/f_0)^2} \qquad f_0 = \frac{1}{2\pi\tau} = \frac{2.16}{4\,a^2}\,K_{TH}/C_{vol}$$

$$= \frac{0.54\,K_{TH}}{\rho\, C_{gm}\, a^2} = \frac{0.54\,D}{a^2}$$

SUSPENSIONS TO SUPPORT $m = 10\,Kg$ WITH FOUR WIRES

$$\frac{mg}{4} = S_{max}\pi a^2 \qquad a^2 = \frac{mg}{\pi 4 S_{max}}$$

QUARTZ $a^2 = 1.56\times10^{-3}\ cm^2$ $a = 3.95\times10^{-2}\ cm$ => 15.5 mils = 31 MIL DIAMETER

TUNGSTEN $a^2 = 3.9\times10^{-5}\ cm^2$ $a = 6.24\times10^{-3}\ cm$ = 2.5 mils 5 MIL DIAMETER

SAPPHIRE $a^2 = 7.8\times10^{-5}\ cm^2$ $a = 8.83\times10^{-3}\ cm$ 3.5 mils 7 MIL DIAMETER

TRANSITION FREQUENCY f_0

QUARTZ = 2.98 Hz TUNGSTEN = 8.7×10^{3} Hz

SAPPHIRE 630 Hz

RELAXATION SPECTRUM

			ϕ(1Hz)	ϕ(f > 10Hz)
QUARTZ	$\phi(f) = \dfrac{(6.9\times10^{-7})f}{1+.113\,f^2}$	=>	6.2×10^{-7}	$\dfrac{6.1\times10^{-6}}{f}$
TUNGSTEN	$\phi(f) = \dfrac{(7.24\times10^{-5})f}{1+(f/8.73\times10^{3})^2}$	=>	7.3×10^{-5}	$7.3\times10^{-5} f$
SAPPHIRE	$\phi(f) = \dfrac{1.44\times10^{-4}\,f}{1+(f/6.3\times10^{2})^2}$	=>	1.44×10^{-4}	$1.44\times10^{-4} f$

PENDULUM Q AT RESONANCE f_{op}

$$Q = \frac{1}{\varphi(f_{op})}$$

$I_w = \pi a^4/4$

THE EFFECTIVE RELAXATION FOR THE PENDULUM

$$\varphi_p(\omega) = \underset{\text{SINGLE WIRE}}{\underset{\uparrow}{\varphi(\omega)}}\; n\; \frac{\sqrt{\frac{mg}{n} Y I}}{2 m g l} = \underset{\text{SINGLE WIRE}}{\underset{\uparrow}{\varphi(\omega)}}\; \frac{n^{1/2} a^2}{4l}\left(\frac{Y\pi}{mg}\right)^{1/2}$$

THE RELAXATION AT 1 Hz, $l = 25$ cm

$$\phi_p\,(\text{QUARTZ}) = \underset{\text{SINGLE WIRE}}{\varphi(\omega)}\; \underset{E_{\text{SPRING}}/E_{\text{GRAV}}}{\underline{1{,}478 \times 10^{-2}}} \times 2$$

FOR OUR TWO FLEXURE BOTH ENDS FLEX IN A BIFILAR SUSPENSION

$$\phi_p\,(\text{TUNGSTEN}) = \underset{\text{SINGLE WIRE}}{\varphi(\omega)}\; 8.36 \times 10^{-4} \times 2$$

$$\varphi_r\,(\text{SAPPHIRE}) = \underset{\text{SINGLE WIRE}}{\varphi(\omega)}\; 1.9 \times 10^{-3} \times 2$$

THE Q AT 1 Hz

$$\left.\begin{aligned} Q_{\text{QUARTZ}} &= 5.0 \times 10^6 \\ Q_{\text{TUNGSTEN}} &= 8.0 \times 10^6 \\ Q_{\text{SAPPHIRE}} &= 1.8 \times 10^6 \end{aligned}\right\}$$

MAXIMUM BARRING OTHER LOSSES INTO THE SUPPORTS

THE SPECTRUM OF THE THERMAL POSITION FLUCTUATIONS

$$x^2(f) = \frac{4 k_B T\, \omega_0^2\, \varphi_p(\omega)}{m\,\omega\left[(\omega_0^2 - \omega^2)^2 + \omega_0^4 \varphi_p^2(\omega)\right]} \quad \text{cm}^2/\text{Hz}$$

EXPRESSING THIS IN TERMS OF THE RESONANCE Q

$$\frac{\varphi_p(\omega)}{\varphi_r(\omega_0)}\,\frac{1}{Q} = \varphi_r(\omega)$$

$$x^2(f) = \frac{4 k_B T\, \omega_0^2/Q\; \frac{\varphi_p(\omega)}{\varphi_p(\omega_0)}}{m\,\omega\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega_0^4}{Q^2}\left(\frac{\varphi_p(\omega)}{\varphi_r(\omega_0)}\right)^2\right]}$$

THE THERMAL NOISE FOR THE TWO MATERIALS

$$\left.\frac{\varphi_p(\omega)}{\varphi_p(\omega_0)}\right|_{\text{QUARTZ}} = \frac{\omega_0}{\omega} \qquad \frac{\varphi_p(\omega)}{\varphi_p(\omega_0)} = \frac{\omega}{\omega_0}$$

