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Calculation of the Modulation Frequency for the 40m Power Recycling Interferometer

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1 Purpose

The purpose of this document is to define a numerical value for the RF modulation frequency based on recycling cavity length.

2 Scope

The document provides results of the recent measurement of cavity lengths, calculations of the modulation frequency and estimation of the errors.

3 Measurements of Length

To find the length of the inline and perpendicular cavities the distances between several reference points inside the vacuum envelope were measured. These points were chosen at the tops of suspension towers because of easy access. For details see LIGO 40m Logbook # 42, p.18w.

The distances between the centers of the reflecting surface of the beam splitter and HR-coated surfaces of the vertex mirrors are

$$s_1 = 224.9\,\mathrm{cm}, \quad (1)$$
$$s_2 = 172.1\,\mathrm{cm}. \quad (2)$$

The geometrical length of the inline and perpendicular cavities is different from s_1 and s_2 by the distance between the beam splitter and the recycling mirror. This distance is determined by the available space in the beam splitter chamber and is chosen to be

$$s_0 = 25.0\,\mathrm{cm}.$$

Thus the geometrical length of the inline and perpendicular cavities are

$$s_1 + s_0 = 249.9\,\mathrm{cm}, \quad (3)$$
$$s_2 + s_0 = 197.1\,\mathrm{cm}. \quad (4)$$

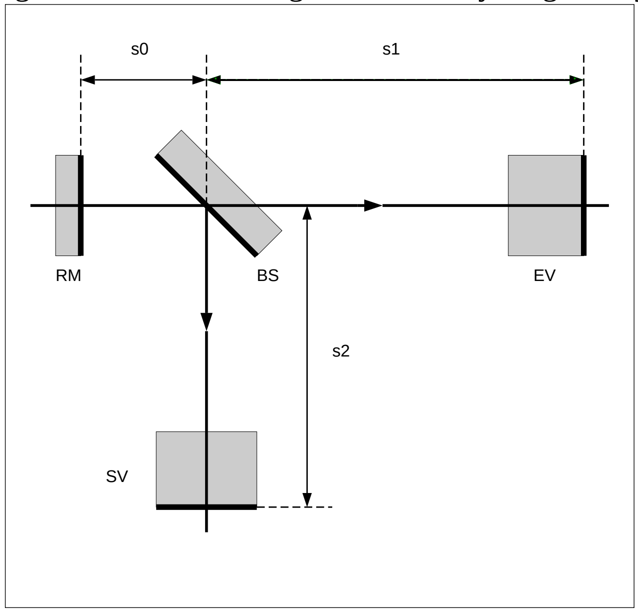
4 Effect of Glass

The optical length is different from the geometrical length due to refraction of the laser beam in the glass.

The index of refraction of fused silica at a wavelength of λ_0 = 514.5 nm is

$$n = 1.46.$$

Figure 1: Different Lengths in the Recycling Cavity



The substrates for both the recycling mirror (RM) and the beam splitter (BS) are 1-inch thick. The thickness of the test masses (TM) is 3.5 inches. Thus the thicknesses of different optical elements are

$$H_{RM} = 2.5 \text{ cm}, \tag{5}$$

$$H_{BS} = 2.5 \text{ cm}, \tag{6}$$

$$H_{TM} = 8.9 \text{ cm}. \tag{7}$$

In the perpendicular cavity the light goes through the recycling mirror and the test mass. The total distance that the light travels in the glass in the perpendicular cavity is $(H_{RM} + H_{TM})$. Thus the correction due to the refraction index for the perpendicular cavity is

$$\Delta_2 = (n-1)(H_{RM} + H_{TM}) = 5.2 \text{ cm}.$$

For the inline cavity, in addition to the recycling mirror and the test mass the light travels through the beam splitter. The correction is

$$\Delta_1 = (n-1)(H_{RM} + H_{TM}) + \left(\sqrt{n^2 - \sin^2\theta} - \cos\theta\right) H_{BS}.$$

Since the beam is incident on the beam splitter is at an angle $\theta = 45^0$ to the incident beam

$$\Delta_1 = 6.6 \text{ cm}.$$

5 Recycling Cavity Length and Asymmetry

The optical lengths for the inline and perpendicular cavities are

$$l_1 = s_1 + s_0 + \Delta_1 = 256.5 \text{ cm}, \tag{8}$$

$$l_2 = s_2 + s_0 + \Delta_2 = 202.3 \text{ cm}. \quad (9)$$

The recycling cavity length is

$$l = \frac{1}{2}(l_1 + l_2) = 229.4 \text{ cm}.$$

The asymmetry length is

$$\delta l = l_1 - l_2 = 54.2 \text{ cm}.$$

6 Resonance Conditions in the Recycling Cavity

The condition for the carrier and sidebands to resonate in the recycling cavity is

$$l = N_0 \frac{\lambda_0}{2}, \quad (10)$$

$$l = N_1 \frac{\lambda_1}{2} + \frac{\lambda_1}{4}. \quad (11)$$

where λ_0 and λ_1 are the wavelength of the carrier and the upper sideband. N_0 and N_1 are two integer numbers.

The frequency of the upper sideband is different from the frequency of the carrier by the modulation frequency

$$f_1 = f_0 + f_{mod}.$$

The above resonance conditions define the modulation frequency

$$f_{mod} = (N_1 - N_0)\frac{c}{2l} + \frac{c}{4l}.$$

The lowest modulation frequency is obtained when $N_1 = N_0$:

$$f_{mod} = \frac{c}{4l}.$$

For the recycling cavity with the length $l = 229.4$ cm the lowest modulation frequency is

$$f_{mod} = 32.7 \text{ MHz}.$$

sometimes it is interesting to know the wavelength of one modulation cycle

$$\lambda_{mod} = 4l = 917.6 \text{ cm}.$$

7 Accuracy of the Calculations

The largest error in the calculation comes from the measurement error. The error in the measurements is due to the fact that the optics are not easily accessible and therefore other reference points have to be found. This error is estimated to be 6 cm. The drawing itself is not accurate and contributes an error, at most of a few millimeters. The error in the drawing is negligible compared to the error of the measurements. The measurement error results in an error in the modulation frequency of approximately 0.5 MHz. Thus the final result is

$$f_{mod} = 32.7 \pm 0.5 \text{ MHz}$$