

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

- LIGO -

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Suspension Design Requirements
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This is an internal working note
of the LIGO Project.

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1 INTRODUCTION

1.1. Purpose

This Design Requirements Document (DRD) for Suspension System (SUS) identifies the information necessary to define the SUS subsystem and quantify its relationship to other subsystems.

1.2. Scope

SUS will develop and provide the suspension system for all the suspended components of Core Optics Component (COC) and Input/Output Optics (IOO).

1.3. Definitions

SUS is the system which suspends, protects, damps, and actuates the optics.

1.4. Acronyms

- LOS1: Large Optics Suspension 1
- LOS2: Large Optics Suspension 2
- SOS: Small Optics Suspension

Acronyms for names of subsystems should be referred to [8] “LIGO DETECTOR Workbook”, LIGO-M940005-C-D (p. 13).

1.5. Applicable Documents

1.5.1. LIGO Documents

- [1] “Alignment Sensing/Control Design Requirements Document”, LIGO-T952007-01-I
- [2] “Core Optics Components Requirements (1064 nm)”, LIGO-E950099-01-D
- [3] “Framework of Range Requirement of Suspension Actuator”, LIGO-T960070-00-D
- [4] “Frequency, Intensity and Oscillator Noise in the LIGO”, LIGO-T960019-00-D
- [5] “Interferometer Requirements Flowdown to SUS”, LIGO-T950061-01-D
- [6] “Length Sensing and Control Design Requirements Document”, LIGO-T960058-00-D
- [7] “LIGO EMI Control Plan and Procedures”, LIGO-E960036-A-E
- [8] “LIGO DETECTOR Workbook”, LIGO-M940005-C-D
- [9] “LIGO Naming Conventions”, LIGO-E950111-A-E
- [10] “LIGO Project System Safety Management Plan”, LIGO-M950046-F
- [11] “LIGO Science Requirements Document”, LIGO-E950018-02-E
- [12] “LIGO Vacuum Compatibility, Cleaning Methods and Procedures”, LIGO-E960022-00-D
- [13] “Mirror-Orientation Noise in a Fabry-Perot Interferometer Gravitational Wave Detector”, LIGO-P940012-00-R

- [14] “Mode cleaner Noise Sources”, LIGO-T960165-00-D
- [15] “Naming Convention and Interface Definition for SUS”, LIGO-T950060-00-D
- [16] “Response of Pendulum to Motion of Suspension Point”, LIGO-T960040-00-D
- [17] “Seismic Isolation Design Requirements Document”, LIGO-T960065-02-D
- [18] “Suspension Design Requirements”, LIGO-T950011-14-D
- [19] “Suspension Losses in the Pendula of Laser Interferometer Gravitational-Wave Detectors”, LIGO-P940011-00-R
- [20] “Suspension Preliminary Design”, LIGO-T960074-00-D
- [21] “Suspension Test Plan”, LIGO-T960086-00-D
- [22] “Thermally Excited Vibrations of the Mirrors of Laser Interferometer Gravitational-Wave Detectors”, LIGO-P940003-00-R
- [23] “Thermal Noise in HAM OS”, LIGO-T960090-00-D

1.5.2. Non-LIGO Documents

2 GENERAL DESCRIPTION

2.1. Specification Tree

This document is part of an overall LIGO detector requirement specification tree. This particular document is circled in Fig. 1.