QUARTZ

$$x^2(f) = \frac{4K_BT\,\omega_0^3}{Q m \omega^2\left[(\omega_0^2-\omega^2)^2 + \frac{\omega_0^6}{Q^2\omega^2}\right]} \xrightarrow[\omega \gg \omega_0]{} \frac{4K_BT\,\omega_0^3}{Q m \omega^6}$$

TUNGSTEN

$$x^2(f) = \frac{4K_BT\,\omega_0}{Q m\left[(\omega_0^2-\omega^2)^2 + \frac{\omega_0^2\omega^2}{Q^2}\right]} \xrightarrow[\omega \gg \omega_0]{} \frac{4K_BT\,\omega_0}{Q m \omega^4}$$ VISCOUS DAMPING

QUARTZ DOES NOT OBEY THE ZENER MODEL IT GIVE $\varphi(F) = \text{CONSTANT}$ 40HZ $\longrightarrow$ 700HZ J. KOVALIK DATA

QUARTZ WILL THEREFORE EXHIBIT $x^2(f)_{\omega \gg \omega_0} \simeq \dfrac{4K_BT\,\omega_0}{Q m \omega^5}$

TYPICALLY GET INTERNAL $Q_{\text{QUARTZ}} = 4\times10^5$

BOTH FLEXURES ✓

PENDULUM $Q_p \simeq \dfrac{E_{\text{STORED GRAVITY}}}{E_{\text{STORED FLEXURE}}}\, Q_{\text{MAT}} = \dfrac{1}{1.5\times10^{-2}}\, 4\times10^5 = 2.7\times10^7 \times \frac{1}{2}$

$= 1.3\times10^7$

SCALING FOR OVERALL THERMAL NOISE WITH THE ZENER MECHANISM

$$\varphi = \frac{\Delta f/f_0}{1+(f/f_0)^2} \qquad f_0 = \frac{0.540}{a^2} \qquad \varphi_{\text{PEAK VALUE}}(f/f_0)\Big|_{f/f_0=1} = \frac{\Delta}{2}$$

BUT $a^2 = \dfrac{mg}{n\pi S_{max}}$ n = NUMBER OF FIBERS

$$f_0 = \frac{0.540\, n\pi S_{max}}{mg} \qquad \Delta = \frac{Y\alpha^2 T}{C_m \rho}$$

FOR A PENDULUM

$$\varphi_p(\omega) = \varphi_{wire}(\omega)\, \frac{n^{1/2} a^2}{4\ell}\left(\frac{Y\pi}{mg}\right)^{1/2} = \frac{\varphi_{wire}(\omega)}{4\ell}\, \frac{mg\, n^{1/2}}{n\pi S_{max}}\left(\frac{Y\pi}{mg}\right)^{1/2}$$

$$\varphi_p(\omega) = \frac{\varphi_{wire}(\omega)\,(Y\pi mg)^{1/2}}{4\pi\ell\, S_{max}\, n^{1/2}}$$

$$\varphi_p(\omega) = \frac{Y\alpha^2 T}{4 C_m \rho S_{max}\ell}\left(\frac{Ymg}{n\pi}\right)^{1/2}\left[\frac{f/f_0}{1+(f/f_0)^2}\right]$$

LIMITING CASES IF $f/f_0 \ll 1$ VISCOUS CASE

$$\varphi_p(\omega) = \left(\frac{Y\alpha^2 T}{4 C_m \rho S_{max}}\right)\left(\frac{mg f}{0.540\, n\pi S_{max}}\right)\left(\frac{Ymg}{n\pi}\right)^{1/2}$$

$$\left(\frac{Ymg}{\pi n}\right)^{3/2} \frac{\alpha^2 T f}{2 K_{TH} S_{max}^2 \ell}$$

VISCOUS LIMIT $f/f_0 \ll 1$ $x(f) \propto m^{1/4}/n^{3/4}$

LEADS TO $x_{THERM}(f) \propto \frac{1}{f^2}$

AT THE PEAK $f/f_0 = 1$

$$\varphi_p(\omega) = \frac{Y^{3/2}\alpha^2 T}{8 C_m \rho S_{max}\ell}\left(\frac{mg}{n\pi}\right)^{1/2}$$

QUARTZ @ FOR $n = 10^{-4}$ gm 6.5×10^{7}

INDEPENDENT OF FREQUENCY

LEADS TO $x_{THERM}(f) \propto \frac{1}{f^{5/2}}$

$x(f) \sim \frac{1}{(mn)^{1/4}}$

ABOVE PEAK $f/f_0 \gg 1$

$$\varphi_p(\omega) \simeq \frac{K_{TH}}{8(C_m\rho)^2}\, \frac{Y^{3/2}\alpha^2 T}{\ell f}\left(\frac{n\pi}{mg}\right)^{1/2}$$

LEADS TO $x_{THERM}(f) \propto \frac{1}{f^2}$

$x(f) \sim n^{1/4}/m^{3/4}$

4 POLE REPRESENTATION

$$F_1 = \alpha_{11} F_2 + \alpha_{12} V_2$$
$$V_1 = \alpha_{21} F_2 + \alpha_{22} V_2 \quad = \left(\alpha_{ij}\right)\begin{pmatrix} F_2 \\ V_2 \end{pmatrix}$$

