Specifications for the Baffle Serrations in the LIGO Beam Tubes (LIGO Technical Report LIGO-T960012-00-R)

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This document specifies the distributions of serrations to be used on the three families of baffles in the LIGO beam tubes.

I. OVERVIEW

In a meeting at Caltech on December 12, attended by Albert Lazzarini, Kip Thorne, Rai Weiss, and Stan Whitcomb, it was decided that the LIGO beam-tube baffles should come in three types. We shall call these *Full-Serration or* **FS** *Baffles*, *Reduced-Serration or* **RS** *Baffles*, and *Shallow-Serration or* **SS** *Baffles*.

The FS baffles will be placed in the central regions of the beam tube, where the Fresnel-zone widths for 4km interferometers are largest.

The RS baffles will be placed near the end and corner stations, where the Fresnel-zone widths are narrower. (By switching from FS to RS baffles near the end and corner stations, we are able to reduce the total number of baffles at Hanford plus Livingston by about 48, thereby saving roughly $48,000.)

Near the Hanford midstation, if the FS baffles are placed close enough together to hide the wall as seen by midstation mirrors, then as seen by 4km interferometers, the serrations of successive FS baffles will shadow each other seriously and thereby will lose their ability to average down the diffraction noise. To avoid this shadowing, near the midstation the FS baffles will be placed only at beam-tube supports; and to hide the wall as seen from the midstation, we will place SS baffles between the FS baffles.

The details of placement of the FS, RS, and SS baffles are worked out and spelled out in a Mathematica notebook entitled *Baffle Locations for LIGO* [1].

In the present document and two addenda (which take the form of ascii data files [2,3]), we spell out the baffle heights and serration distributions for the FS, RS, and SS baffles. In the following sections, we shall deal with each baffle type in turn. The baffle specifications are as indented items using italic type and bold-face labels, and are discussed in the text in Roman type.

II. FS BAFFLES

A. Shape and Mean Height of Serration Teeth

The serrations consist of saw teeth with straight edges. The purpose of these saw teeth is to reduce the diffraction noise from scattered light in the LIGO beam tube, by averaging the oscillatory light-wave field along the edge of a tooth. This averaging reduces the noise by a factor $2/(\pi N_z)$, where N_z is the number of Fresnel zones subtended by the tooth edge (more precisely, the number of Fresnel zones of the light that propagates from one interferometer mirror to the tooth edge and then, upon diffraction, on to the other mirror.)

The width of a Fresnel zone, in the beam tube's vertical plane (i.e. the transverse distance over which the light phase changes by π) is

$$w_F = \frac{\lambda L \beta}{8Y} = \frac{\beta}{Y/50\text{cm}}\, 0.1\text{cm}\ , \tag{1}$$

where λ (assumed 1.06μm) is the wavelength of light, L (assumed 4km) is the interferometer arm length, Y is the distance from the main beam's central axis, and

$$\beta = \frac{4l(L-l)}{L^2} = 1 \quad \text{at tube midpoint}$$
$$\simeq \frac{l}{L/4} \quad \text{near tube ends} \tag{2}$$

is a parameter that depends on the baffle's distance l from the nearest interferometer mirror. The serration teeth have variable height (see the next section).

FS1 *The mean height ΔH of a serration tooth in the beam tube's vertical plane is chosen to be*

$$\Delta H = 0.8\text{cm}\ , \tag{3}$$

so it will subtend $N_z = 8$ Fresnel zones for FS baffles near the tube midpoint, and $N_z = 8/\beta > 8$ zones for FS baffles away from the midpoint.

B. Distribution of Serration Peaks and Valleys: Parameters and Formulae

If the interferometer's main beam is significantly off-center from the tube's central axis, then the pattern of Fresnel zones on the baffle tops manages, without help, to ensure that light which diffracts off different locations on a baffle superposes incoherently. However, for nearly centered beams, there is danger of coherent superposition. By making the heights of the serration peaks and valleys variable, we can ensure that such coherent superposition never occurs. Forthermore, by rotating the randomly serrated baffles relative to each other, we can ensure incoherence from one baffle to the next. The baffle rotation is discussed in our specification document for baffle locations [1]; the randomness of the serrations is discussed here.