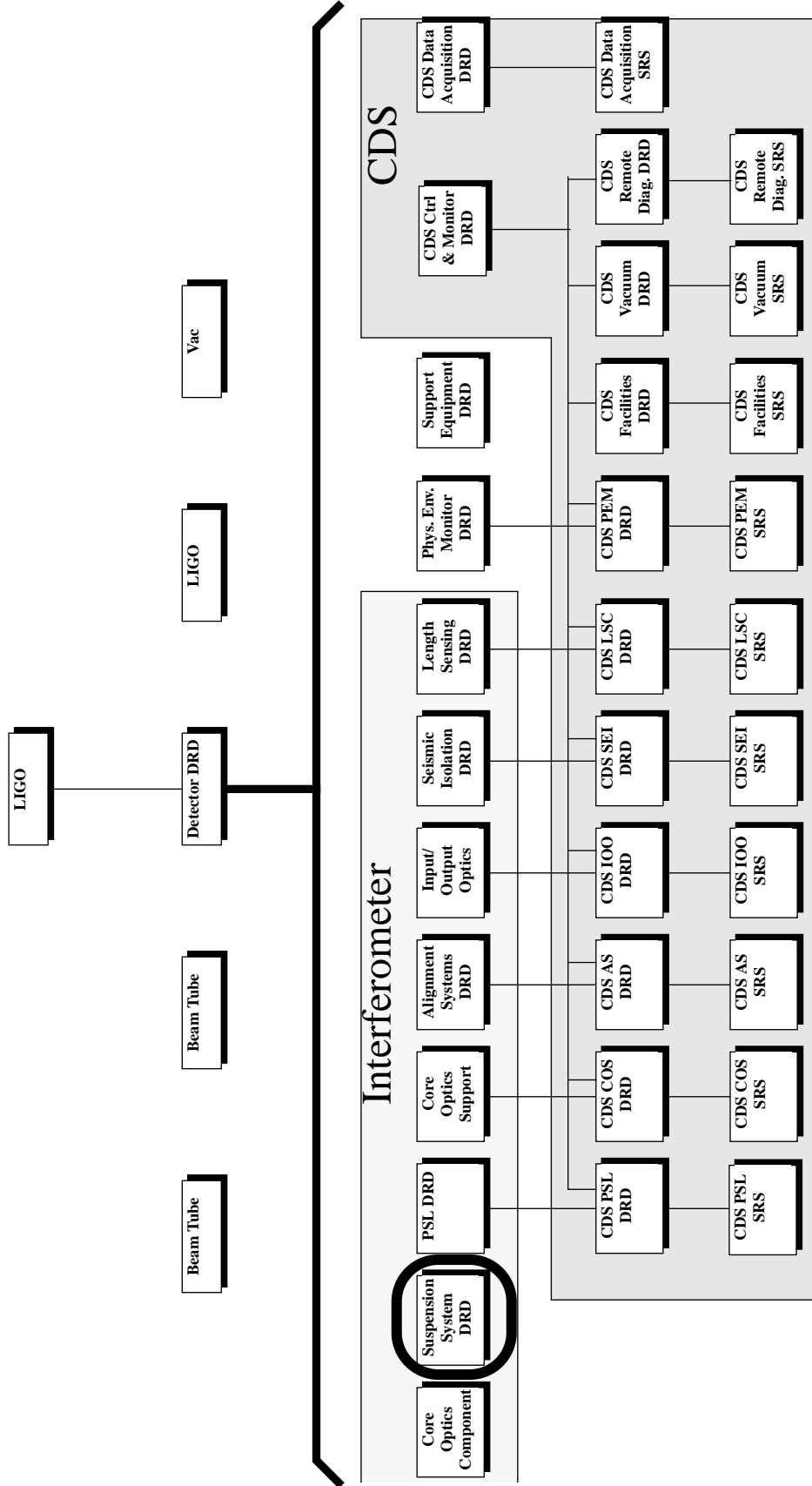


Figure 1: Overall LIGO detector requirement specification tree. SUS DRD is circled.

2.2. Product Perspective

SUS relates to the rest of the system as shown in Fig. 2.

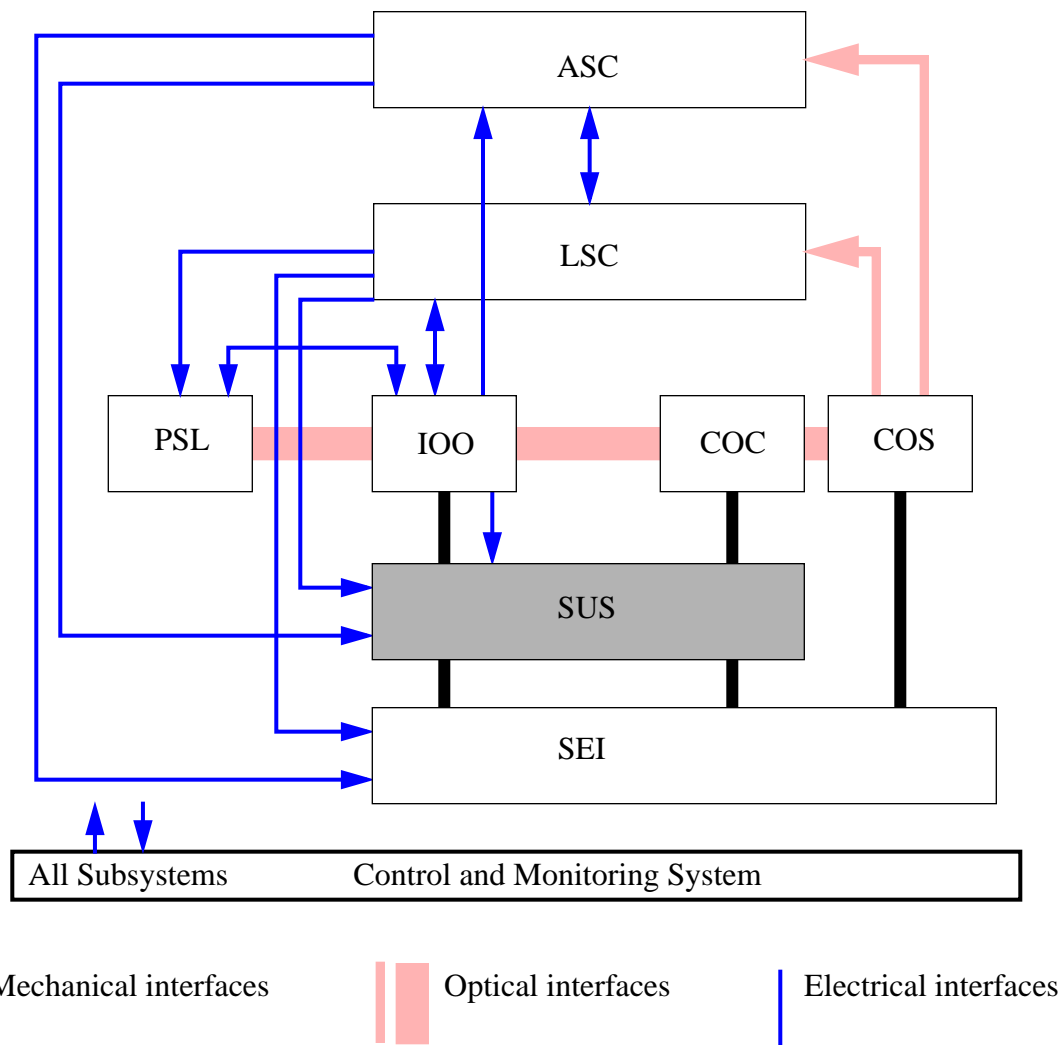


Figure 2: Relationship of SUS to the rest of the detector subsystem. SUS is shaded.