FOR TRANSMISSION LINE RESONANCES OF ELEMENT BELLOWS OR ELASTOMER

$$\alpha_{11} = \cos\frac{\omega}{\omega_{00}} \qquad \omega_{00} = \omega_0 (1+\delta^2)^{1/4} e^{i\frac{\tan^{-1}\delta}{2}}$$
$$= \alpha_{22}$$

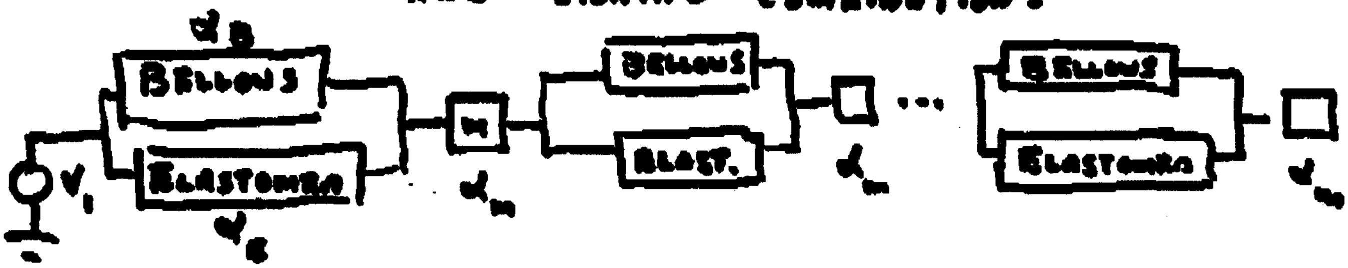
ELASTOMER MODULI

$$G = G'(1+i\delta)$$

$$\alpha_{21} = -\frac{1}{i m \omega_{00}} \sin\frac{\omega}{\omega_{00}}$$

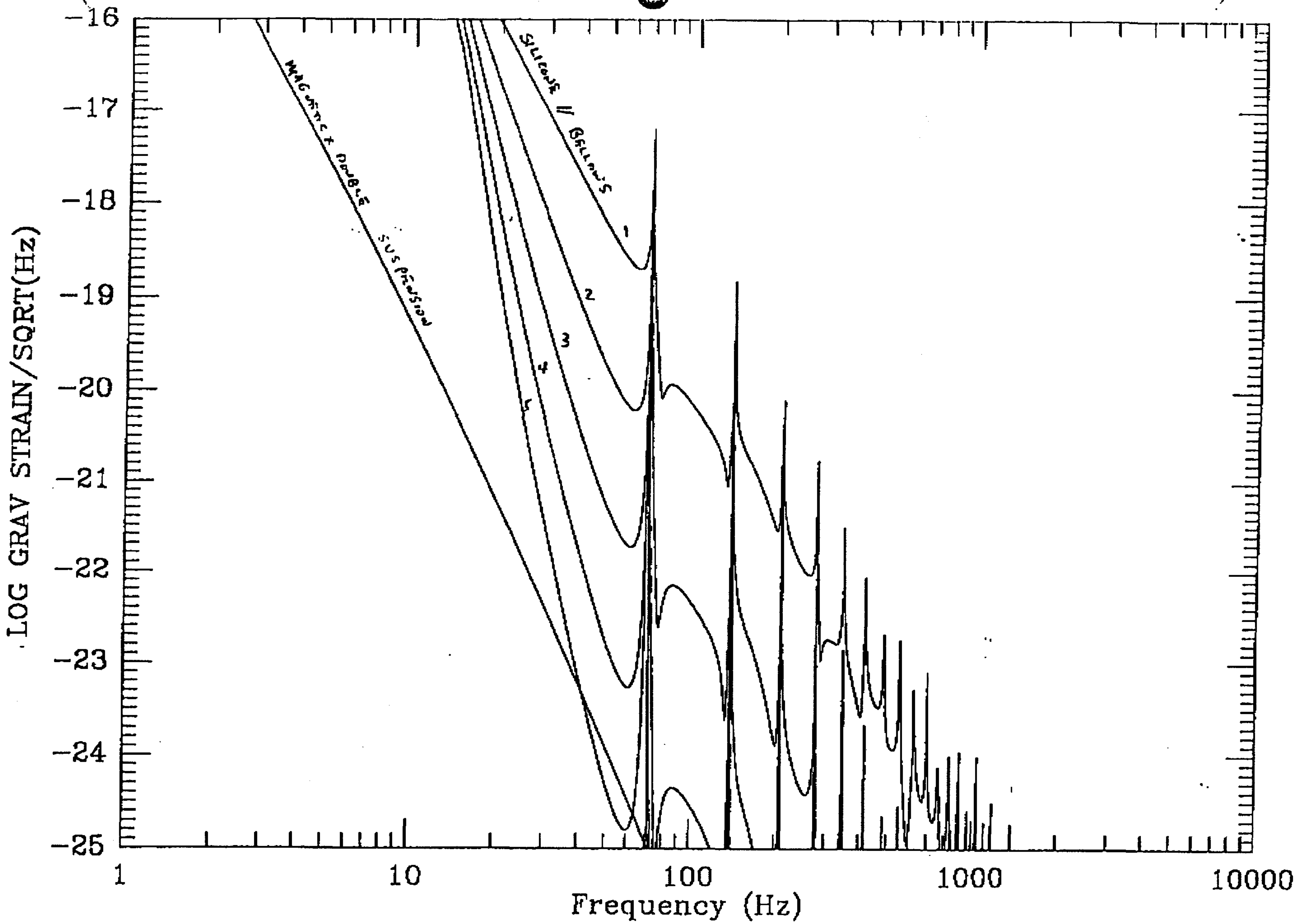
$$\alpha_{12} = \frac{i m \omega_{00}}{i\sqrt{km}} \sin\frac{\omega}{\omega_{00}}$$