Each serration peak or valley (and adjacent piece of serration tooth) contributes to the diffracted light field an amount whose phase factor is $\exp[i(\pi s_{\rm pv}/w_F)+\text{const}]$, where $s_{\rm pv}$ is the height of the peak or valley above the mean baffle top, as measured in the beam tube's vertical plane. To ensure that this phase factor is random from one peak to the next, and one valley to the next, we choose $s_{\rm pv}$ from a probability distribution with rms fluctuations $\pm\delta H/2$, so the phase factor $\Phi_{\rm pv} = \pi s_{\rm pv}/w_F$ has rms fluctuations distributed over the range $-\pi(\delta H/2)/w_F$ to $+\pi(\delta H)/2$, i.e. a total range

$$\delta\Phi_{\rm pv} = \pi\delta H/w_F. \tag{4}$$

FS2 *The scale of serration height fluctuations, measured in the beam tube's vertical plane, is chosen to be*

$$\delta H = 0.2\text{cm} \tag{5}$$

so that for centered beams (the dangerous case) the rms range of phase fluctuations is $\delta\Phi_{\rm pv} \geq 2\pi$ (with a value 2π for baffles at the beam-tube midpoint).

The probability distribution from which the peak and valley heights is drawn is chosen to be Gaussian:

FS3 *The probability to find a peak with height $s_{\rm p}$ above the mean baffle top in range $ds_{\rm p}$ is*

$$\frac{dP}{ds_{\rm p}} = \frac{1}{\sqrt{2\pi(\delta H/2)}} \exp\left(\frac{-(s_{\rm p} - \Delta H/2)^2}{2(\delta H/2)^2}\right) , \tag{6}$$

and

FS4 *The probability to find a valley with height $s_{\rm v}$ above the mean baffle top in range $ds_{\rm v}$ is*

$$\frac{dP}{ds_{\rm v}} = \frac{1}{2\pi(\delta H/2)} \exp\left(\frac{-(s_{\rm v} + \Delta H/2)^2}{2(\delta H/2)^2}\right) . \tag{7}$$

This Gaussian distribution has the nice feature that, when we make the additional specification

FS5 *The baffle edges all have the same pitch angle $\mu \equiv$ (angle between tooth edge and mean baffle top, measured in tube's vertical plane; cf. Fig. 2 below) independently of the heights of the adjacent peaks and valleys,*

then for a centered main beam and axisymmetric mirrors (the most dangerous case of all), the diffraction noise amplitude becomes a Fourier transform of the Gaussian, which in turn is a Gaussian,

$$(\text{noise amplitude}) \propto \exp\left[-\frac{1}{2}\left(\frac{\pi\delta H/2}{w_F}\right)^2\right] . \tag{8}$$

This gives much better noise suppression than would a uniform probability distribution.

The diffraction noise decreases with increasing pitch angle μ, though not terribly rapidly (typically no faster than $\propto 1/\sqrt{\tan\mu}$).

FS6 *We specify the constant pitch angle to be*

$$\mu = \arctan(8/3) = 69^{\rm o} , \tag{9}$$

so the mean azimuthal length of a baffle half tooth is

$$\bar{b} = (3/8)\Delta H = 0.3\text{cm} ; \tag{10}$$

cf. Fig. 2 below.

A specific realization of the above serration specifications has been generated using a Mathematica notebook that accompanies this report [4]; those serrations are listed in an ascii file [2] that is printed as Addendum FS to this report. The format of that serration table is explained in the next two sections (Secs. II C and II D).

C. Baffle Segments and Their Fabrication

FS7 *The baffle surfaces are cones that lean away from the nearest mirror, with at an opening angle of $55 \pm 2^{\rm o}$ to the vertical plane ($35 \pm 2^{\rm o}$ to the beam-tube axis).*

FS8 *Each baffle is to be fabricated by but-welding together four baffle segments. When unrolled into the flat sheets from which they are fabricated, these baffle segments are identical pieces of an anulus with the basic shape shown in Fig. 1(a).*

FS9 *Although the baffle segments are identical, their faces must be alternated, so the serration peaks (or valleys) at the ends of adjacent segments match up at the welds.*

FS10 *When each baffle is mounted into the beam tube, the serration teeth at the baffle ends must not be allowed to overlap each other and thereby shadow each other. To avoid this, one set of teeth in the overlap region should be cut off.*

FS11 *The outer radius of each baffle segment is specified, as shown in Fig. 1(a), to be 42.38 inches.*

(Note: We specify all lengths in the baffle plane in inches, and all lengths in the tube's vertical plane in centimeters.) This outer radius is taken from Albert Lazzarini's specifications for the prototype baffles, and can be altered if so desired, so long as the specified heights in the beam tube's vertical plane are unchanged.