2.3. Product Functions

The main functions of SUS are:

- Suspend a test mass to allow it to move freely horizontally for detection of gravitational waves.
- Isolate an optical component from ground motion by suspending the component.
- Damp the optical component's motion in position and orientation using the local suspension's sensors and actuators.
- Provide control inputs for applying forces and torques to the suspended component in

response to signals from the LSC and ASC systems.

- Suppress the noise related to the SUS system to the desired level.
- Protect the optical components by limiting motion from external disturbance.
- Hold the optical components firmly during installation.
- Reduce the effect of stray/scattered light from the optical component.

2.4. General Constraints

- The initial LIGO must have a sensitivity specified in [11] “LIGO Science Requirements Document”, LIGO-E950018-02-E, which SUS must not preclude.
- LIGO must operate continuously, therefore SUS must be designed with high reliability and low mean time to repair.
- LIGO interferometers have strict vacuum-compatibility requirements which constrain the material choices for the SUS components to those materials compatible with [12] “LIGO Vacuum Compatibility, Cleaning Methods and Procedures”, LIGO-E960022-00-D.

2.5. Suspension type

There will be three types of suspension systems depending on the size of the suspended optical component: Large Optics Suspension 1 (LOS 1), Large Optics Suspension 2 (LOS 2), and small optics suspension (SOS). A list of suspended optical components for each suspension system is shown in Table 1.

Chapter 3 describes LOS1&2 and Chapter 4 describes SOS.

Table 1: List of suspended optical components

<i>SUS type</i>	<i>Suspended Optical Components (Subsystem)</i>
LOS 1	Test Mass (CO), Recycling Mirror (CO), Large Folding Mirror (CO), Large Mode Matching Mirror (IOO), Faraday Isolator (IOO) ^a
LOS 2	Beamsplitter (CO)
SOS	Mode Cleaner Mirror (IOO), Small Mode Matching Mirror (IOO), Small Folding Mirror (IOO), Small Pick-off (IOO)

a. An adapter ring will be fitted to the Faraday Isolator to fit it into the standard assembly.

3 LARGE OPTICS SUSPENSION 1&2

Under construction! See [18] “Suspension Design Requirements”, LIGO-T950011-14-D for the latest information.

4 SMALL OPTICS SUSPENSION

4.1. Assumptions

4.1.1. Assumptions in SUS

4.1.1.1 Single Pendulum

The suspension system employs a single pendulum as opposed to a multi-stage pendulum.

4.1.1.2 Damping by Suspension's Sensor

A suspended component is always (before and after the lock acquisition) damped using the sensor signals from its respective suspension assembly.

4.1.1.3 Suspension's Actuator

The suspension's actuator is used to correct all the fluctuations during operation.

4.1.2. Assumption in Other Subsystems

4.1.2.1 Size of Optics

The size, wedge, and optical clear aperture of the suspended optics (IOO) are listed in **TBD**.

4.1.2.2 Mode Cleaner Output Frequency Noise

Allowed frequency noise of the light coming out of the mode cleaner (IOO) is $1 \times 10^{-4} \text{ Hz} / \sqrt{\text{Hz}}$ at 100 Hz with $f^{-0.5}$ dependence above 100 Hz and f^{-2} dependence below 100 Hz (See [6] "Length Sensing and Control Design Requirements Document", LIGO-T960058-00-D).

4.1.2.3 Beam Spot Offset

The requirement of the beam spot offset from the center of the optic is 3 mm for mode cleaner mirrors (ASC).

4.1.2.4 Level of Mode Cleaner

The requirement of the level of the mode cleaner (IOO) is 3×10^{-4} rad.

4.1.2.5 Internal Mode Loss of Bare Substrate

The required internal mode loss of the bare substrate of the mode cleaner mirrors (IOO) is **TBD**.

4.2. Specific Requirements

4.2.1. Requirements Flowdown

Performance requirements of SOS are derived from system requirements of the interferometer noise and detector availability. Fig. 3 shows the noise and availability requirements flowdown to the SUS subsystem. The requirements for SOS are shaded.