USE PARALLEL AND SERIAL COMBINATIONS

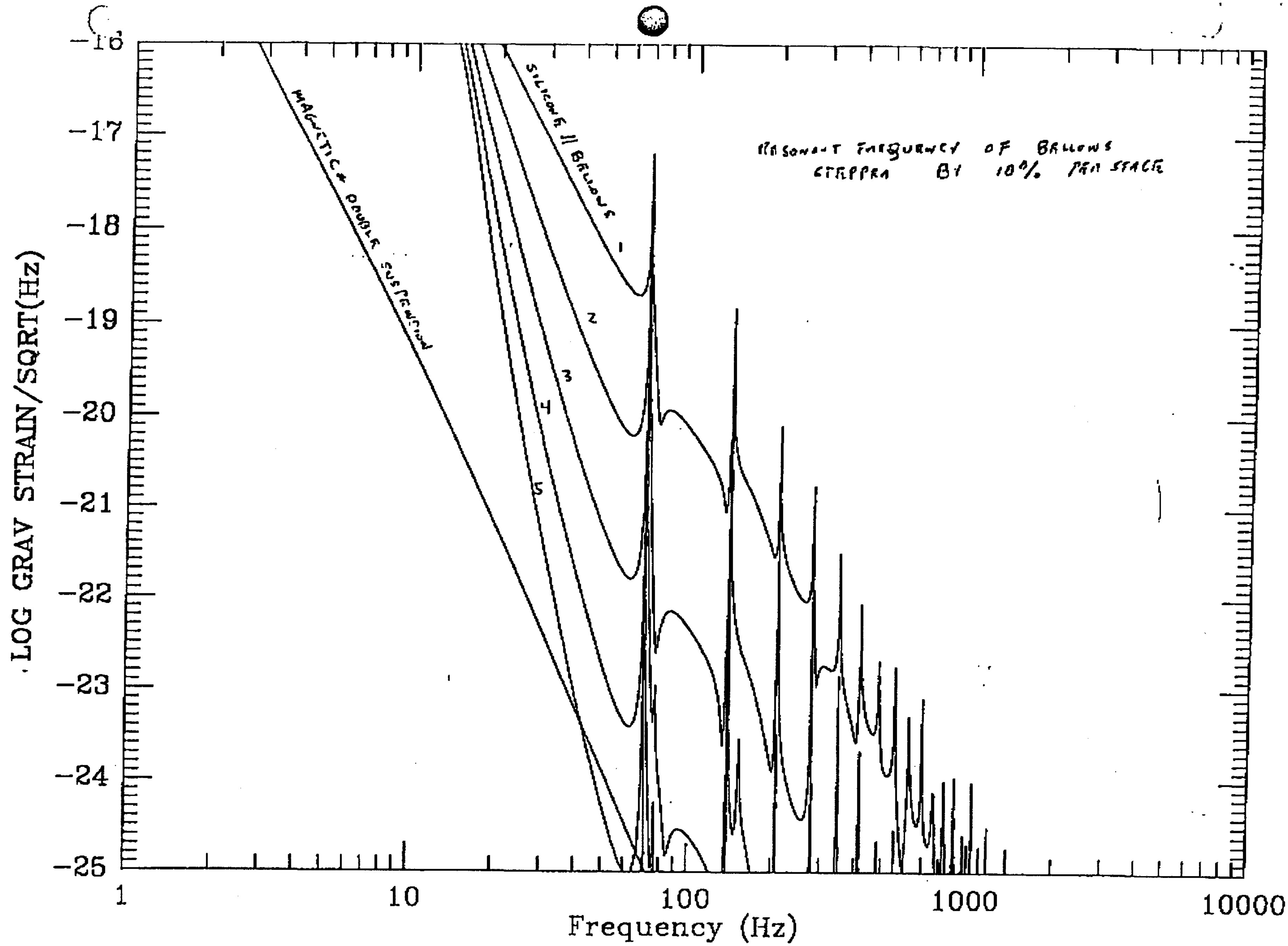


MATERIAL	THICKNESS (PAIL) MILS	O.D. INCH	I.D. INCH	SPAN (OD-ID)/2 INCH	MASS CONV gm	K CONV dynes/cm	Km (gram/cm)	LENGTH CONV in/CONV	N_c	LENGTH INCH	m gm	K dyne/cm	f_0 SELF Hz
STANDARD-THOMSON BRASS FORMED	4.5	1.5	.875	.313	1.9	7.4×10^6	1.4×10^7	.375	MAX 14 5	5.25 1.9	27 9.5	5.3×10^5 1.5×10^6	22.3 63.3
STANDARD-THOMSON BRASS FORMED	5.5	3.0	2.0	.50	8.7	1.1×10^7	9.6×10^7	.5	MAX 16 4	8.00 2.0	139 35	6×10^6 2.4×10^7	33.1 132
EG&G SEALOL 316 L WELDED	2.5	1.5	.750	.375	0.8	1.81×10^7	1.45×10^7	MAX, 3 SPAN .113	18	2.0	15	1.0×10^6	41.
EG&G SEALOL TITANIUM WELDED	2.5	1.5	.750	.375	0.45	1.0×10^7	4.5×10^6	.113	18	2.0	8.1	5.56×10^5	112
EG&G SEALOL 316 L WELDED	2.5	3.025	2.275	.375	2.1	2.2×10^7	4.6×10^7	.113	18	2.0	38	1.2×10^6	28
EG&G SEALOL TITANIUM WELDED	2.5	3.025	2.275	.375	1.2	1.16×10^7	1.4×10^7	.113	18	2.0	22	6.4×10^5	27
EG&G SEALOL TITANIUM WELDED	5.0	7.00	6.05	.475	7.3	4.7×10^7	3.4×10^8	.113	18	2.0	131	2.6×10^6	22.4