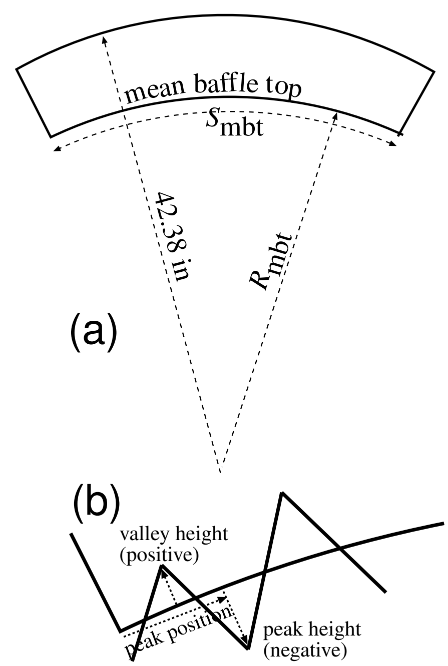

FIG. 1. (a) The shape and dimensions of a baffle segment, before serrations are added. (b) The parameters that characterize the serration distribution in the table given in Addendum FS [2].

We use the phrase "mean baffle top" to describe the arc, in the baffle plane, that is mid way between the baffle peaks and valleys.

FS12 *The radius $R_{\rm mbt}$ of the mean baffle top, in the baffle plane, must be chosen in such a way that the height $H_{\rm max}$ of the highest baffle peak, in the tube's vertical plane, is*

$$H_{\rm max} = 9.0 \pm 0.5{\rm cm}\ . \tag{11}$$

The tolerance of 0.5cm (to accommodate inaccuracies in the 35° baffle tilt and in the baffle's attachment to its base) was agreed upon in the December 12 meeting.

The highest baffle peak turns out to be a distance $\Delta H/2 + 2.2\delta H/2 = 0.62$cm above the mean baffle top (in the tube's vertical plane). Correspondingly,

FS13 *We specify the height for the mean baffle top (in the tube's vertical plane) to be*

$$H_{\rm mbt} = H_{\rm max} - \frac{\Delta H}{2} - 2.2\frac{\delta H}{2} = 8.38 \pm 0.50{\rm cm}\ . \tag{12}$$

If we were to allow 0.30cm of vertical height for the baffle's base and attachment of the baffle to it, then the radius of the mean baffle top in the baffle plane (Fig. 1 would be

$$R_{\rm mbt} = 42.38{\rm in} - \frac{H_{\rm mbt} - 0.30{\rm cm}}{(2.54{\rm cm/in})\sin(35^{\rm o})} = 36.84{\rm in} \tag{13}$$

FS14 *The total azimuthal length of each baffle segment, along its mean baffle top, is*

$$S_{\rm mbt} = 35.13{\rm in}\ . \tag{14}$$

This is the length specified in the second column of the serration table (Addendum FS [2]). When combined with our estimate (13) of the radius of the mean baffle top in the baffle plane, this gives the same angular size $35.13/36.84 = 0.953{\rm rad} = 54.6^{\rm o}$ for a baffle segment, in the baffle plane, as was used in the prototype baffles.

D. Table and Tolerances of Serration Peaks and Valleys

We specify our specific realization of the serration distribution (Addendum FS [2]) as a table of peak and valley azimuthal positions and peak and valley heights, defined along and relative to the mean baffle top, in the manner of Fig. 1(b). Random errors in the peak and valley positions and heights will not substantially affect the diffraction noise, so somewhat arbitrarily we specify tolerances of 0.01inch:

FS15 The positions and heights of the serration peaks and valleys should be as given in Addendum FS [2] to within accuracies ±0.010inch.

These tolerances can be relaxed if they are a problem.

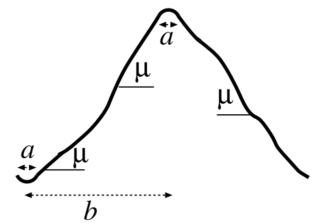

FIG. 2. The geometry of an imperfect serration tooth.

