

"Energy Dissipation in the Violin Modes of Suspension of Fused Silica Pendula," by V. B. Braginsky, V. P. Mitrofanov, and K. V. Tokmakov, Report No. 4 (undated).

Distributed by S. Whitcomb 2/21/95

PLEASE PASS ALONG PROMPTLY

COPY 1

A. Abramovici
J. Camp
J. Carri
D. Durance
A. Gillespie
S. Kawamura
A. Kuhnert
T. Lyons

COPY 2

J. Mason
S. Meshkov
M. Rakhmanov
R. Savage
V. Schmidt
L. Sievers
R. Spero
H. Yamamoto

cc: B. Barish
A. Lazzarini
F. Raab
G. Sanders
R. Vogt
S. Whitcomb -original
MIT Science Group
Project File
Science File

V.B. Braginsky, V.P. Mitrofanov, K.V. Tokmakov
Moscow State University, Department of Physics

ENERGY DISSIPATION IN THE VIOLIN MODES
OF SUSPENSION OF FUSED SILICA PENDULA

The content :

1. Fused silica pendula with intermediate mass.
2. Design of the double pendulum and technology of manufacturing of the ridges for the fused silica fiber welding.
3. Damping of the violin modes caused by the losses in the ridges and the support structure.
4. The results of study of some loss mechanisms.
5. Conclusion.

1. Fused silica pendula with intermediate mass

We present here results of study of the energy dissipation in the violin modes of the all fused silica pendulum suspension fibers. The pendulum bob mass was $M = 1.6 \text{ kg}$. We used the fused silica fibers with a diameter about $150 \mu\text{m}$ and a length about 20 cm . The stress in the fiber was near 100 kg/mm^2 . The fiber was welded to the upper slab with mass of 1.8 kg .

A change of the pendulum mass was from 30 g to 1.6 kg so that the tension in the fiber was near ultimate value permitted to obtain the maximum increase in Q -factor of the violin modes Q_v , compared to the intrinsic Q_{SiO_2} of fused silica [1,2,3]:

$$Q_v^{-1} \approx \frac{2}{l} \sqrt{\frac{YI}{T}} \left[1 + \frac{(n\pi)^2}{2l} \sqrt{\frac{YI}{T}} \right] Q_{\text{sil}}^{-1}, \quad (1)$$

where $I = \pi D^4/64$ - the area momentum of inertia, Y - the Young's modulus, T - the tension, n - the harmonic number.

The fiber material losses Q_{sil}^{-1} were measured using a part of the fused silica fiber welded to the slab at one end and broken down at the other end, so it was left free. The measured Q 's of the different harmonics of a piece of the fiber bending modes are plotted in fig.1. Fiber was fabricated of the high quality fused silica of the KS - type with minimum quantity of impurities. A maximum of thermoelastic losses for our fibers is at the characteristic frequency near 40 Hz , so that at the frequencies higher than 1 kHz thermoelastic damping gives a small contribution to the total losses.

Substituting the pendula parameters into Eq.(1), we find the expected value of the violin modes $Q \approx 2 \cdot 10^9$. Note that Eq.(1) is valid for the fiber with constant cross section

along its length. Calculation of the losses for the fiber with a changing diameter (the fiber diameter is larger near the endpoints is more complicated. But our experiments with the pendula fabricated from alkali-silica glass showed that we can use this equation in order to estimate the violin modes losses with an accuracy near 50%.

2. Design of the double pendulum and technology of fabrication of the ridges for the fused silica fiber welding

If the mass of the suspended body is increased, damping of the violin modes (caused by exciting of vibrations in support structure) also increases. Suppose that the pendulum is suspended from the surface of a seminfinite absolutely elastic body. In this case the damping of the violin modes is caused by the emission of elastic waves into the body (longitudinal, transverse and Rayleigh waves). It is possible to obtain an expression for the damping of the violin modes due to exciting of the elastic waves in the support structure using a mathematical calculation published in the reference [4]:

$$Q_{v.sup}^{-1} = \frac{\alpha M \omega_p^2 \omega_v (1-\eta/2)}{2 \rho c_t^3} \approx 10^{-9} \quad (2)$$

where ρ is the density of the elastic medium, c_t is the velocity of shear waves, η is the Poisson's ratio, $\alpha = 0.597$ is the coefficient, calculated in the reference [4].

