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Subject: Preliminary Data Analysis

Number of Pages: (Including Cover) 15

Message: Dear Prof. Tanner

Here is a copy of my writeup on a preliminary analysis of Axion expt data. I would welcome questions/comments

Best wishes

Edward Daw

Axion Note

16th May 1996

Preliminary Analysis of the Axion Power Sensitivity from the Early 1996 Axion Data

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Abstract

The purpose of this axion note is to give a status report on estimating the sensitivity of our experiment. The data sample is production data from February 1996 through the present.

Note

These results are preliminary and should not be distributed or discussed outside the collaboration. This note is a status report, intended as a reference on my analysis software, and a tool to facilitate discussion. The note assembles contributions from myself and other Axion collaborators. I welcome comments and suggestions from all of you.

1. Preliminary Steps

Figure 1.1 is a typical 'trace' from the output of the FFT, taken on February 12th. On the vertical axis is power per 125Hz bin, mixed down from frequencies near the cavity resonance by the axion receiver electronics. There are 400 bins per FFT trace, plotted on the horizontal axis, each 125Hz wide. Power from the centre frequency of the cavity TM_{010} mode is mixed down to the centre of the trace.

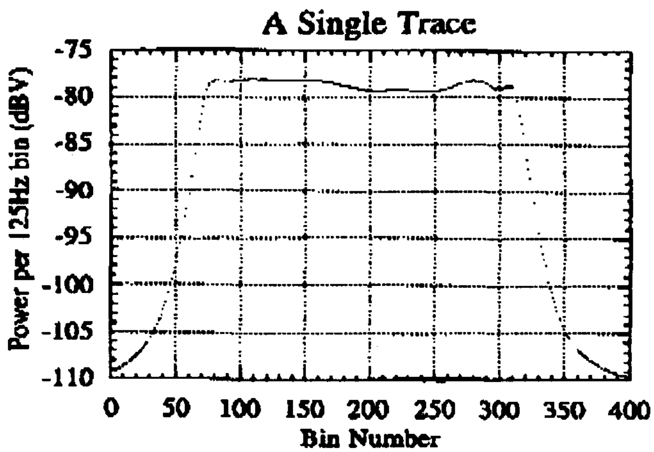


figure 1.1

The FFT unit typically takes a few thousand power spectra (I will refer to these simply as spectra) and makes a linear average of these to form the 'trace' which is written to disk. The information relevant to this analysis is contained in the fluctuations about the mean power in each bin. To focus on these fluctuations, we subtract from each trace a 'long average'. The long average is the average of the 5 traces acquired immediately before the trace being processed, the trace itself, and the 5 traces acquired immediately after it.

At this stage the size of the fluctuations is dependent on the gain of the receiver, which is susceptible to drifts of several dB. To remove this dependence, the fluctuations are next divided by the long average. After this processing, the fluctuations, which I will call 'deltas', are dimensionless. They represent the ratio of the fluctuations in the noise power to the average noise power, at any stage in the receiver chain.

Finally, note from Figure 1.1 that the first 100 bins and the last 100 bins in a trace are dominated by the receiver filter skirts. Also, they are far from the cavity resonant frequency. Hence, we discard these 200 bins and do not consider them further in the remaining analysis.

Figure 1.2 shows a single trace after this preliminary processing. Recall that the deltas are the ratio of the fluctuation about the mean power, to the average power in a 125 Hz frequency bin. The deltas are plotted on the vertical axis, for each of the 200 bins in the receiver passband.