More serious are tolerances on the the straightness of the tooth edges and sharpness of the peaks and valleys. The tolerances on the these are driven by our desire to reduce the diffraction noise by a factor $2/(\pi N_z) \lesssim 1/10$ via averaging the light field along a tooth edge. To guarantee a reduction by at least a factor 10, we impose two constraints (see Fig. 2):

FS16 *The cotangent of the serrations' pitch angle μ (the angle of the tooth edge to the mean baffle top), averaged over 1/10 of a half tooth, should be constant along the tooth to within a precision $\Delta \cot\mu = \pm 0.10 \cot\mu$.*

This specification is the same whether imposed in the tube's vertical plane or in the baffle plane, since $\cot\mu$ is the same in the two planes up to a multiplicative constant $0.57 = \sin 35^{\rm o}$.

FS17 *The radii of curvature of the teeth's rounded peaks and valleys, in the baffle plane, should be $\lesssim 1/64$inch $\simeq 0.04$cm.*

This specification corresponds to an azimuthal length $a \simeq 0.06$cm for the teeth's rounded peaks and valleys (in both the baffle plane and the tube's vertical plane; cf. Fig. 2). Since the azimuthal length of a half tooth is $b \simeq 0.3$cm [Eq. (10)], this specification allows half of each rounded peak or valley to contribute to the noise a fraction $\frac{1}{2}a/b \sim 1/10$ of that which its adjacent half tooth contributes, before the "$2/(\pi N_z)$" averaging.

E. Shadowing of the Tube Wall

For our realization of the FS baffle serrations (Addendum FS [2]), the difference in height between the highest peak and the lowest valley is

$$H_{\rm max} - H_{\rm min} = \Delta H + 2.4\delta H = 1.28{\rm cm} \tag{15}$$

in the tube's vertical plane. Correspondingly, the lowest valley is at a height

$$H_{\rm min} = 9.0 - 1.28 = 7.72 \pm 0.5{\rm cm} \tag{16}$$

above the tube wall.

At the meeting of 12 December it was agreed that we should insist that test mass mirrors not be able to see the lowest 0.5 cm of a baffle, because of glint due to its attachment to its base. This means that in computing the baffle separation [1], the effective height of a FS baffle in the tube's vertical plane should be taken to be

$$h_{\rm FS} = 7.72 - 0.5 - 0.5 = 6.72{\rm cm}\ . \tag{17}$$

The first factor 0.5cm is for the tolerance in the baffle height (due in large measure to tolerance in 35° tilt). The second factor 0.5cm is for hiding the baffle's base.

III. RS BAFFLES

A. Choice of Tooth Heights for FR Baffles

As one moves from the tube midpoint toward a corner or end station, the width w_F of the Fresnel zones decreases [Eqs. (1) and (2)], and correspondingly, to achieve the same noise reduction as near the midpoint, one can make do with shorter serration teeth. This, in turn, allows the baffles to be farther apart [1], thereby reducing the number of baffles. At the December 12 meeting it was decided, for simplicity of installation, to use just two two serration depths, those of Full-Serration (FS) baffles and those of Reduced Serration (RS) baffles.

The following considerations dictate the tooth heights for the RS baffles:

Let l be the distance from the nearest mirror, at which the transition from FS baffles to RS baffles is made. Then we choose the heights of the serration teeth for the RS baffles in such a way that an RS tooth at location l produces the same diffraction noise as an FS tooth at the tube midpoint. To guarantee this, we need that the RS tooth at l subtend the same number of Fresnel zones, $N_z = 8$, as does the FS tooth at the tube midpoint. Since the zones at l are closer together by a factor $\beta(l)$ than at the tube midpoint [Eqs. (1) and (2)], *the tooth heights for the RS baffles are to be smaller by a factor* $\beta(l) = 4l(L-l)/L^2$ *than for the FS baffles.* Correspondingly, the effective height $h_{\rm RS}$ of the serration baffles, in hiding the tube walls, is greater than that of the FS baffles by

$$h_{\rm RS}(l) = h_{\rm FS} + 1.28{\rm cm}\left(1 - \frac{4l(L-l)}{L^2}\right)\ ; \tag{18}$$

cf. Eqs. (15), (16), and (17).

