LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Radiation Pressure Noise in LIGO

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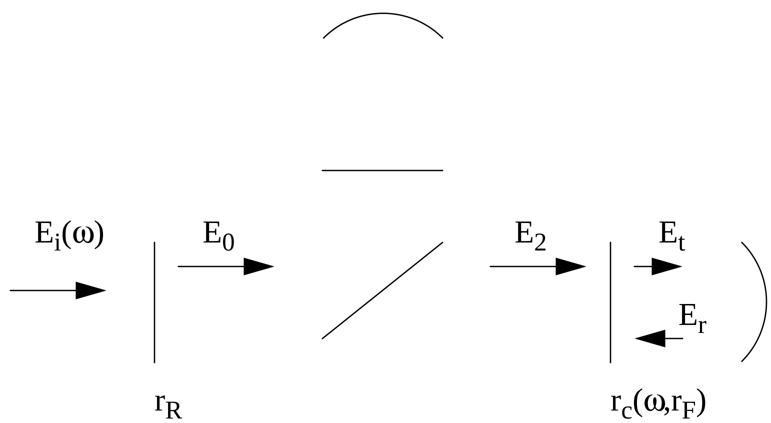
1 INTRODUCTION

In this note we calculate the displacement noise caused by fluctuating radiation pressure on the LIGO test masses. This allows us to derive a constraint on allowed beam splitter imbalance.

2 FIELD AMPLITUDES



We apply the usual circulating field analysis to derive the fields that exert radiation pressure on the test masses. Since we are interested in the in-band displacements of the masses, we need the frequency dependence of the electric field intensity variations in both the arm and recycling cavities.

2.1. Carrier

$$E_0 = E_i t_R - r_c r_R E_0$$

$$E_0 = \frac{E_i t_R}{1 + r_C r_R}$$

$$\text{then}^1 \; E_2 = f_2 E_0 = \frac{f_2 E_i t_R}{1 + r_R r_0} \frac{1 + i\frac{\omega}{\omega_c}}{1 + i\frac{\omega}{\omega_{cc}}}$$

where f_2 is the beam splitter fractional amplitude transmission to the in-line arm, r_0 is the arm cavity DC carrier reflectivity, and

1. Frequency, Intensity and Oscillator Noise in the LIGO, LIGO-T960019-00-D, pg. 5-7

$$\omega_{CC} = \frac{1 + r_R r_0}{1 + r_R} \omega_C ,$$

where ω_c and ω_{cc} are the arm cavity and coupled cavity poles, respectively.

$$\text{Next } E_T = E_2 t_F + E_T \left(1 - \frac{2i\omega L}{c}\right) r_F$$

where L is the arm cavity length.

$$E_T = \frac{E_2 t_F}{(1 - r_F)\left(1 + \frac{i\omega}{\omega_C}\right)}$$

$$= f_2 E_i \frac{t_R t_F}{(1 + r_R r_0)(1 - r_F)} \frac{1}{1 + \frac{i\omega}{\omega_{CC}}}$$

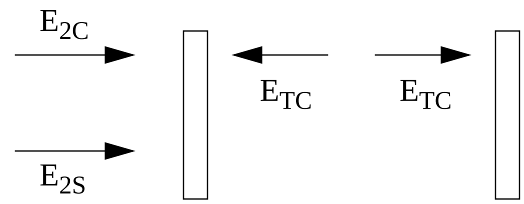
2.2. Sideband

The sidebands undergo no frequency filtering. Also for the sidebands $r_c = 1$ so that

$$E_{2s} = f_2 E_i J_1(\Gamma) \frac{t_R}{1 - r_R \cos\alpha}$$

where α is determined by the recycling cavity asymmetry and sideband modulation frequency.

3 MOTION OF ARM CAVITY FROM RADIATION PRESSURE



Arm Cavity

We are concerned with in-band fluctuations in the radiation pressure, assumed to be due to intensity noise, which affect the arm cavity length. The momentum *p(ω)* of the electric fields incident on the test masses results in a disturbance *x(ω)* of the test mass from its equilibrium position:

$$m\omega^2 x(\omega) = \frac{dp(\omega)}{dt} = \frac{2P_{incident}(\omega)}{c}$$

$$\text{Thus } x_2(\omega) = \frac{2f_2^2}{m\omega^2 c}[E_{2C}^2 + E^2{}_{2S} - E^2{}_{TC} - E_{TC}^2]$$

$$= \frac{4E_i(\omega)E_{DC}t_R^2 f^2{}_2}{m\omega^2 c}\left[\frac{\cos(\omega t + \varphi_R)}{(1 + r_R r_0)^2}\left(\frac{\omega_{CC}}{\omega_C}\right) + \frac{2\cos(\omega t)J_1^2(\Gamma)}{(1 - r_R\cos\alpha)^2} - \frac{2t_F^2\cos(\omega t + \varphi_A)}{(1 - r_F)^2(1 + r_R r_0)^2}\left(\frac{\omega_{CC}}{\omega}\right)\right]$$

We see that the above expression has contributions from 1) the carrier in the recycling cavity, 2) the sidebands in the recycling cavity, and 3) the carrier in the arm cavity. The terms have distinct gains, filtering factors, and phase shifts with respect to the incident light.

With the following constants:

$r_R = 0.98$
$r_F = 0.985$
$r_0 = -0.985$
$\omega_{cc} = 0.015\ \omega_c = 2\pi\ (1.6\ \text{Hz})$
$J_1(\Gamma) = 0.2$
$m = 10\ \text{kg}$
$\cos\alpha = 0.99$

we find:

$$x_2 = \frac{2E_{DC}E_i(\omega)f_2^2}{\omega^2}(2.6\cdot 10^{-11})\left[13\cos(\omega t + \varphi_R) + 87\cos(\omega t) - 3240\left(\frac{100}{f}\right)\cos(\omega t + \varphi_A)\right]$$

This expression shows the relative contribution of the radiation pressure components on the cavity displacement. The arm cavity light dominates over most of the spectrum.

With f_{cc} ~ 1.5 Hz, ϕ_A (f=100 Hz) = π/2. Thus the second and third components are 90 degrees out of phase.

$$\text{Since } 2E_{DC}\,E_i(\omega) = \left[\frac{\Delta I(\omega)}{I}\right] P_i$$

$$\text{We have } x_2 = \frac{f_2^2}{(2\pi \cdot 100)^2}\left(\frac{\Delta I(\omega)}{I} P_i\right) 2.6 \cdot 10^{-11}\,[3240]\; m \qquad @\ f = 100\ \text{Hz}$$

With P_i = 5 watt and ΔI / I = 10^{-8} / $Hz^{1/2}$

$$x_2 = 1 \cdot 10^{-20} f_2^2\,(m/\sqrt{Hz})$$

This is far below the LIGO initial detector sensitivity level and thus is not a driver for a beam splitter balance requirement.

4 SUMMARY

Strain noise from variations in the radiation pressure incident on the LIGO test masses is dominated by the carrier buildup in the arm cavities. The effect is small for the initial detector and does not impose requirements on the beam splitter.