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- LIGO -
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Length Control RMS Deviations from Resonance

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1 INTRODUCTION

The function of the LIGO Length Sensing and Control (LSC) subsystem is to maintain the interferometer lengths sufficiently close to exact resonance so that the LIGO primary sensitivity requirements are satisfied. This note derives the allowable RMS deviations from exact resonance allowed.

The LIGO primary sensitivity requirements are[1]

- x (100 Hz) = 1 x 10^{-19} m / $Hz^{1/2}$
- x (10 kHz) = 4 x 10^{-18} m / $Hz^{1/2}$

The lengths considered are

- L_- : Arm cavity differential mode
- l_- : Michelson cavity differential mode
- L_+ : Arm cavity common mode
- l_+ : Michelson cavity common mode

We assume the following set of IFO parameters in these calculations:

Table 1: Input Parameters

Symbol	Value	Description
P_{in}	6 W	IFO input power
P_{DC}	450 mW	GW port sideband power
Pex	130 mW	GW port excess power
G_{RC}	30	recycling cavity gain
r_0	0.98	arm cavity carrier reflectivity
r_F	0.985	arm cavity input mirror reflectivity
r_R	0.98	recycling mirror reflectivity
$\Delta I / I$	1 x 10^{-8} / $Hz^{1/2}$	Intensity Noise
k	6.7 x 10^6 m^{-1}	light wave number

1. LIGO Science Requirements Document : LIGO-E950018-02-E

2 ALLOWED RMS DEVIATIONS FROM RESONANCE

2.1. Common Mode Lengths

A common mode deviation from resonance causes a decrease in the amplitude of the cavity electric field. The resultant loss of sensitivity will set the allowed rms deviation.

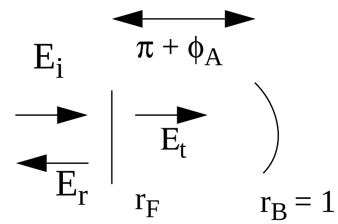
We have:

$$\frac{S}{N} \sim \frac{E_0 E_1}{\sqrt{P_1 + P_{ex}}}$$

where E_0 is the arm cavity field, E_1 is the sideband recycling cavity field, P_1 is the sideband power at the GW photodetector, and P_{ex} is excess noise power. We conservatively assume that P_{ex} dominates the shot noise and examine the effect of a *common mode* rms offset on E_0 and E_1.

We first consider the arm cavity:

$$r_c = \frac{E_r}{E_i}$$



We have:

$$E_t = E_i t_F + E_t \; r_F (1 + i\phi_A)$$

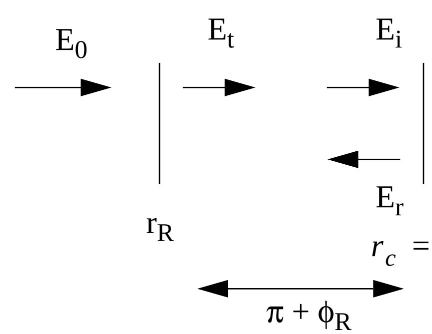
and

$$E_r = E_i r_F - E_t t_F (1 + i\phi_A)$$

Thus

$$r_C = \frac{E_r}{E_i} = r_F - \frac{t_F^2 (1 + i\phi_A)}{1 - r_F (1 + i\varphi_A)}$$

Now the recycling cavity:



$$r_c = \frac{E_r}{E_i}$$

$$E_t = \frac{E_0 t_R}{1 + r_R r_C (1 - i\varphi_R)}$$

$$= \frac{E_0 t_R}{1 + r_R\left[r_F - \frac{t_F^2(1 + i\phi_A)}{1 - r_F(1 + i\varphi_A)}\right](1 - i\varphi_R)}$$

$$= \frac{E_0 t_R}{1 + r_R\left[\frac{r_0 + \frac{(r_F - t_F^2)i\varphi_A}{1 - r_F}}{1 + \frac{i\varphi_A}{1 - r_F}}\right](1 - i\varphi_R)}$$

where $r_0 = -r_F + \frac{t_F^2}{1 - r_F}$, as usual.

Thus for $\varphi_A \ll 1 - r_F$, we can write

$$E_t \sim \frac{E_0 t_R}{1 + r_R\left[\left(r_0 + r_F\frac{i\varphi_A}{1 - r_F}\right)\left(1 - \frac{i\varphi_A}{1 - r_F}\right) - i\varphi_R r_0\left(1 - \frac{i\varphi_A}{1 - r_F}\right)\right]}$$

$$= \frac{E_0 t_R}{(1 + r_R r_0)\left[1 + \frac{i2\varphi_A}{(1 - r_F)(1 + r_R r_0)r_0} - \frac{i\varphi_R r_0}{1 + r_R r_0}\right]}$$

where we have neglected terms of order ϕ^2.

Finally, with E_R the recycling cavity field on resonance, we have:

$$E_t = \frac{E_R}{1 + i\left[\frac{2\varphi_A}{(1 - r_F)(1 + r_R r_0)r_0} - \frac{\varphi_R r_0}{1 + r_R r_0}\right]}$$

The above expression gives the recycling cavity field in terms of the arm and recycling cavity phase deviation from resonance.

We require < 0.5% degradation in GW signal to noise due to deviation from resonance.