If FS baffles were used instead of RS at distances less than l, then the total number of FS baffles needed to cover those distances would be

$$N_{\rm FS}(l) = \frac{H - h_{\rm FS}}{h_{\rm FS}} \ln(l/l_0)\,, \tag{19}$$

where l_0 is the distance from the mirror to the adjacent gate valve (location of first baffle), and $H = 108$cm is the greatest height above the tube wall, in the mirror plane, at which we wish the wall to be hidden from view; cf. Fig. 1 and Sec. I of the Mathematica Notebook in which the baffle locations are computed [1]. By switching from FS to RS baffles, we change the number of baffles at distances less than l to

$$N_{\rm RS}(l) = \frac{H - h_{\rm RS}(l)}{h_{\rm RS}(l)} \ln(l/l_0)\,, \tag{20}$$

This represents a net reduction in the number of baffles near each end or corner mirror by

$$\Delta N(l) = N_{\rm FS}(l) - N_{\rm RS}(l) \tag{21}$$

We choose l [and correspondingly the tooth-height reduction factor $\beta(l)$] in such a way as to maximize the number of baffles saved, $\Delta N(l)$. A numerical calculation based on Eqs. (17)–(21), together with $L = 4$km and $l_0 = 11.75$m (the average for end and corner stations), gives $l = 300$m, which means that the switch from FS to RS baffles occurs at a distance

$$l_{\rm switch} = 300 - 11.75 \simeq 288{\rm m} \tag{22}$$

from the end and corner gate valves. Correspondingly, the heights of the RS baffles should be reduced from those of the FS baffles by a factor

$$\beta(300{\rm m}) = 0.2775\ , \tag{23}$$

and the number of baffles saved near each corner or end mirror is $\Delta N(300{\rm m}) = 6.3$ according to Eq. (21) (but actually $\Delta N = 6$ since it must be an integer). Since there are 8 end and corner mirrors at Hanford plus Livingston, this is a net saving of $8 \times 6 = 48$ baffles.

B. Distribution of Serration Peaks and Valleys

The serration peaks and valleys for RS baffles are chosen to have the same Gaussian distributions as for FS baffles, and for the same reasons, with the only change being the reduction of the mean height and variances by the factor 0.2775. This leads to the following specifications:

RS1 *The mean height ΔH of a RS serration tooth in the beam tube's vertical plane is chosen to be*

$$\Delta H = 0.8 \times 0.2775 = 0.222{\rm cm}\ . \tag{24}$$

RS2 *The scale of serration height fluctuations, measured in the beam tube's vertical plane, is chosen to be*

$$\delta H = 0.2 \times 0.2775 = 0.0555{\rm cm}\ . \tag{25}$$

RS3–RS5 *Identical to specifications* **FS3–FS5** *for FS baffles.*

The tolerance of 1/64 inch that we have placed on the radii of curvature of the FS peaks and valleys is about as tight as is practical, so to avoid a tighter tolerance, we keep the azimuthal placement of the peaks and valleys the same for RS baffles as for FS baffles. Correspondingly, we must reduce the pitch angle of the RS teeth below that of the FS teeth:

FS6 *We specify the constant pitch angle of the RS teeth to be*

$$\mu = \arctan(8 \times 0.2775/3) = 36.5^{\mathrm{o}} \tag{26}$$

in the tube's vertical plane.

We pay a modest and tolerable price in noise for this more gentle pitch.

C. Baffle Segments and Their Fabrication

The basic design and fabrication of the baffle from baffle segments is the same for RS baffles as for FS,

RS7–RS12 *Same specifications as* **FS7–FS12**.

However, because the depth of the serrations has been reduced, the height of the mean baffle top can be correspondingly increased:

RS13 *We specify the height for the mean baffle top (in the tube's vertical plane) to be*

$$H_{\mathrm{mbt}} = H_{\max} - \frac{\Delta H}{2} - 2.2\frac{\delta H}{2} = 8.83 \pm 0.50\mathrm{cm}\ . \tag{27}$$

If we were to allow 0.30cm of vertical height for the baffle's base and attachment of the baffle to it, then the radius of the mean baffle top in the baffle plane (Fig. 1 would be

$$R_{\mathrm{mbt}} = 42.38\mathrm{in} - \frac{H_{\mathrm{mbt}} - 0.30\mathrm{cm}}{(2.54\mathrm{cm/in})\sin(35^{\mathrm{o}})} = 36.53\mathrm{in} \tag{28}$$

On the other hand, the azimuthal length of each baffle segment remains the same for RS as for FS baffles:

RS14 *Same specification as* **FS14**.