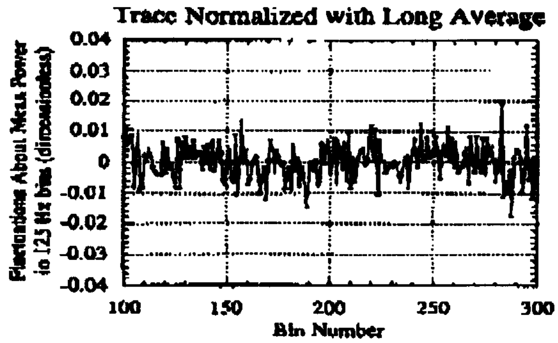


figure 1.2

Let me emphasize at this stage that these deltas are dimensionless, since after processing each is the ratio a power fluctuation to an average power, and both quantities are in Watts. An important quantity for our analysis is the rms size of these fluctuations for a single trace, $\sigma(\delta)$.

$$\sigma(\delta) = \sqrt{\sum_{i}^{n_b} \frac{\delta_i^2}{n_b}} \qquad (1.1)$$

where n_b is the number of bins and δ_i is the delta from the ith bin. There happens to be a simple relationship between $\sigma(\delta)$ and N, the number of spectra used in the linear average taken by the FFT to form the trace. This relationship comes from the radiometer equation.

$$\frac{P_S}{\delta P_N} = \frac{P_S}{P_N}\sqrt{N} \qquad (1.2)$$

where P_S is the power level of a signal, P_n is the average noise power and δP_N is rms size of the noise power fluctuations. Canceling the signal power, we have

$$\frac{\delta P_N}{P_N} = \frac{1}{\sqrt{N}} \qquad (1.3)$$

$$\text{or,} \quad \sigma(\delta) = \frac{1}{\sqrt{N}} \qquad (1.4)$$

because $\sigma(\delta)$ is the rms noise fluctuation normalized to the average noise power.

At this stage I do a quick check that our normalized data obeys equation 1.4. I take a single data file, containing 259 traces, perform the normalization procedure and evaluate $1/\sigma^2(\delta)$ for each trace. Figure 1.3 is the resultant histogram.

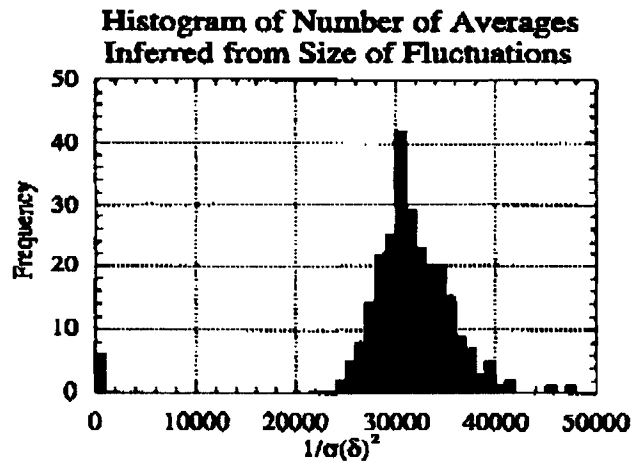


figure 1.3

The distribution is centred on the 30000, which was the number of averages for this trace, as you would expect from equation 1.4. Non-Poisson noise sources or non-stationary noise sources would result in a distribution peaked at smaller values of N. We therefore have confidence that we do not have a big contribution from spurious non-Poisson or non-stationary noise. The remainder of the analysis, which depends on stationary Poisson noise, is validated.

2. Signal Power

Consider the same normalized spectrum shown in figure 1.2, except we overlay a fake candidate peak. This is shown in figure 2.1

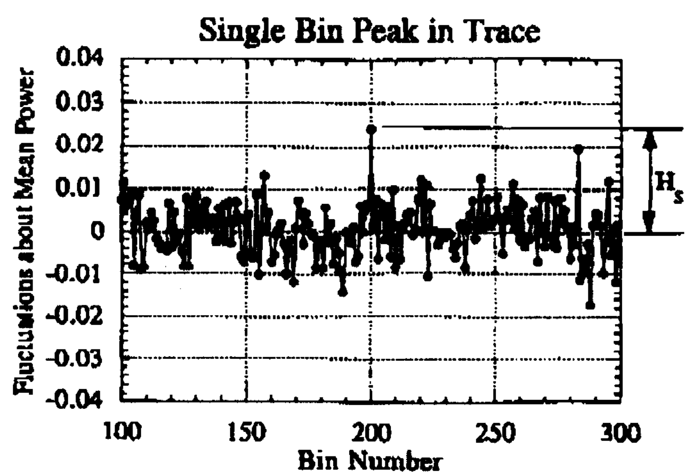


figure 2.1

What power in Watts results in a peak of height H_S on this dimensionless scale? We use the definition of $\sigma(\delta)$:

$$\sigma(\delta) = \frac{\sigma P_N}{\overline{P_N}} \quad (2.1)$$

The average power $\overline{P_N}$ can be expressed in terms of the receiver noise temperature T_N, the bin width at the FFT B, and Bolzmann's constant k_B.

$$\overline{P_N} = k_B T_N B \quad (2.2)$$

$$\text{hence } \sigma P_N = \sigma(\delta) k_B T_N B \quad (2.3)$$

$$\text{and the signal power is given by } P_S = \frac{H_S}{\sigma(\delta)} \sigma(\delta) k_B T_N B \quad (2.4)$$

$$\text{or } P_S = H_S k_B T_N B \quad (2.5)$$

How does the system noise temperature affect the sensitivity? Consider: if an axion gives rise to power P_S at some frequency, the height H_S of the signal seen in the normalized trace is inversely proportional to the system noise temperature.

We explore the issue of the effect of system noise temperature by considering two otherwise equivalent traces at two different system noise temperatures. In both cases, a fake axion peak at the same conversion power is overlaid on the middle bin.

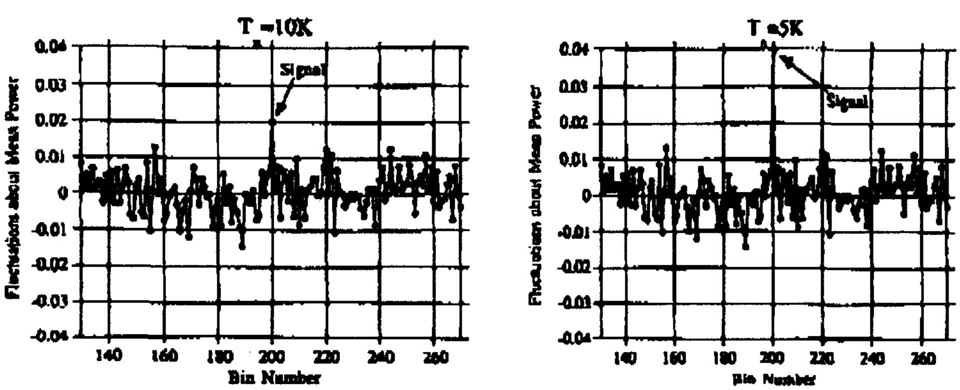


figure 2.2

As doubling the noise temperature doubles both the average noise power and the rms noise power fluctuations, the rms of the deltas, $\sigma(\delta)$, is the same in the two cases. However, the normalized signal height is degraded by a factor of two with a doubling of T_N. This is because the rms noise power fluctuation doubles, but the signal power remains the same.

It seems somehow more direct to have traces where a fixed signal power leads to a peak height independent of T_N with the rms noise fluctuation proportional to the system noise temperature. How do we achieve this? Just multiply the deltas by $k_B T_N B$. After this operation the traces from figure 2.2 are transformed into those in figure 2.3 below.