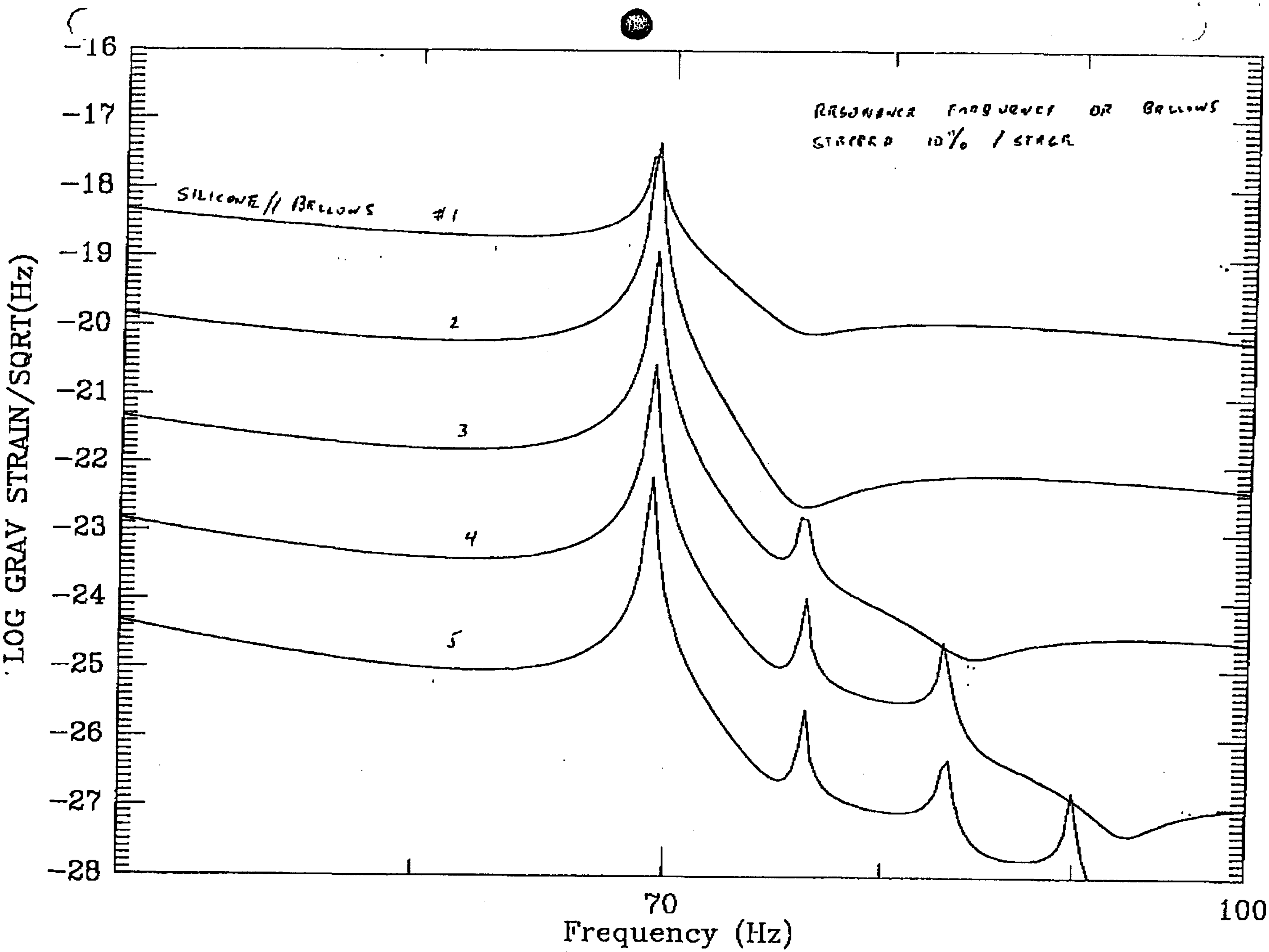
CONCLUSIONS: EXPENSIVE (5×) WELDED BELLOWS DO NOT HAVE ADVANTAGE OVER BRASS BELLOWS UNTIL SIZES GET ABOVE OD ~ 2"



LOG GRAV STRAIN/SQRT(Hz)
-16
-17
-18
-19
-20
-21
-22
-23
-24
-25
1
10
100
1000
10000
Frequency (Hz)
1
2
3
4