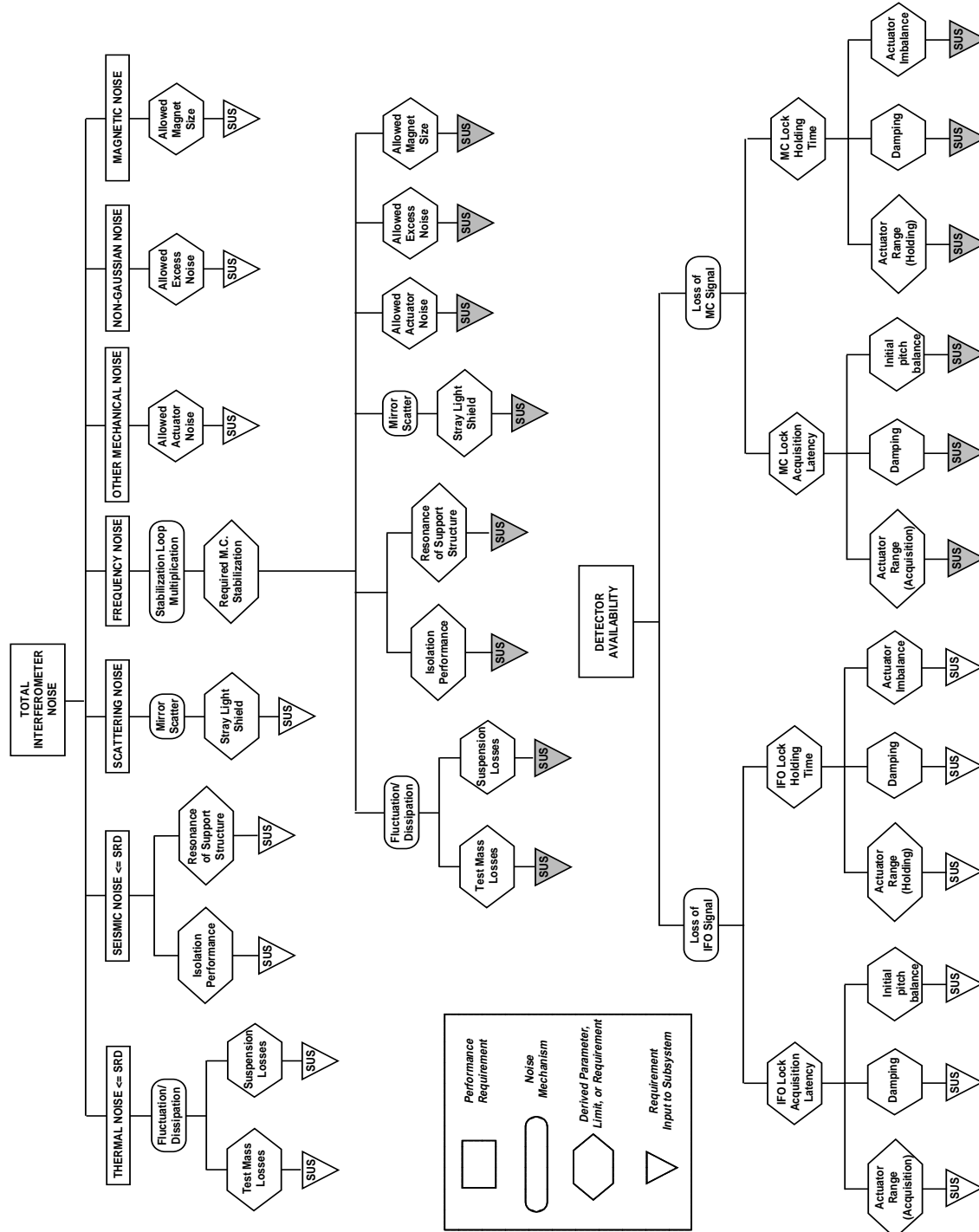


Figure 3: Interferometer noise and detector availability requirements flowdown to SOS.

4.2.2. Detector Availability

4.2.2.1 Range

The actuator range for SOS is required to provide:

- continuous operation of the LSC and IOO system,
- smooth acquisition of the LSC and IOO system,
- proper initial alignment, and
- continuous operation of the ASC system.

Table 2 shows the requirement of the suspension range for displacement (operation and acquisition mode) and orientation of the mass. (See [3] “Framework of Range Requirement of Suspension Actuator”, LIGO-T960070-00-D).

Table 2: Requirement of the SOS suspension actuator range.

<i>Mode</i>		<i>Range ($f < 10$ Hz)</i>
Displacement	Operation	$5 \mu\text{m}_{\text{pp}}$
	Acquisition	$80 \mu\text{m}_{\text{pp}}$
Orientation		$3 \text{ mrad}_{\text{pp}}$ in pitch $1 \text{ mrad}_{\text{pp}}$ in yaw

4.2.2.2 Damping

The magnitude and quality of damping of the suspended component for SOS is required to provide:

- stable operation of the LSC and IOO system,
- smooth acquisition of the LSC and IOO system, and
- negligible up-conversion noise of the spurious interferometer.

In the transfer function from (horizontal, vertical, and pitch/yaw) motion of the suspension point to (horizontal and pitch/yaw) motion of the suspended mass, the maximum allowed amplitude around the resonant frequencies over the amplitude at DC is required to be less than 3.

4.2.2.3 Actuator Imbalance

The four face actuators of SOS are required to be balanced to ensure:

- No significant cross-coupling from the LSC and IOO signal to orientation of the optics.
- No significant cross-coupling from the ASC signal to displacement of the optics.

The maximum variation of force exerted by four face actuators is required to be less than 0.01.

4.2.2.4 Initial Pitch Imbalance

The initial pitch imbalance of the optical component is required to be less than 0.5 mrad.