The real hard support for the pendulum is far from the absolutely elastic bulk. It is a lot of connected bodies. With this tipe of support, we could not achieve the violin modes Q_v better than $2 \cdot 10^7$.

In order to reduce the losses of the violin modes due to the support structure, we used a double pendulum that is shown in figure 2. The upper pendulum acts as a low pass filter for the vibrations of the fiber. The upper 1.8 kg cylinder of fused silica was suspended by two loops of 250 μm diameter tungsten wires. A ridge (5 mm long by 2.5 x 2.5 mm in cross section) was fabricated on a side surface of the cylinder by diamond grinding. The fiber was welded to the ridge. A drawing of the fused silica cylinder with a ridge is shown in figure 3. At first, two parallel grooves were fabricated by a diamond cutter with a diameter of 6 mm. Then a part of a crosspiece was removed by a diamond cutter with a diameter of 16 mm.

Such procedure causes an additional damping of intrinsic modes of the fused silica cylinder due to the surface losses, for example. The Q of the first longitudinal mode of the cylinder was $1.2 \cdot 10^6$ until the ridge was fabricated (the cylinder was made of not so good kind of fused silica). After fabrication Q did not change, in the limits of accuracy of the measurement. Therefore the contributed losses were no more than $5 \cdot 10^{-8}$.

3. Damping of the violin modes caused by the losses in the ridges and the support structure

The ridge on the support slab and on the pendulum test mass to which the fiber was welded can be considered as a intermediate oscillator - a rod. The bending modes of the rod are excited by the violin oscillations of the fiber (m_r is its effective mass, ω_r is the natural frequency, Q_r^{-1} is the damping. According to the model of coupled oscillators, the damping of

the violin modes due to the losses in the ridges is following (under condition $\omega_r \gg \omega_v$ and frequency independent Q_r):

$$Q_{v.r}^{-1} = \frac{2M\omega_p^2}{m_r\omega_r^2} Q_r^{-1} \quad (3)$$

It is easy to see that the ridge is bound to have a high natural frequency in order to minimize the losses in the ridge. Hence it is necessary to make the ridge as short as possible. However it is difficult to weld the fiber to the short ridge without generation of the cracks in the cylinder due to thermal gradients. As a result of a large body of experiments we found that the optimum length of the ridge (including a thick part of the fused silica fiber) was near 15 mm. In this case the measured frequency of the first bending mode of the ridge was $\omega_r \approx 2\pi \cdot 10^4$ Hz, the $Q_r = (5-7) \cdot 10^5$. It is lower than the intrinsic losses in fused silica. The dominant losses in the ridges are the surface losses because it is difficult to polish them well. For our pendula and the ridges with the effective mass $m_r \approx 0.1g$ we obtain the estimate $Q_{v.r}^{-1} \approx 1.5 \cdot 10^{-9}$.

Note that we could not obtain the measured Q 's of the violin modes higher than 10^7 as long as we did not succeed in fabricating "good" (short and smooth) ridges.

The damping of the violin modes due to the support structure can be calculated on the bases of the model of coupled oscillators. If the model $H_{sup} \approx \text{const}$ is valid, we have:

$$Q_{v.sup}^{-1} \approx \frac{2 M \omega_p^2 \omega_{sup} Q_{sup}^{-1}}{m_{sup} \omega_v^3 (1 - \omega_{sup}^2 / \omega_v^2)^2} , \quad (4)$$

For a case of $Q_{sup}^{-1} \approx \text{const}$:

$$Q_{v.sup}^{-1} \approx \frac{2 M \omega_p^2 \omega_{sup}^2 Q_{sup}^{-1}}{m_{sup} \omega_v^4 (1 - \omega_{sup}^2 / \omega_v^2)^2} \quad (4a)$$

Equations (4) and (4a) are not valid, if $\omega_v \approx \omega_{sup}$ where the losses can be significant because of resonance effect.