LOG GRAV STRAIN/SQRT(Hz)
Frequency (Hz)
-16
-17
-18
-19
-20
-21
-22
-23
-24
-25
1
10
100
1000
10000
MAGNETIC & DOUBLE SUSPENSION
SILICONE // BELLOWS
1
2
3
4
5
RESONANT FREQUENCY OF BELLOWS
STEPPED BY 10% PER STAGE


RESONANCE FREQUENCY OF BELLOWS
STEPPED 10% / STAGE
SILICONE // BELLOWS #1
2
3
4
5
-16
-17
-18
-19
-20
-21
-22
-23
-24
-25
-26
-27
-28
LOG GRAV STRAIN/SQRT(Hz)
70
100
Frequency (Hz)

ELASTOMER PARAMETERS

1) ANISOTROPY $Y \sim 3G$

MAXIMUM STRAIN VERTICAL $\frac{\Delta l}{l} \sim \frac{g}{l\,\omega_0^2}$

SIZE l AND MAX ALLOWED $\Delta l/l$ VS x_{RMS}

Δl (cm)	f_0 (Hz)
6	2
2.8	3
1.55	4
1	5
.7	6
.5	7

2) DRIFT

BEST $\delta x(t) \sim \Delta l \;\; 6 \times 10^{-3} \ln t_{DAYS}$ 4% 1 YR

WORST $\delta x(t) \sim \Delta l \;\; 1.7 \times 10^{-2} \ln t_{DAYS}$ 11% 1 YR