4.2.3. Interferometer Noise

The required frequency noise of the light coming out of the mode cleaner is

$\tilde{v} = 1 \times 10^{-4} \text{ Hz}/\sqrt{\text{Hz}}$ at 100 Hz with $f^{-0.5}$ frequency dependence above 100 Hz and f^{-2} frequency dependence below 100 Hz. Therefore displacement noise requirement per MC mirror is

$$\tilde{x}_{\text{MC}} = \frac{1}{\sqrt{3}} \cdot \frac{l_{\text{MC}} \cdot \tilde{v}}{v_0} = 2.4 \times 10^{-18} \text{ m}/\sqrt{\text{Hz}} \text{ at 100 Hz, where } l_{\text{MC}} \text{ is the length of the mode}$$

cleaner and v_0 is the frequency of the light. We allocate a portion of this displacement noise to each noise source.

4.2.3.1 Transfer Function of Suspension

The transfer function of the suspension for SOS is required to provide sufficient isolation above 40 Hz to meet the required frequency noise of the light coming out of the mode cleaner. Table 3 shows the requirements of the transfer function of the suspension system from (horizontal, vertical, and pitch/yaw) motion of the suspension point to (horizontal, vertical, and pitch/yaw) motion of the suspended mass. Wire violin modes are excluded. Coordination of the requirements between the suspension transfer function and the stack isolation is explained in [17] ‘‘Seismic Isolation Design Requirements Document’’, LIGO-T960065-02-D.

Table 3: Requirements matrix of the transfer function of the SOS suspension system from motion of the suspension point to motion of the suspended mass ($f > 40\text{Hz}$).

<i>Transfer Function</i>	<i>To</i>	<i>Horizontal (m)</i>	<i>Vertical (m)</i>	<i>Pitch (rad)</i>
<i>From</i>				
<i>Horizontal (m)</i>		$< \left(\frac{f_p}{f}\right)^2 \text{ m/m}$ $f_p = 1.0 \text{ Hz}$	N/A	$< \alpha \times \left(\frac{f_p}{f}\right)^2 \text{ rad/m}$ $\alpha = 30$
<i>Vertical (m)</i>		$< 3 \times 10^{-5} \times \left(\frac{f_v}{f}\right)^2 \text{ m/m}$	$< \left(\frac{f_v}{f}\right)^2 \text{ m/m}$ $f_v = 16 \text{ Hz}$	$< \beta \times \left(\frac{f_v}{f}\right)^2 \text{ rad/m}$ $\beta = 1 \times 10^{-2}$

4.2.3.2 Resonance of Suspension Support Structure

The resonance of the SOS suspension support structure shall not preclude meeting the required frequency noise of the light coming out of the mode cleaner. Table 4 shows the requirements for the frequency and Q of the resonance of the suspension support structure. See [23] “Thermal Noise in HAM OS”, LIGO-T960090-00-D.

Table 4: Requirements for resonances of the SOS suspension support structure.

<i>Mode</i>	<i>SOS</i>
Resonant Frequency	> 150 Hz
Q	< 300

4.2.3.3 Thermal Losses

We allocate 70% of the allowed MC displacement noise ($1.6 \times 10^{-18} \text{ m}/\sqrt{\text{Hz}}$ at 100 Hz per MC mirror) to thermal noise. There are three kinds of thermal losses with regard to the suspension system (See Appendix B for details):

- internal mode losses due to the suspension attachments,
- suspension (pendulum, pitch/yaw, and vertical mode) losses,
- eddy current damping losses due to interaction between the magnets and the external metal.

Table 5 shows the requirements of the (average effective) thermal loss of the suspension system.

Table 5: Requirements of the (average effective) thermal loss of the SOS suspension system.

<i>Damping Mode/ Mechanism</i>	<i>Allocation to the allowed MC displacement noise</i>	<i>Loss</i>
Internal mode	50%	$< 9.6 \times 10^{-6}$
Pendulum	20%	$< 9.0 \times 10^{-6}$
Pitch	20%	$< 5.3 \times 10^{-4}$
Yaw	20%	$< 6.8 \times 10^{-4}$
Vertical	10%	$< 9.9 \times 10^{-2}$
Eddy Current Damping	30%	$< 2.0 \times 10^{-5} \times \left(\frac{f}{100\text{Hz}} \right)$
Total	70%	N/A

4.2.3.4 Control Noise

We allocate 30% of the allowed MC displacement noise ($7.3 \times 10^{-19} \text{ m}/\sqrt{\text{Hz}}$ at 100 Hz per MC mirror) to control noise. Table 6 shows the requirements of the control noise per mass expressed in displacement or orientation motion. A beam spot offset of 3 mm are assumed for the pitch/yaw requirement.

Table 6: Requirements of the SOS control noise per mass ($40 \text{ Hz} < f < 100 \text{ Hz}$).

<i>Mode</i>	<i>Allocation to the allowed MC displacement noise</i>	<i>Control Noise</i>
Displacement	25%	$< 6.1 \times 10^{-19} \times \left(\frac{100\text{Hz}}{f}\right)^2 \text{ m}/\sqrt{\text{Hz}}$
Pitch/Yaw	10% each	$< 8.2 \times 10^{-17} \times \left(\frac{100\text{Hz}}{f}\right)^2 \text{ rad}/\sqrt{\text{Hz}}$
Total	30%	N/A

4.2.3.5 Magnet Strength

The strength of the magnets used for actuation is required to cause no more than 30% of the required frequency noise of the light coming out of the mode cleaner.