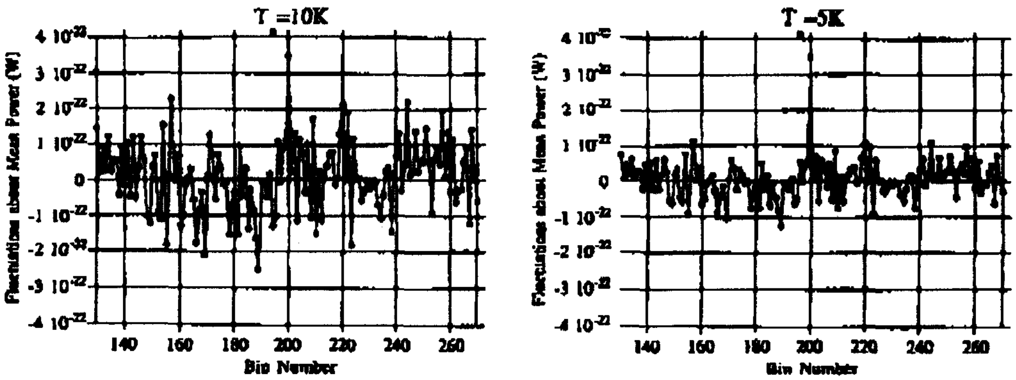


figure 2.3

Note that the fluctuations are now in units of Watts. Particularly note that the deltas are now power (in Watts) referenced to the input of the first cryogenic amplifier. If the rms of the fluctuations in noise power at the input of the first stage cryogenic amplifier is 10^{-22}W, then in the processed data stream, the rms fluctuation in the deltas is 10^{-22}W. This correspondence permits a direct and simple estimation of our axion power sensitivity.

3. Sensitivity from a Single Trace

I now give an example of determining power sensitivity from the rms noise fluctuations (which I will hereafter refer to as σ^W). I decide to search for a power excess in one bin of a single trace in file FFT_108. The receiver noise temperature for these data was 10.3K. In a typical trace, $\sigma(\delta)$ was around 6×10^{-3}. Thus

$$\sigma^W = k_B T_N B \sigma(\delta) = 1.4 \times 10^{-23} \times 10.3 \times 125 \times 0.006 \approx 10^{-22}\,\mathrm{W} \quad (3.1)$$

Knowledge of σ^W gives us a scale for fluctuations in a single trace. Suppose we decide to do a search for peaks of height $3\sigma^W$. From 3.1, this would correspond to a search for signals of power 3×10^{-22}W or greater.

For comparison, I'll calculate the power that a KSVZ type axion would deposit on the centre of the cavity TM_{010} mode resonance. The power deposited in the cavity from axion to photon conversion can be calculated using:

$$P_{a\rightarrow\gamma} = 2.4\times10^{-26}\,\mathrm{W}\left(\frac{V}{220l}\right)\left(\frac{B}{8.5T}\right)^2 C\left(\frac{g_\gamma}{0.97}\right)^2\left(\frac{\rho_a}{0.5\times10^{-24}\,\mathrm{g/cm^3}}\right)\left(\frac{\omega_a}{2\pi\mathrm{GHz}}\right)Q_c \quad (3.2)$$

Where V=220l, B=7.5T, C=0.56, g=0.97, $\rho_a = 0.5\times10^{-24}\,\mathrm{gcm^{-3}}$, ω_a=2π700MHz, Q_C=90,000 I obtain a power level of 3.5×10^{-22}W from axion to photon conversion. Apparently a KSVZ axion depositing power in a single bin would have been picked up in this data file as a peak more than $3\sigma^W$ above the mean power.

This exercise illustrates some concepts and procedures we will need when we calculate the sensitivity for a large data sample. Our sensitivity to single-bin axions at 98% confidence from a large data sample is calculated in section 7.

4. Combining Traces

The change in TM_{010} mode frequency between adjacent traces is typically one tenth of the width of the receiver passband. As a result, there may be many traces that contain a data point in the same 125Hz bin of frequency space. We must decide on a scheme by which these data points may be combined in a manner which maximizes our sensitivity. I wish to discuss three factors which determine the weight given to a data point in the combined data. Firstly, the system noise temperature T_N when the trace was taken. Secondly, the number of spectra averaged, N. And thirdly, the difference between the frequency of the data bin and the centre frequency of the TM_{010} mode.