Thus $|E_t|^2 / |E_R|^2 = 0.99$ from ϕ_A

$= 0.995$ from ϕ_R since both carrier and sideband field decrease

Thus:

$$\left[\frac{2\varphi_A}{(1 - r_F)(1 + r_R r_0)r_0} - \frac{2\varphi_R r_0}{1 + r_R r_0}\right]^2 = 0.01$$

and

$$\frac{2\varphi_A}{(1-r_F)(1+r_R r_0)} = \frac{2\varphi_R}{(1+r_R r_0)} = 0.1$$

With:

r_0 = -0.98, r_F = 0.985, r_R = 0.98

We derive:

$\phi_A = 3 \times 10^{-5}$ rad and $\phi_R = 2 \times 10^{-3}$ rad

and with $\phi_A = 2\,k\,L_+$ and $\phi_R = 2\,k\,l_+$

we find finally:

2.1.1. L_+

$L_+ = 2.5 \times 10^{-12}$ m

2.1.2. l_+

$l_+ = 1.6 \times 10^{-10}$ m

2.2. Differential Mode Lengths

2.2.1. Contrast Defect

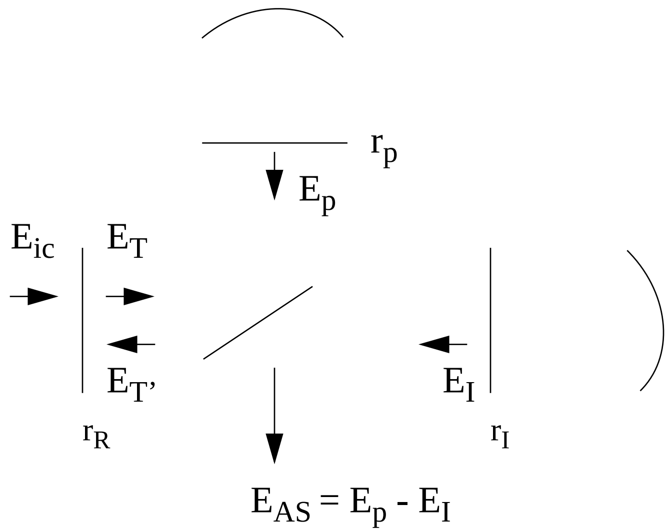
We first consider the coupling of a differential length change to excess shot noise at the dark port.

r_P, r_I : complex arm reflectivities

r_R : recycling mirror reflectivity

E_{AS} : carrier field at dark port

l_I, l_P : round-trip length in recyc cavity for carrier



we have[1]:

$$E_{AS} = \frac{\frac{E_{ic}}{2} t_R (r_P(1-ikl_p) - r_I(1-ikl_I))}{1 + \frac{r_R}{2}(r_P(1-ikl_p) + r_I(1-ikl_I))}$$

with[2] $r_{P,I} = \dfrac{r_0 + i\frac{2kL_{P,I}}{1-r_F}}{1 + i\frac{2kL_{P,I}}{1-r_F}}$ and $L_P + L_I = l_P + l_I = 0$ (we assume zero common mode length)

$$E_{AS} = \frac{\frac{E_{ic}}{2} t_R ik\left(\frac{2(L_P - L_I)}{1-r_F} - r_0(l_P - l_I)\right)}{1 + r_R r_0}$$

1. Frequency, Intensity and Oscillator Noise in the LIGO : LIGO T960019-00D, pg. 5-6
2. Ibid, app. 1

$$= \frac{E_{RC}}{2} ik\left(\frac{2(L_P - L_I)}{1 - r_F} - r_0(l_P - l_I)\right)$$

We require $P_{AS} = |E_{AS}|^2 \sim 1\%\ P_{DC}$ (P_{DC} is the GW port total power)

With $P_{RC} = G_{RC}\ P_{in} = 180$ W and $P_{DC} = 600$ mW[1], we find:

$130\ L_- = l_- = (2 * 0.006 / 180.)^{1/2} / k$ Thus:

2.2.1.1 l-

$l_- = 1.3 \times 10^{-9}$ m

2.2.1.2 L-

$L_- = 1.0 \times 10^{-11}$ m

2.2.2. Intensity Noise Coupling

We now consider intensity noise coupling to a differential length offset.

2.2.2.1 L_-

The noise coupling to L_- is[2]

$\Delta x = (\Delta I / I)\ x_{rms} = (\Delta I / I)\ L_-$

With the assumptions that:

- Intensity noise stabilization gives $(\Delta I / I) = 1 \times 10^{-8} / Hz^{1/2}$
- $\Delta x < 0.1$ LIGO sensitivity

we find $L_- = 10^{-12}$ m / $Hz^{1/2}$

2.2.2.2 l_-

The noise coupling to l_- is[3]

$\Delta x = (1/130)\ (\Delta I / I)\ x_{rms} = (1/130)\ (\Delta I / I)\ l_-$

we find $L_- = 1.3 \times 10^{-10}$ m / $Hz^{1/2}$

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1. Shot noise in the Length Error Signals: LIGO-T960042-00-D
2. Frequency, Intensity and Oscillator Noise in the LIGO : LIGO T960019-00D, sec. 6.3
3. Frequency, Intensity and Oscillator Noise in the LIGO : LIGO T960019-00D, app. 3

3 SUMMARY

We have investigated the coupling of rms deviations from length control resonances to decreased stored power, shot noise and intensity noise. The results which give the tightest constraints are:

Table 2: Requirements on RMS Deviations from Resonance

Length	*Allowed RMS deviation*	*Noise Mechanism*	*GW S/N degradation*
L_-	1×10^{-12} m	intensity noise -> strain noise	0.5 %
L_+	4×10^{-12} m	loss of 1% arm stored power	0.5 %
l_-	1.3×10^{-10} m	intensity noise -> strain noise	0.5 %
l_+	5×10^{-10} m	loss of 1% arm stored power	0.5 %