Depending on the relation between the frequencies ω_v and ω_{sup} we used the model, giving higher estimate of giving damping. For the violin modes frequencies $\omega_v > 2\pi \cdot 10^3$ Hz a predicted value of $Q_{v.sup}^{-1}$ is not more than 10^{-11} if the damping is caused both by the losses of the pendulum mode of the upper cylinder and losses of the intrinsic longitudinal mode.

4. The results of study of some loss mechanisms

The measured Q 's of the violin modes are plotted in figure 4 the best achieved values Q are:

$$\begin{array}{ll} f_1 = 1292 \text{ Hz} & Q_1 = 8.1 \cdot 10^7 \\ f_2 = 2598 \text{ Hz} & Q_2 = 9.5 \cdot 10^7 \\ f_3 = 5080 \text{ Hz} & Q_3 = 6.5 \cdot 10^7 \end{array}$$

The statistical errors of the measurement were near 5%. The measurements were made at low pressure (near $2 \cdot 10^{-6}$ Torr) so that gas damping was $Q_{gas}^{-1} \approx (5 \pm 2) \cdot 10^{-10}$.

The main problem is a discrepancy between the predicted value of the damping of the violin modes according Eq.1 and the measured value by a factor 5 at least. Note that different pendula with the fibers made of the same kind of fused silica with the same technology had dispersion of Q_v from $5 \cdot 10^7$ to $9.5 \cdot 10^7$. It evidently means that we did not take into account some other damping mechanisms that are ^{not} described by the relationship (1). The surface losses in the fiber and the losses

caused by the intrinsic stresses in the fiber are among them.

In order to reduce these losses the fiber of the pendulum was heated *in situ* inside the vacuum chamber. A heater was installed on a stainless steel tube that surrounded the fused silica fiber. Such heating usually removes adsorbed molecules (especially water) from the surface layers and thereby avoids additional losses of the energy. In our case heating of the fiber at $T \approx 250^{\circ}\text{C}$ during 3 hours did not give increase of Q of the violin modes.

In order to reduce the intrinsic stresses in the fiber it was heated in the open flame of an oxygen burner at the temperatures $T_1 \approx 800^{\circ}\text{C}$ or $T_2 \approx 1200^{\circ}\text{C}$. A quality of the fiber and its surface can be determined from the mean value of its breaking strength σ_{br} (at this strength the fiber was broken within 10 second after loading). The results of the measurements are the following:

Before heating	$\sigma_{br} \approx 170 \text{ kg/mm}^2$
After heating at $T \approx 800^{\circ}\text{C}$	$\sigma_{br} \approx 75 \text{ kg/mm}^2$
After heating at $T \approx 1200^{\circ}\text{C}$	$\sigma_{br} \approx 50 \text{ kg/mm}^2$.

In the process of welding of the fiber to the body of the pendulum or to the support slab evaporation of quartz takes place. Quartz vapors are deposited on the fiber. The most intense sedimentation is at the distance of 2 cm from the place of heating (fig.5). There is the friable layer of deposited quartz on the fiber surface. This layer is responsible for the losses of elastic energy.

Note that the influence of the quartz vapor sedimentation on the damping of the violin modes is stronger than on the damping of the bending modes, because the deposited layer is localized in the place of the concentration of strains for the

violin modes. Until present, we did not find the method of removing of the quartz vapor sedimentation from the fiber surface.