As discussed in section 2, the initially dimensionless deltas are multiplied by kT_NB so they are the power fluctuations (in units of Watts) referenced to the input of the first cryogenic amplifier. Hence the rms deviation for the deltas is proportional to the system noise temperature. From the radiometer equation, the rms noise fluctuations are also proportional to $N^{-1/2}$. Thus the rms noise fluctuations in watts, σ^W, can be used to derive both N and T_N. Recall that after processing, the height of an axion induced signal at constant power is independent of the noise temperature. Hence the signal to noise ratio is inversely proportional to T_N and directly proportional to $\sqrt{N}$. This looks right.

Next, I discuss the effect of the position of the TM010 mode on the weighting of a data point from a trace. I define h as the ratio of the signal height in a bin from a-γ conversion, to the signal height seen if the bin is at the centre of the TM010 mode resonance. Thus if the bin is at resonance, h=1. If the bin is far off resonance, h=0. Between these two extremes, h is a Lorentzian in the difference between the frequency of the bin and the frequency of the resonance. Recall that in our experiment, we set the first local oscillator to centre the cavity resonance in the middle of the trace. If n is the bin number (see Fig 2.1 etc.), the bins are 125Hz wide and the cavity resonance is of width Γ, then h is given by:

$$h(n)=\frac{1}{1+\frac{4(125)^2(n-200)^2}{\Gamma^2}} \qquad (4.1)$$

The factor of 4 comes from the definition of Γ as the Lorentzian full width at half height.

I now compute the optimal weighted average for combining contributions to a single bin from two traces. Let the relevant bin in trace 1 have h=h_1 and let the trace have rms fluctuations $\sigma^W=\sigma_1^w$. Similarly, for the relevant bin from the second trace, h=h_2 and $\sigma^W=\sigma_2^w$. For the weighted average, I add delta from trace 1 to delta from trace 2 multiplied by c, a weighting factor. I am interested h and σ^W for the combined data. They are given by

$$h_{tot}=h_1+ch_2 \qquad (4.2)$$

$$\sigma_{tot}^w=\sqrt{\sigma_1^{w2}+c^2\sigma_2^{w2}} \qquad (4.3)$$

To maximize sensitivity in the combined data point, I maximize $f(c)=h_{tot}/\sigma_{tot}^w$ with respect to c. Figure 4.1 is a family of f(c) curves, for different σ and h.

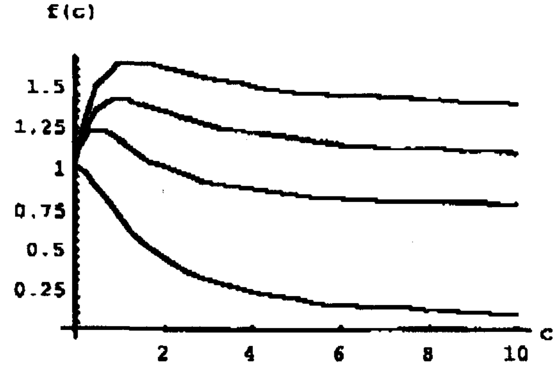


figure 4.1

$$\text{The maxima are at } c = \frac{h_2 \sigma_1^{w2}}{h_1 \sigma_2^{w2}} \quad (4.4)$$

Note that this is equivalent to the more familiar procedure of multiplying delta from the first trace δ_1 by h_1/σ_1^{w2} and δ_2 by h_2/σ_2^{w2}. Does this weighting scheme extend to the case of contributions from an arbitrary number of traces?