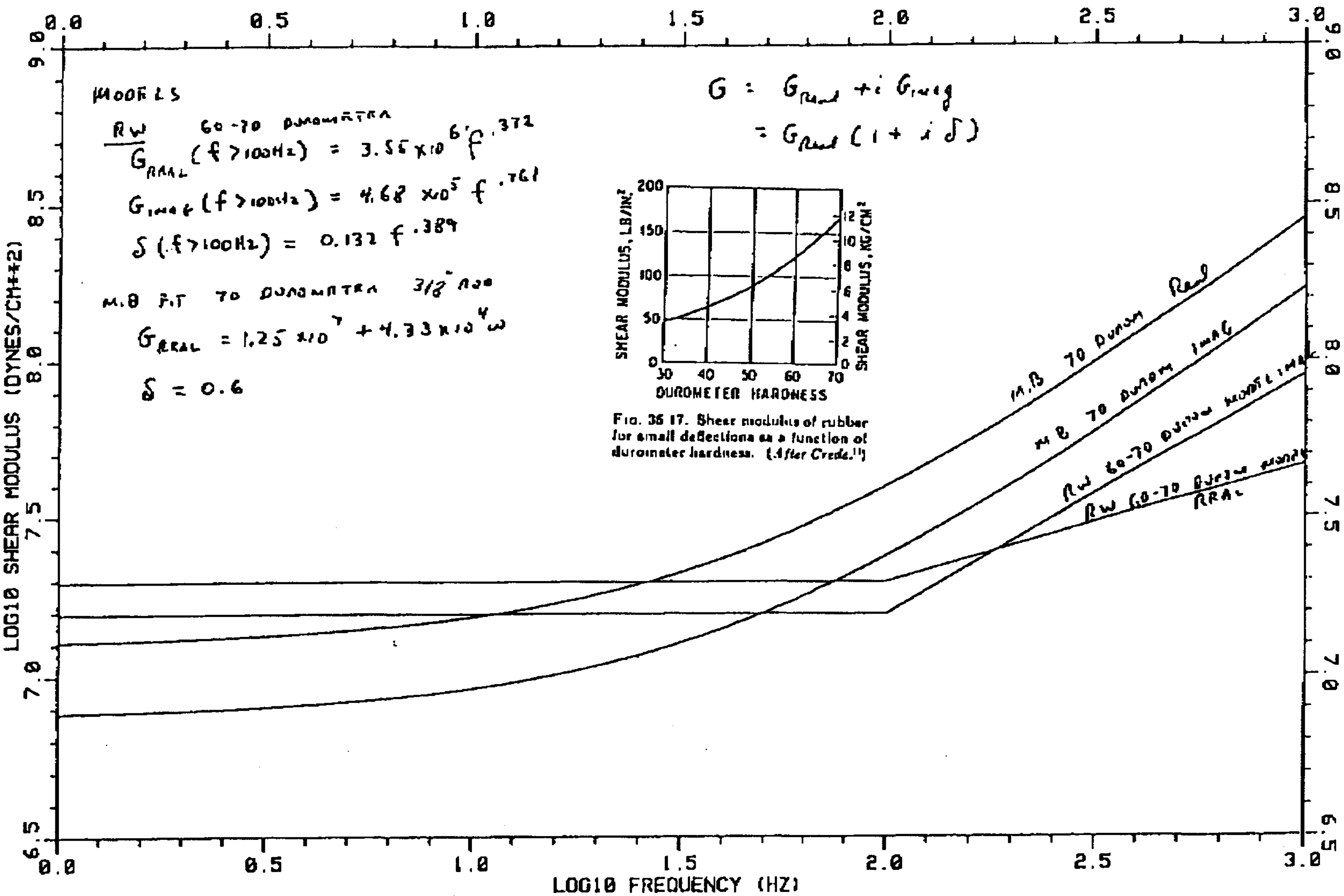
TEMPERATURE DEPENDENCE

BEST $\Delta G/G/K \simeq \Delta Y/Y/K = .006$

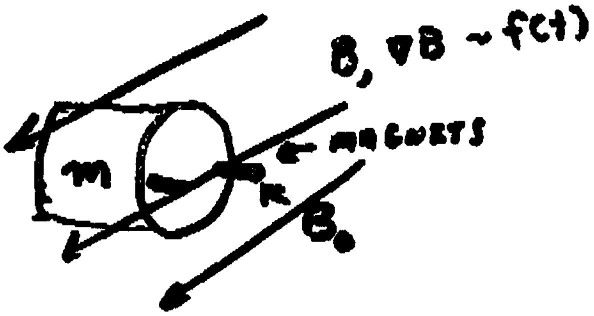
$\Delta\delta/\delta/K = .002$

WORST $\Delta G/G/K = \Delta Y/Y/K = .01$

$\Delta\delta/\delta/K = .02$



VITON SHEAR MODULI REAL AND IMAGINARY



MAGNET VOLUME = V
POLE FIELD = B_0
$\mu_D = B_0 V$

EXTERNAL B INTERACTING WITH MAGNET μ_D

$$B^2(f) \sim \frac{4\tilde{\tau}_0 B^2_{RMS}}{1 + (2\pi\tilde{\tau}_0 f)^2}$$

FLUCTUATION FORCE EXPRESSED IN $h(f)$

$$h(f) = \frac{B_0 V N B_{RMS}}{4\pi^3 m \tilde{\tau}_0^{1/2} h L} \frac{1}{f^3}$$

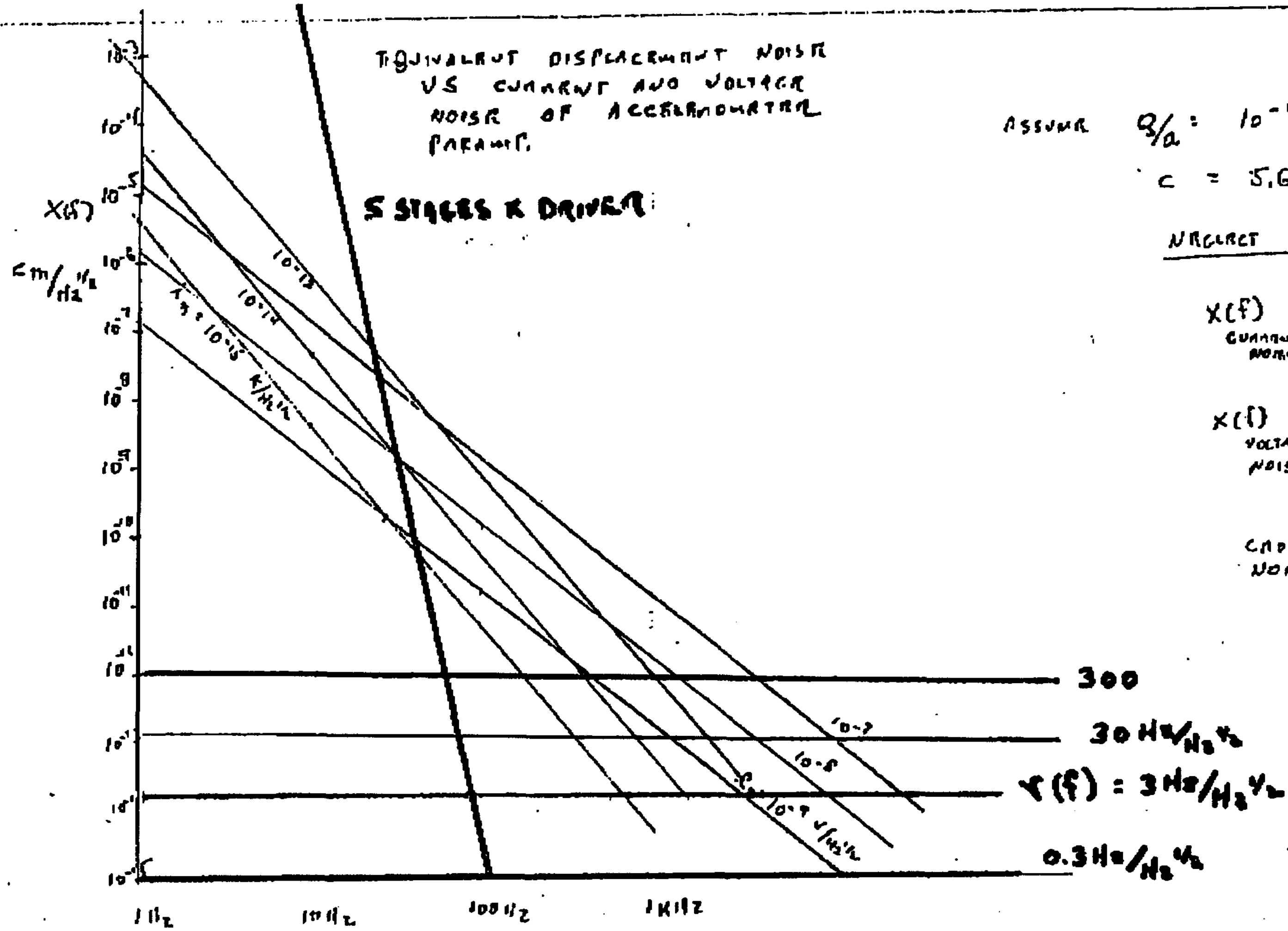
$B_0 = 500$ gauss
$V = 10^{-1}$ cc
$N = 2$
$B_{rms} = 10^{-4}$ gauss