Another additional damping mechanism for the violin modes is caused by the particles of dust adhered to the fiber after its fabrication. The particle of dust can be modeled by a small rod, attached to the fiber. Such rod (for example 500 μm long by 5 μm in diameter) can be treated as an oscillator with the effective mass $m_d \approx 2 \cdot 10^{-8} \text{g}$ and the resonant angular frequency $\omega_d \approx 2\pi \cdot 10^4 \text{c}^{-1}$. In this case one can estimate the losses of the violin modes due to attached oscillator with damping $Q_d^{-1} \approx 10^{-2}$:

$$Q_{v.d}^{-1} \approx \frac{m_d \omega_v^2}{m_v \omega_d^2} Q_d^{-1} \approx 3 \cdot 10^{-9} \quad (5)$$

This estimate can be illustrated by the following experiment. The fiber of the pendulum was blown by the nonfiltered air and the Q 's of the violin modes were measured. The Q 's obtained before and after blowing are given in the Table

f_v	Q_v before blowing	Q after blowing
1238 Hz	$4.4 \cdot 10^7$	$3.3 \cdot 10^7$
2522 Hz	$4.5 \cdot 10^7$	$3.1 \cdot 10^7$
3547 Hz	$4.3 \cdot 10^7$	$2.5 \cdot 10^7$

Then the flame of the oxygen burner was applied to the fiber. The particles of dust were burnt out and one could estimate a bright points (about 20 in our case). This is in accordance with the estimate given by (5). So the level of dust in the laboratory could limit the achieved Q of the violin modes.

Conclusion

Our investigations showed that the Q 's of the violin modes of the intermediate size prototypes test masses fabricated of fused silica are near the value $Q_y \approx 10^8$ for the stresses in the fiber $\sigma \approx 10^{10} \text{ dyn/cm}^2$. We consider that the Q 's were limited not by the losses in the material-fused silica, but by the conditions of fabrication of the pendulum (primarily by the deposition of quartz vapors and dust).

References

- 1) P.R. Saulson, *Phys. Rev. D* 42 (1990) 2437.
- 2) V.B. Braginsky, V.P. Mitrofanov, K.V. Tokmakov, *Phys. Lett. A* 186 (1994) 18-20.
- 3) A. Gillespie and F. Raab, *Phys. Lett. A* 178 (1993) 351.
- 4) G.M.L. Gladwell, *J. Sound Vib.* (1968) 8 (2), 215-228.