I will prove by induction that the extension of the above scheme to an arbitrary number of traces is valid. First assume that the method is optimal for the weighted average of deltas from k traces, where the bin from the i^{th} trace is parameterised by h_i and σ_i. Then h_{total} for the weighted sum of k traces is

$$h_{total} = h_1 \times \frac{h_1}{\sigma_1^{w2}} + h_2 \times \frac{h_2}{\sigma_2^{w2}} + \ldots + h_k \times \frac{h_k}{\sigma_k^{w2}} = \sum_i^k \frac{h_i^2}{\sigma_i^{w2}} \quad (4.5)$$

$$\text{Similarly, } \sigma_{total}^{W} = \sqrt{\sigma_1^{w2} \times (\frac{h_1}{\sigma_1^{w2}})^2 + \sigma_2^{w2} \times (\frac{h_2}{\sigma_2^{w2}})^2 + \ldots + \sigma_k^{w2} \times (\frac{h_k}{\sigma_k^{w2}})^2} = \sqrt{\sum_i^k \frac{h_i^2}{\sigma_i^{w2}}} \quad (4.6)$$

I now treat the delta from the weighted average as a single data point, and combine it with a delta from a k+1th trace, parameterised by h_{k+1} and σ_{k+1}. The weighting factor for delta from the combined traces is $h_{total}/\sigma_{total}^{w\,2} = 1$. The weighting factor for the k+1th delta is $h_{k+1}/\sigma_{k+1}^{w\,2}$ The overall h and σ^W are

$$h_{overall} = (\sum_i^k \frac{h_i^2}{\sigma_i^{w2}}) \times 1 + h_{k+1} \times \frac{h_{k+1}}{\sigma_{k+1}^{w\,2}} = \sum_i^{k+1} \frac{h_i^2}{\sigma_i^{w2}} \quad (4.7)$$

$$\text{Similarly, } \sigma_{overall}^{w} = \sqrt{(\sqrt{\sum_i^k \frac{h_i^2}{\sigma_i^{w2}}})^2 \times 1^2 + \sigma_{k+1}^{w\,2} \times (\frac{h_{k+1}}{\sigma_{k+1}^{w\,2}})^2} = \sqrt{\sum_i^{k+1} \frac{h_i^2}{\sigma_i^{w2}}} \quad (4.8)$$

These are of the same form as h and σ^W for the weighted average of k deltas. Hence we have shown that, given that our proposed method for combining deltas from k traces, the optimal weighted average of the resultant delta and that from the k+1th trace yields the same expression for the weighted average for k+1 traces. So if the scheme is optimal for k traces, it is also optimal for k+1 traces. Finally, we know it is optimal for k=1, hence our weighting scheme optimizes sensitivity in a weighted average of contributions from an arbitrary number of traces. This concludes the proof.

5. Signal Sensitivity in the Combined Data

How do we calculate the sensitivity in data combined as described in the previous section? Let us re-examine the calculation of sensitivity for a single trace in section 3. Recall that I posited that a-γ conversion resulted in excess power Ps being present in a single bin in the trace. I also showed that the size of the noise fluctuations was $\sigma^w = k_B T_N B \sigma(\delta)$. The power of a signal at the n_S sigma level in a single trace is $n_S\sigma^W$ Watts . I will extend this calculation to allow for the fact that, if the cavity Lorentzian is not centered on the bin in question, only a fraction hP_a of the available power from a-γ conversion will be deposited in the cavity. The signal level is modified, $P_S \to hP_a$, and the signal to noise ratio is:

$$\frac{P_S}{\sigma^w} \to \frac{hP_a}{\sigma^w} = \frac{P_a}{(\sigma^w/h)} \quad (5.1)$$

So if we're sensitive to signals in the cavity at a level $n_S\sigma^W$, then we're sensitive to axions at a level $n_S\sigma^W/h$. Notice that if a bin is far off resonance, h=0 and we're very insensitive to axions. If we're right on resonance, the sensitivity is maximized. So, this seems right.