4.2.3.6 Excess Noise

TBD

4.2.4. Size Constraints

The size constraints for SOS are:

- SOS must accommodate a small optical component and must satisfy a condition of optical clear aperture.
- SOS must provide a proper vertical position for its suspended component.

The size and the optical clear aperture of small optical components and the height of the beam are

summarized in Table 7.

Table 7: Size and optical clear aperture of suspended components.

<i>Physical Quantity</i>	<i>Value</i>
Diameter of Suspended Component	7.62 cm
Thickness of Suspended Component ^a	2.54 cm
Weight of Suspended Component	0.25 kg
Required Optical Clear Aperture	ϕ32 mm
Wedge Angle of Suspended Component	0.5° horizontally configured
Beam Height	140 mm

a. The thickness is measured at the thickest part of the substrate.

5 GENERAL REQUIREMENTS

This chapter describes general requirements which are common to LOS1&2 and SOS.

5.1. General Characteristics

5.1.1. Reliability

Mean Time Between Failures (MTBF) should be **TBD**.

5.1.2. Maintainability

Mean Time To Repair (MTTR) should be less than **TBD**.

5.1.3. Environmental Conditions

5.1.3.1 Natural Environment

5.1.3.1.1 *Temperature and Humidity*

TBD

5.1.3.1.2 *Atmospheric Pressure*

TBD

5.1.3.1.3 *Seismic Disturbance*

TBD

5.1.3.2 *Induced Environment*

5.1.3.2.1 *Electromagnetic Radiation*

Per [7] “LIGO EMI Control Plan and Procedures”, LIGO-E960036-A-E

5.1.3.2.2 *Acoustic*

TBD

5.1.3.2.3 *Mechanical Vibration*

TBD

5.1.4. *Transportability*

All items shall be transportable by commercial carrier without degradation in performance. As necessary, provisions shall be made for measuring and controlling environmental conditions (temperature and accelerations) during transport and handling. Special shipping containers, shipping and handling mechanical restraints, and shock isolation shall be utilized to prevent damage. All containers shall be movable for forklift. All items over 100 lbs. which must be moved into place within LIGO buildings shall have appropriate lifting eyes and mechanical strength to be lifted by cranes.

5.2. Design and Construction

5.2.1. Materials and Processes

5.2.1.1 *Finishes*

- Metal components must have quality finishes on all surfaces, suitable for vacuum finishes.
- All materials must have non-shedding surfaces.

5.2.1.2 *Materials*

All materials used inside the vacuum must comply with [12] “LIGO Vacuum Compatibility, Cleaning Methods and Procedures”, LIGO-E960022-00-D.

5.2.1.3 *Processes*

5.2.1.3.1 *Welding*

TBD

5.2.1.3.2 Annealing

TBD

5.2.1.3.3 Cleaning

All materials used inside the vacuum chambers must be cleaned in accordance with [12] “LIGO Vacuum Compatibility, Cleaning Methods and Procedures”, LIGO-E960022-00-D.

5.2.2. Component Naming

All components shall be identified using [9] “LIGO Naming Conventions”, LIGO-E950111-A-E.

5.2.3. Workmanship

TBD

5.2.4. Interchangeability

All LOS1 components must be interchangeable among LOS1 systems except for those components that depend on wedge value and orientation. All SOS components must be interchangeable among SOS systems except for components that depend on wedge value and orientation.

5.2.5. Safety

This item shall meet all applicable NSF and other Federal safety regulations, plus those applicable State, Local and LIGO safety requirements. A hazard/risk analysis shall be conducted in accordance with guidelines set forth in section 3.3.2. of [10] “LIGO Project System Safety Management Plan”, LIGO-M950046-F.

No special considerations for human safety are presented by the suspension design, except for the normal precautions taken with electronics. The suspension actuator electronics will be capable of delivering voltages up to 150 Volts.

The following special precautions must be taken to assure safety of the equipment:

- the suspension hardware and the suspended component are ultrahigh-vacuum components and must be handled according to approved LIGO procedures to prevent contamination
- the magnet/standoff assemblies are extremely delicate and must be protected during all operations prior to mounting the suspended component into the suspension structure
- tools fabricated from metal or other hard substances should be kept away from the suspended component’s polished faces to prevent scratching or marring of these surfaces
- suspended components must always be properly locked into the safety cage before any movement of the suspension structure is attempted

5.2.6. Human Engineering

The suspension design must allow for coarse alignment of the suspended component relative to the suspension support structure to be accomplished on a clean bench outside the vacuum chamber. The alignment procedure is delicate and requires that personnel aligning an optic have comfortable access to the components. Alignment procedures within the vacuum chamber will be restricted to adjustments of the orientation of the suspension support structure. Fine alignment will be accomplished through the suspension actuators.

5.3. Documentation

5.3.1. Specifications

TBD

5.3.2. Design Documents

- Suspension Final Design

5.3.3. Engineering Drawings and Associated Lists

TBD

5.3.4. Technical Manuals and Procedures

5.3.4.1 Procedures

Procedures shall be provided for, at minimum,

- Initial installation and setup of equipment
- Normal operation of equipment
- Normal and/or preventative maintenance
- Troubleshooting guide for any anticipated potential malfunctions

5.3.4.2 Manuals

TBD

5.3.5. Documentation Numbering

All documents shall be numbered and identified in accordance with the LIGO documentation control numbering system LIGO document **TBD**

5.3.6. Test Plans and Procedures

Test plan is documented in [21] “Suspension Test Plan”, LIGO-T960086-00-D.