Quartz material quality factor

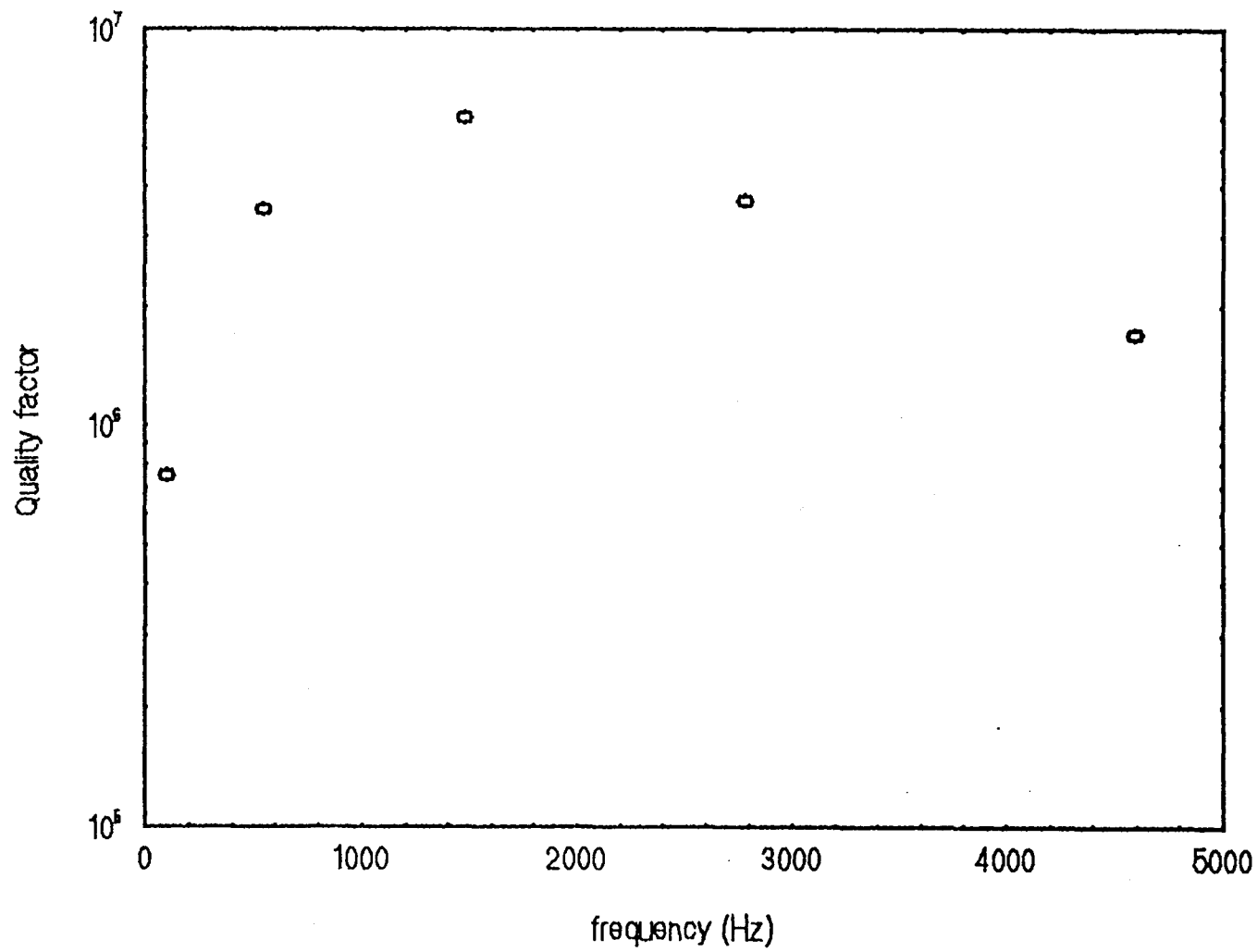
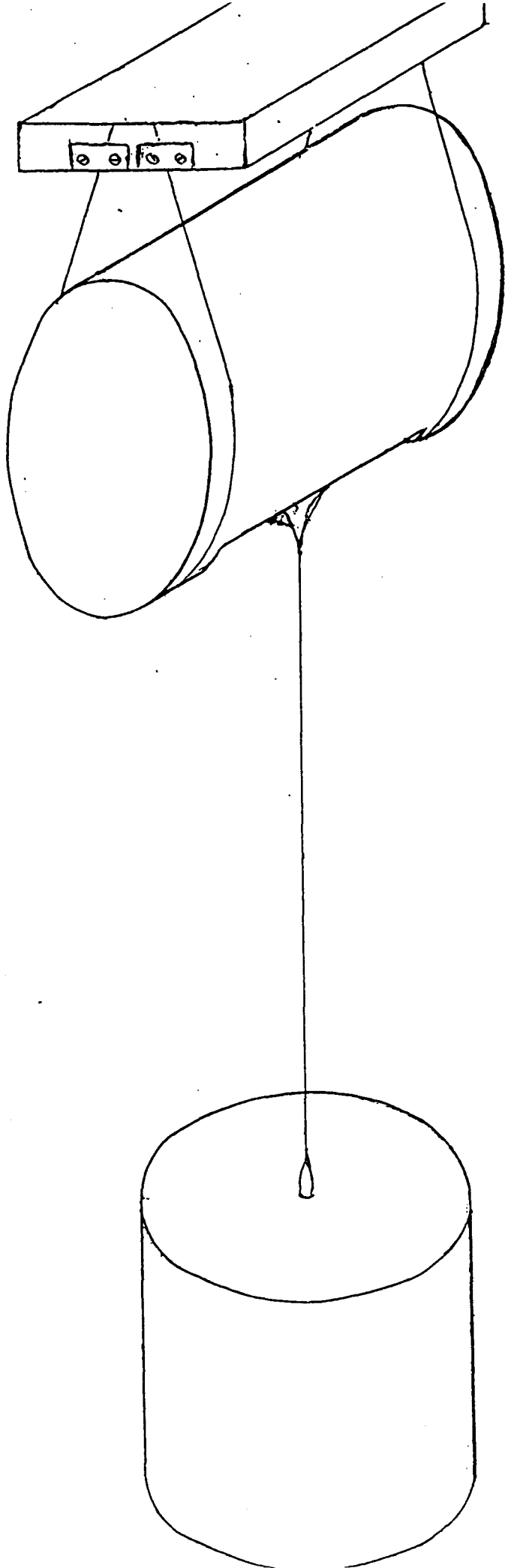