I now make the obvious extension to the case where we are looking at a bin in the combined data stream instead of a single trace. The signal to noise ratio in the combined data is

$$\frac{P_S}{\sigma^w_{total}} = \frac{h_{total}P_a}{\sigma^w_{total}} = \frac{P_a}{(\sigma^w_{total}/h_{total})} = \frac{P_a}{\left(\frac{1}{\sqrt{\sum_i^n \frac{h_i^2}{\sigma_i^{w2}}}}\right)} \quad (5.2)$$

In exact analogy to the single trace calculation above, the power in Watts corresponding to $n_S\sigma^W$ peaks in the combined data stream is:

$$P_{sensitivity}(W) = \frac{n_S}{\sqrt{\sum_i^n \frac{h_i^2}{\sigma_i^{w2}}}} \quad (5.3)$$

This formula also looks right. If the h's are all small, the minimum power you are sensitive to is large, as expected. If the noise fluctuations σ^W are all small (as is the case with low noise temperatures and lots of spectra averaged in each trace), the minimum power you are sensitive to is small, as expected.

Eventually, we need to search for peaks. This search is conceptually easier if the deltas are in units of the standard deviation. We can put delta in this form by dividing the delta from the weighted average by sigma for that bin.

$$\frac{\delta_j}{\sigma_j^w} = \frac{\sum_i^n \frac{h_i \delta_i}{\sigma_i^{w^2}}}{\sqrt{\sum_i^n \frac{h_i^2}{\sigma_i^{w^2}}}} \qquad (5.4)$$

Here index j labels jth bin in the combined data streams and index i labels parameters pertaining to the ith trace contributing to the jth bin.

6. Algorithm for Combining the Raw Data

The following is a summary of the method by which I combine the raw data into a single data stream and derive our sensitivity from it. The algorithm is implemented in C.

Normalize each raw trace with the long average and crop off bins outside the crystal filter passband

For each trace read/measure:
1) Γ, the width of the TM010 resonance.
2) T_N, the system noise temperature.
3) N, the number of averaged spectra per trace.
4) $\sigma(\delta)$, the dimensionless rms fluctuations

Multiply the fluctuations $\sigma(\delta)$ in each raw trace by $k_B T_N B$.

Make 2 combined data streams.

1. $\sum_i^n \frac{h_i \delta_i}{\sigma_i^{*2}}$ 2. $\sum_i^n \frac{h_i^2}{\sigma_i^{*2}}$

where δ_i is the fluctuation from the ith trace
h_i is the lorenzian height in the bin in question
σ_i are the rms fluctuations for the ith trace

Sensitivity in jth bin

$$P_{sensitivity}(W) = \frac{n_c}{\sqrt{\sum_i^n \frac{h_i^2}{\sigma_i^{*2}}}}$$

Data Stream for Peak Search

$$\frac{\delta_j}{\sigma_j^*} = \frac{\sum_i^n \frac{h_i \delta_i}{\sigma_i^{*2}}}{\sqrt{\sum_i^n \frac{h_i^2}{\sigma_i^{*2}}}}$$

7. Sensitivity for a Single Bin Peak Search

By sensitivity I mean the minimum power level at which axion to photon conversion could deposit excess power in our cavity that would be detected in our data analysis at 98% confidence. The sensitivity of the experiment to axions depends not only on the total axion to photon conversion power, but also on the particular hypothesis you take for the axion line shape. I will first take the simplest case, where I assume that all the power is deposited in a single 125Hz bin.