$h = 5$ meters
$\tilde{\tau}_0 = 10$ sec
$m = 10$ kg

MEASURE $B(f)/h$
REDUCE X_{RMS}

INTRINSIC NOISE
BARKHAUSEN MAGNETIC
DOMAIN RELAXATION

ELECTROSTATIC CONTROL



Assume $Q/a = 10^{-8}$ coulombs/cm/sec^2

$c = 5.6 \times 10^{-9}$

Neglect thermal noise

$$X(f)_{\text{current noise}} = \frac{i_n(f)}{(Q/a)(2\pi)^3 f^3}$$

$$X(f)_{\text{voltage noise}} = \frac{e_n(f)\,C}{Q/A\,(2\pi)^2 f^2}$$

Crossing frequency between noise sources

$$f_{cross} = \frac{i_n(f)}{e_n(f)\,2\pi C}$$

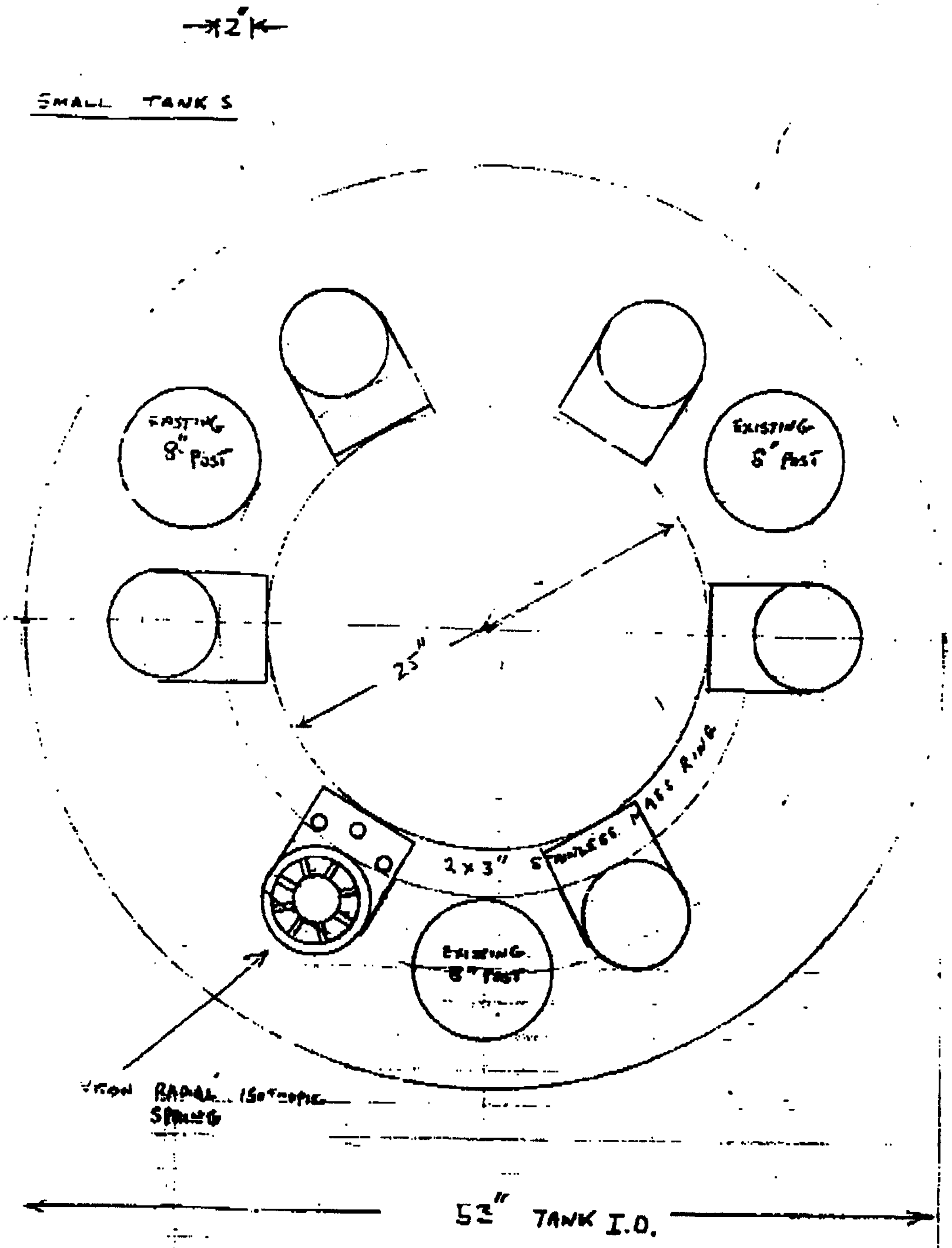
$$a(f) = \frac{e_n(f)\,C}{Q/A}$$

$F_{max\ driver} = 6 \times 10^6$ dynes

$X_D = \frac{16}{f^2}$ cm $\qquad M/\text{stage} = 10$ Kg

TOP VIEW SCALE 1/10

HEXAGONAL SPRING CONFIGURATION



S.S RING MASS 75Kg 31" O.D 2"x3"

ALUMINUM OPTICAL TABLE MASS 71Kg 31" DIAMETER

RING COMPONENTS AND BRACKETS 6061 ALUMINUM