Fig. 1



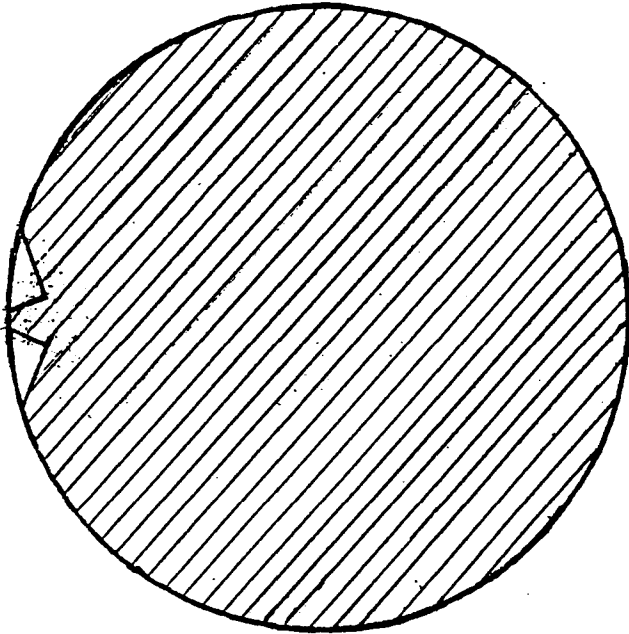
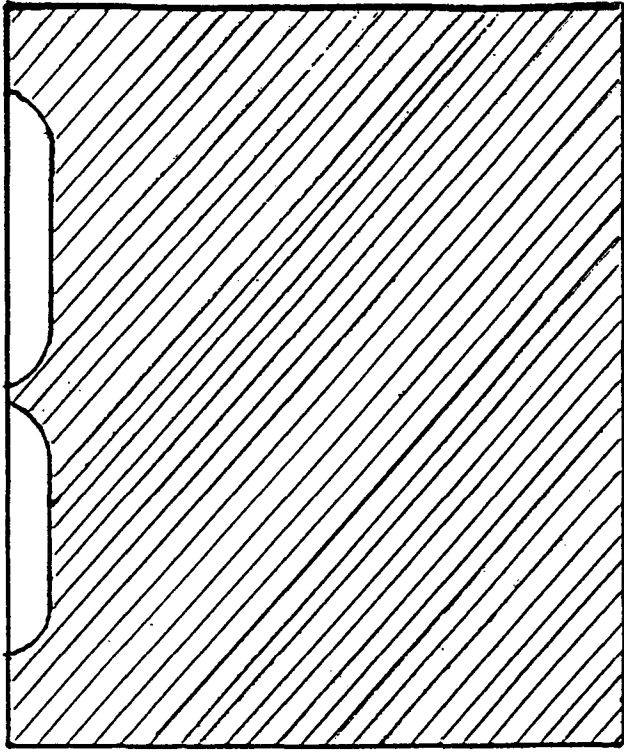


Fig. 3.

Quality factor of the violin modes of different pendula. Suspension mass 16 kg.