D. Table and Tolerances of Serration Peaks and Valleys

We have computed a specific realization of the RS serration peak and valley distribution, using a Mathematica notebook [5]. The resulting distribution is tabulated in Addendum RS [3]. We specify the same somewhat arbitrary 0.01inch tolerances on this distribution as for the FS baffles:

RS15 *The positions and heights of the RS serration peaks and valleys should be as given in Addendum RS [3] to within tolerances* ± 0.010*inch.*

The tolerances on the straightness of the tooth edges and sharpness of peaks and valleys are the same for RS as for FS baffles:

RS16–RS17 *Same specifications as* **FS16–FS17**.

E. Shadowing of the Tube Wall

For our realization of the RS baffle serrations (Addendum RS [3]), the difference in height between the highest peak and the lowest valley is

$$H_{\max} - H_{\min} = \Delta H + 2.4\delta H = 0.36\mathrm{cm} \tag{29}$$

in the tube's vertical plane. Correspondingly, the lowest valley is at a height

$$H_{\min} = 9.0 - 0.36 = 8.64 \pm 0.5\mathrm{cm} \tag{30}$$

above the tube wall. This means that the effective height of an RS baffle in the tube's vertical plane, for wall-shadowing purposes, is

$$h_{\mathrm{RS}} = H_{\min} - 0.5 - 0.5 = 7.64\mathrm{cm}\ , \tag{31}$$

in accord with Eq. (18). Here as for FS baffles, the first factor 0.5cm is for the tolerance in the baffle height, and the second is for hiding the baffle's base.

IV. SS BAFFLES

A. The Role of the SS Baffles

As was discussed briefly in the introduction, near the Hanford midstation, the FS baffles must be placed far enough apart that they do not seriously shadow each others' serrations, as seen by 4km interferometers. To hide the wall as seen from midstation mirrors, SS baffles must be placed between the FS baffles; and to avoid the SS baffles' shadowing the FS serrations as seen by 4km interferometers (and avoid the SS baffles' contributing to diffraction noise for any interferometers), the SS baffles must be short enough that their tops are hidden from the end station and corner station mirrors.

B. Serration Distribution

Since the SS baffles do not contribute to diffraction, no diffraction-reducing serration peaks are needed on them. Nevertheless, we recommend that they be given shallow serrations as a protection against glint that will result if errors in installation enable a few SS baffles to be seen by corner station mirrors, toward which they lean. (The serrations prevent light from reflecting specularly back to a corner mirror from the baffle top, except at the top's rounded peaks and valleys.) These glint-reducing serrations can be very shallow indeed; we recommend somewhat arbitrarily that

SS1 *The SS baffles should have uniform serrations with depths in the tube's transverse plane*

$$\Delta H = 0.14\text{cm} \quad (32)$$

and serration-edge lengths (half-tooth lengths) along the mean baffle top

$$b = 2.54\text{cm} \ , \quad (33)$$

corresponding to a tooth pitch angle μ*, in the tube's vertical plane, given by*

$$\tan\mu = \frac{\Delta H}{b} = 0.059 \ . \quad (34)$$

SS2 *When translated into the plane of a baffle segment, this specification implies that, relative to the mean baffle top, the n'th SS peak or valley (with* $n = 0, 1, 2, ...$*) has height* s_n *and distance from the segment's edge* S_n *given by*

$$s_n = (-1)^{n+1} 0.05\text{inch}, \quad S_n = n \times 1.0\text{inch} \ . \quad (35)$$

(The 0.05 inch half height is $\Delta H \sin(35^o)$. The tolerances on these specifications can be whatever is easy to achieve.

C. Baffle Height

The height $H_{\rm max}$ of the SS serration peaks above the tube wall (measured in the tube's vertical plane) is dictated by the demand that the FS baffles hide them from view by the corner and end mirrors of a 4 km interferometer. The SS baffle that can most nearly be seen by a 4km mirror is the one nearest the midstation in the first 19.812 m interval between FS baffles. In order that this baffle be hidden from 4km view, $H_{\rm max}$ must be (cf. Fig. XXX of [1])

$$H_{\rm max} = H_{\rm min\,FS} - 0.5\text{cm} - \frac{F/2}{\mathcal{L}}(H - H_{\rm min\,FS} - 0.5\text{cm}) - 0.3\text{cm} \ , \quad (36)$$

where $H_{\rm min,FS} - 0.5\text{cm} = 7.22\text{cm}$ is the height of the lowest valley of the FS baffles including the -0.5cm allowance for the tolerance in that lowest height, $F/2 = 19.812$m is the distance between the FS baffles that hide the SS peaks, $\mathcal{L} = 1959$m is the distance to the 4km mirror which must not see the SS baffle, $H = 108$cm is the greatest transverse distance that that mirror can be from the tube wall (in the vertical plane), $H - H_{\rm min\,FS} - 0.5$cm is the mirror's greatest transverse distance from a FS baffle valley, and the final -0.3cm is to allow for a tolerance of ± 0.3cm in the SS baffle height (corresponding to a tolerance of $\pm 2^o$ in its 35^o inclination angle to the tube wall). Equation (36) and the above numbers imply that