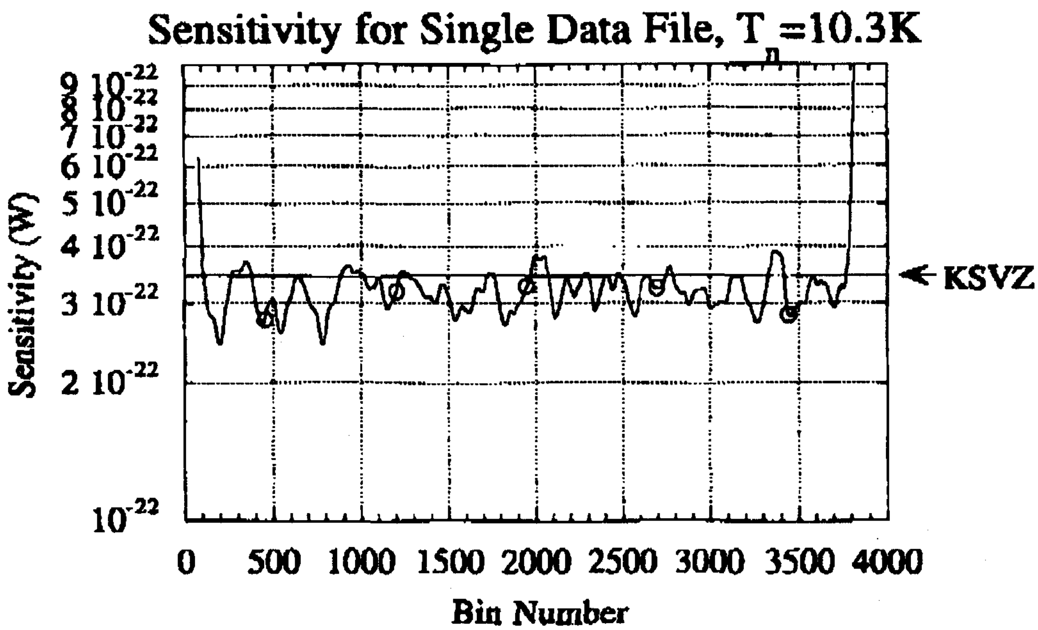
To search for single bin power excess, one would simply go through the combined data and examine all bins where delta was some number of sigma above the mean. These bins would then be re-scanned to see if the power excess is persistent. The search is more sensitive if you look at all bins at the 2 sigma level, than if you only look at the 4 sigma bins, but you also have to think about how many bins you can afford to re-scan. For instance, if you have data from 100MHz of frequency space, that is 800,000 125Hz bins. From Gaussian statistics, you expect 2% of these bins to be upwards 2 sigma fluctuations, so if your search was at the 2 sigma level, you'd be re-scanning 16,000 bins. This is impractical.

Using Chris Hagmann's criterion, I start by requiring that in 100MHz of frequency space, we wish to re-scan 100 bins. Thus, in any single bin, the probability of a re-scan is $100/800{,}000 = 1.25\times10^{-4}$. This probability corresponds to 3.6sigma. So if I look for 3.6sigma bins in a 100MHz bandwidth, I should find 100 of them. If axions deposit power in our detector at the 3.6 sigma level, the probability that then will generate an upwards 3.6 sigma fluctuation is 0.5. This is because the excess power from axions must be added to the receiver noise in the same bin, which may fluctuate above or below the mean power, with equal probability. For a 98% confidence that we will detect the power excess at 3.6 sigma, the power excess from axions must be at the (3.6+2)=5.6sigma level. At this level, in order for us not to detect the excess power from axions, the receiver noise in that bin would have to be a -2sigma fluctuation. The probability of this is 2%. Hence the probability of seeing the 5.6sigma axion in a 3.6 sigma single bin peak search is 98%.

Using the formula (5.3) for sigma from my data combining algorithm, I compute the sensitivity from my combined data, with $n_s=5.6$:

$$P_{[illegible]} = \frac{5.6}{\sqrt{\sum_{i}^{n} \frac{h_i^2}{\sigma_i^{w2}}}} \qquad (7.1)$$

A plot of the sensitivity for a single data file with T_n=10.3K, where the number of averages was 30,000, is shown below. The file is FFT_108, data from Feb 12th.



The KSVZ line is the power from a→γ conversion due to KSVZ axions (see section 3).

8. Conclusion and Future Work

I have developed an analysis program which I believe maximizes sensitivity to interesting signals in the combined data, providing us with a data stream in which to conduct peak searches and a mechanism for deducing our sensitivity. The next stage is to combine all our data into a single stream, decide on what to search for and generate a list of frequencies for re-scanning.

I would welcome comments on the above analysis, suggestions for improvements and ideas for the future. For those of you not at the lab, my telephone numbers and email address are on the cover page.