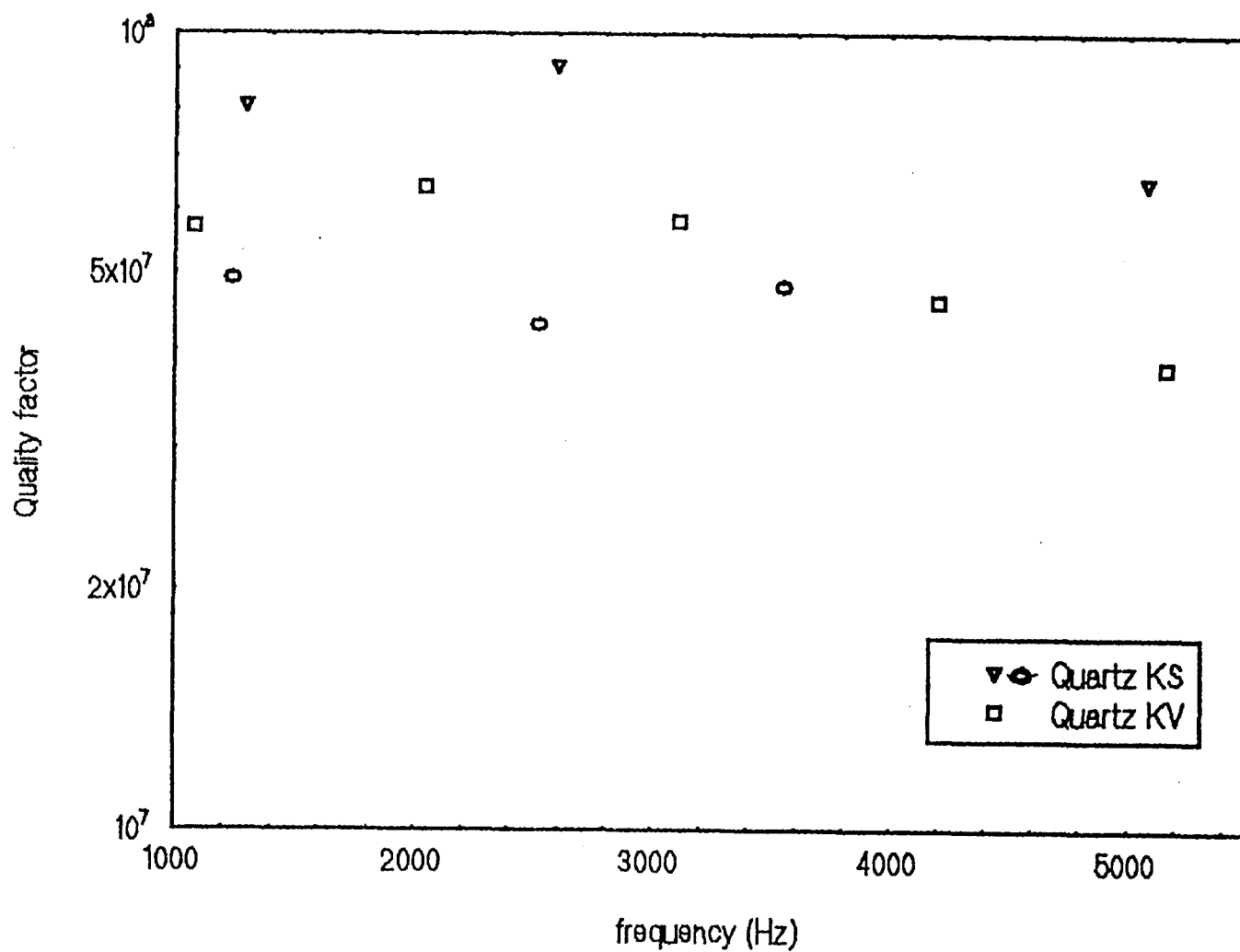
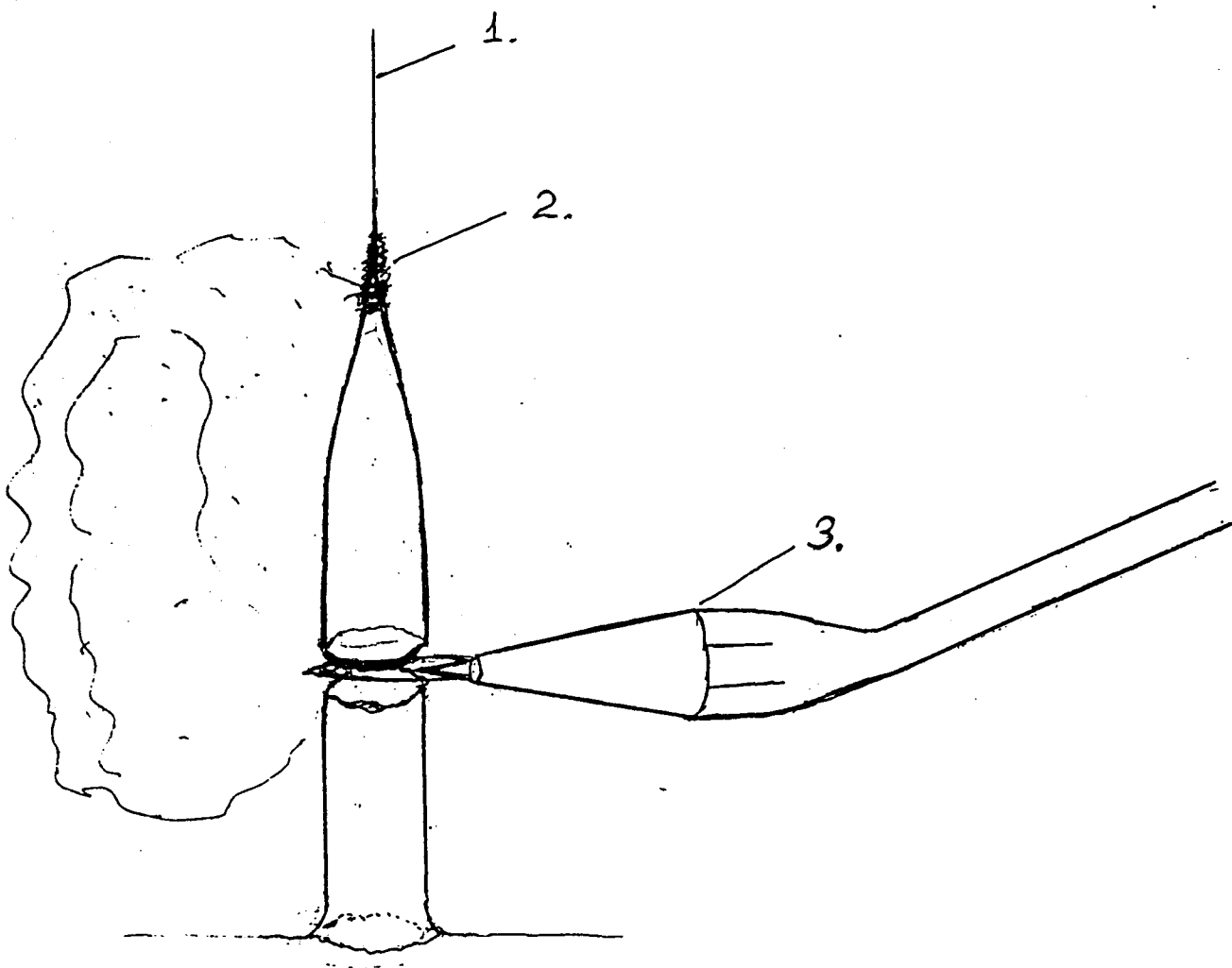


Fig. 4



1. The silica fiber.
2. The place of the most intense sedimentation of quartz.
3. The burner.

Fig. 5.