SS3 *The height of the SS serration peaks above the tube wall, measured in the tube's transverse plane, must be*

$$H_{\rm max} = 5.90 \pm 0.30\text{cm} \quad (37)$$

This corresponds to a height for the mean baffle top in the tube's vertical plane

$$H_{\rm mbt} = H_{\rm max} - \frac{\Delta H}{2} = 5.83 \pm 0.30\text{cm} \ , \quad (38)$$

and to a height for the SS baffle valleys

$$H_{\rm min} = H_{\rm max} - \Delta H = 5.76 \pm 0.30\text{cm} \ . \quad (39)$$

Correspondingly, when computing the locations of the baffles [1], the effective height of a SS baffle in the tube's vertical plane should be taken to be

$$h_{\rm SS} = H_{\rm min} - 0.30 - 0.50 = 4.96\text{cm} \ . \quad (40)$$

The factor -0.30cm is for the tolerance in the baffle height, and the -0.50cm is to hide the baffle base from view.

D. Baffle Segments and Their Fabrication

SS4 *The SS baffles are to be fabricated from four segments in the same manner as the FS and SS baffles. More specifically, specifications* **FS7**–**FS11** *are to be applied to SS baffles, just as they are to FS and RS.*

If we were to allow 0.30 cm of vertical height for an SS baffle's base and the attachment of the baffle to it, then the radius of the mean baffle top in the baffle plane (Fig. 1) would be

$$R_{\rm mbt} = 42.38\text{in} - \frac{H_{\rm mbt} - 0.30\text{cm}}{(2.54\text{cm/in})\sin(35^o)} = 38.58\text{in} \quad (41)$$

In order that the total angular length of an SS baffle segment be about the same as that of the FS baffle segments ($\phi_{\rm mbt\,FS} = S_{\rm mbt\,FS}/R_{\rm mbt\,FS} = 35.13\text{in}/36.84\text{in} = 0.954\text{rad} = 54.6^o$) and the RS baffle segments ($\phi_{\rm mbt\,RS} = S_{\rm mbt\,RS}/R_{\rm mbt\,RS} = 35.13\text{in}/36.53\text{in} = 0.962\text{rad} = 55.1^o$), the total linear length along the SS mean baffle top should be about $S_{\rm mbt} = 0.954 R_{\rm mbt} = 36.81$in. We adjust this to 37in:

SS5 *The total azimuthal length of each SS baffle segment, along its mean baffle top, is*

$$S_{\rm mbt} = 37.00{\rm in}\ , \tag{42}$$

so the baffle segment contains an integral number of half teeth (beginning with a baffle peak and ending with a valley).

ACKNOWLEDGMENTS

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[1] E. E. Flanagan and K. S. Thorne, *Baffle Locations for the LIGO Beam Tube*, a Mathematica Notebook (LIGO Technical Report, 21 December 1995).

[2] E. E. Flanagan and K. S. Thorne, *Full-Serration* **(FS)** *Baffles: A Table of Positions and Heights of Serration Peaks and Valleys*, an ascii file (Addendum FS to the present LIGO Technical Report).

[3] E. E. Flanagan and K. S. Thorne, *Reduced-Serration* **(RS)** *Baffles: A Table of Positions and Heights of Serration Peaks and Valleys*, an ascii file (Addendum RS the present LIGO Technical Report).

[4] E. E. Flanagan and K. S. Thorne, *Computation of Serration Peaks and Valleys for Full-Serration* **(FS)** *Baffles*, A Mathematica Notebook (LIGO Technical Report, 23 December 1995).

[5] E. E. Flanagan and K. S. Thorne, *Computation of Serration Peaks and Valleys for Reduced-Serration* **(RS)** *Baffles*, A Mathematica Notebook (LIGO Technical Report, 23 December 1995).