All procedures shall be developed in accordance with **TBD**.

5.4. Logistics

The design shall include a list of all recommended spare parts and special test equipment required.

5.5. Precedence

This section should list the relative importance of requirements (or goals) to be achieved by the design.

5.6. Qualification

Test and acceptance criteria.

6 QUALITY ASSURANCE PROVISIONS

This section includes all of the examinations and tests to be performed in order to ascertain the product, material or process to be developed or offered for acceptance conforms to the requirements in section 3.

6.1. General

This should outline the general test and inspection philosophy, including all phases of development.

6.1.1. Responsibility for Tests

Who is responsible for testing.

6.1.2. Special Tests

6.1.2.1 Engineering Tests

List any special engineering tests which are required to be performed. Engineering tests are those which are used primarily for the purpose of acquiring data to support the design and development.

6.1.2.2 Reliability Testing

Reliability evaluation/development tests shall be conducted on items with limited reliability history that will have a significant impact upon the operational availability of the system.

6.1.3. Configuration Management

Configuration control of specifications and designs shall be in accordance with the LIGO Detector Implementation Plan.

6.2. Quality conformance inspections

Design and performance requirements identified in this specification and referenced specifications shall be verified by inspection, analysis, demonstration, similarity, test or a combination thereof per the Verification Matrix, Appendix 1 (See example in Appendix). Verification method selection shall be specified by individual specifications, and documented by appropriate test and evaluation plans and procedures. Verification of compliance to the requirements of this and subsequent specifications may be accomplished by the following methods or combination of methods:

6.2.1. Inspections

Inspection shall be used to determine conformity with requirements that are neither functional nor qualitative; for example, identification marks.

6.2.2. Analysis

Analysis may be used for determination of qualitative and quantitative properties and performance of an item by study, calculation and modeling.

6.2.3. Demonstration

Demonstration may be used for determination of qualitative properties and performance of an item and is accomplished by observation. Verification of an item by this method would be accomplished by using the item for the designated design purpose and would require no special test for final proof of performance.

6.2.4. Similarity

Similarity analysis may be used in lieu of tests when a determination can be made that an item is similar or identical in design to another item that has been previously certified to equivalent or more stringent criteria. Qualification by similarity is subject to Detector management approval.

6.2.5. Test

Test may be used for the determination of quantitative properties and performance of an item by technical means, such as, the use of external resources, such as voltmeters, recorders, and any test equipment necessary for measuring performance. Test equipment used shall be calibrated to the

manufacture's specifications and shall have a calibration sticker showing the current calibration status.

7 PREPARATION FOR DELIVERY

Packaging and marking of equipment for delivery shall be in accordance with the Packaging and Marking procedures specified herein.

7.1. Preparation

Equipment shall be appropriately prepared. For example, vacuum components shall be prepared to prevent contamination.

7.2. Packaging

Procedures for packaging shall ensure cleaning, drying, and preservation methods adequate to prevent deterioration, appropriate protective wrapping, adequate package cushioning, and proper containers. Proper protection shall be provided for shipping loads and environmental stress during transportation, hauling and storage.

7.3. Marking

Appropriate identification of the product, both on packages and shipping containers; all markings necessary for delivery and for storage, if applicable; all markings required by regulations, statutes, and common carriers; and all markings necessary for safety and safe delivery shall be provided.

APPENDIX A TRANSFER FUNCTION OF SOS SUSPENSION

A.1. Transfer Function from Horizontal to Horizontal, T_{hh}

The transfer function is required to represent a single pendulum performance.

A.2. Transfer Function from Horizontal to Pitch, T_{hp}

The requirement is obtained by demanding the resultant cavity length variation due to pitch motion with a beam spot offset $d = 3$ mm to be 10% of the transfer function from horizontal to horizontal:

$$d \times T_{hp} < 0.1 \times T_{hh}. \quad (1)$$

A.3. Transfer Function from Vertical to Vertical, T_{vv}

The transfer function is required to represent a single spring performance. Incidentally the resultant vertical motion contributes to the cavity length variation due to a misalignment of the optical axis with the local perpendicular to gravity (3×10^{-4}).

A.4. Transfer Function from Vertical to Horizontal, T_{vh}

The requirement is obtained by demanding the resultant horizontal motion to be 10% of the resultant cavity length variation due to the vertical to vertical transfer function:

$$T_{vh} < 0.1 \times (3 \times 10^{-4}) \times T_{vv}. \quad (2)$$

A.5. Transfer Function from Vertical to Pitch, T_{vp}

The requirement is obtained by demanding the resultant cavity length variation due to pitch motion with a beam spot offset $d = 3$ mm to be 10% of the resultant cavity length variation due to the vertical to vertical transfer function:

$$d \times T_{vp} < 0.1 \times (3 \times 10^{-4}) \times T_{vv}. \quad (3)$$

APPENDIX B THERMAL NOISE FOR SOS

B.1. Internal Mode

We allocate 50% of the allowed MC displacement noise (1.2×10^{-18} m/ $\sqrt{\text{Hz}}$ at 100 Hz per MC mirror) to the internal mode thermal noise.

Thermal noise due to internal vibrations of the mirrors was estimated following the method used in [22] ‘‘Thermally Excited Vibrations of the Mirrors of Laser Interferometer Gravitational-Wave Detectors’’, LIGO-P940003-00-R.

$$\tilde{x}_{\text{MC, int}}(f) = 1.3 \times 10^{-18} \frac{\text{m}}{\sqrt{\text{Hz}}} \cdot \left(\frac{100 \text{ Hz}}{f}\right)^{1/2} \cdot \left(\frac{\phi(f)}{10^{-5}}\right)^{1/2} \quad (4)$$

for the average displacement noise per MC mirror, accounting for the finite spot sizes of the beams on the flat and curved mirrors, and summing over the appropriate internal modes of the mirrors. Therefore the average effective Loss of internal modes due to attachments must be less than 9.6×10^{-6} .

B.2. Pendulum Mode

We allocate 20% of the allowed MC displacement noise ($4.9 \times 10^{-19} \text{ m}/\sqrt{\text{Hz}}$ at 100 Hz per MC mirror) to the pendulum mode thermal noise.

The spectral density of the displacement due to the pendulum thermal noise is:

$$\tilde{x}_{\text{MC, pen}}^2(f) = \frac{4k_{\text{B}}T}{M} \cdot \frac{\omega_0^2 \varphi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4 \varphi^2(f)]} \quad (5)$$

where $k_{\text{B}} = 1.4 \times 10^{-23} \text{ J/K}$, $T = 295 \text{ K}$, $M = 0.25 \text{ kg}$, and $\omega_0 = 1 \times 2\pi$ are Boltzman's constant, the temperature, the mass, and the resonant angular frequency, respectively. $\varphi(f)$ must be less than 9.0×10^{-6} .

B.3. Pitch and Yaw Mode

We allocate 20% of the allowed MC displacement noise ($4.9 \times 10^{-19} \text{ m}/\sqrt{\text{Hz}}$ at 100 Hz per MC mirror) to each of the pitch and yaw mode thermal noise.

Suspension pitch/yaw thermal noise couples with the beam spot offset from the center of mass to be converted into the cavity length variations. Since the requirement for the beam spot offset (IOO) is 3 mm, the required angle fluctuation is $1.6 \times 10^{-16} \text{ rad}/\sqrt{\text{Hz}}$ at 100 Hz per MC mirror both for pitch and yaw.

The spectral density of orientation due to the pitch/yaw thermal noise is:

$$\tilde{\theta}_{\text{MC, pit/yaw}}^2(f) = \frac{4k_{\text{B}}T}{I} \cdot \frac{\omega_0^2 \varphi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4 \varphi^2(f)]} \quad (6)$$

where $k_{\text{B}} = 1.4 \times 10^{-23} \text{ J/K}$, $T = 295 \text{ K}$, $I = 1.0 \times 10^{-4} \text{ kgm}^2$, and $\omega_0 = 0.85 \times 2\pi$ or $0.75 \times 2\pi$ are Boltzman's constant, the temperature, the momentum of inertia, and the resonant angular frequency (for pitch and yaw), respectively. $\varphi(f)$ must be less than 5.3×10^{-4} for pitch and 6.8×10^{-4} for yaw, respectively.

B.4. Vertical Mode

We allocate 10% of the allowed MC displacement noise ($2.4 \times 10^{-19} \text{ m}/\sqrt{\text{Hz}}$ at 100 Hz per MC mirror) to the vertical mode thermal noise.

Vertical thermal noise contributes to the interferometer noise due to a misalignment of the mode cleaner mirrors from the local verticals, that is 3×10^{-4} . Therefore the vertical motion must be less than $8.2 \times 10^{-16} \text{ m}/\sqrt{\text{Hz}}$ at 100 Hz per MC mirror. The spectral density of displacement due to the vertical thermal noise is:

$$\tilde{z}_{\text{MC, vert}}^2(f) = \frac{4k_{\text{B}}T}{M} \cdot \frac{\omega_0^2 \phi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4 \phi^2(f)]} \quad (7)$$

where $k_{\text{B}} = 1.4 \times 10^{-23} \text{ J/K}$, $T = 295 \text{ K}$, $M = 0.25 \text{ kg}$, and $\omega_0 = 16 \times 2\pi$ are Boltzman's constant, the temperature, the mass, and the resonant angular frequency, respectively. $\phi(f)$ must be less than 9.9×10^{-2} .

B.5. Eddy Current Damping

We allocate 30% of the allowed MC displacement noise ($7.3 \times 10^{-19} \text{ m}/\sqrt{\text{Hz}}$ at 100 Hz per MC mirror) to the eddy current damping thermal noise.

The spectral density of displacement due to the eddy current thermal noise in pendulum mode is:

$$\tilde{x}_{\text{MC, edd}}^2(f) = \frac{4k_{\text{B}}T}{M} \cdot \frac{\omega_0^2 \phi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4 \phi^2(f)]} \quad (8)$$

where $k_{\text{B}} = 1.4 \times 10^{-23} \text{ J/K}$, $T = 295 \text{ K}$, $M = 0.25 \text{ kg}$, and $\omega_0 = 1 \times 2\pi$ are Boltzman's constant, the temperature, the mass, and the resonant angular frequency, respectively. $\phi(f)$ must be less than 2.0×10^{-5} at 100 Hz.