New Folder Name Pulsar Searches

Kip Thorne

Pulsar Searches by Schutz's Stepping Method

LHR → LAX
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2:45 PM PDT

A. The Method

1. See Schutz, in the Blair Volume, for the method [pp 434–437]

B. Schutz's Improvement on the Method [Told to me in Cardiff this week]

1. Do the search in 2 or 3 steps:

   For a time τ_1 with some threshold σ_1

   Then for a time τ_2, on only those ~~points~~ sky regions that gave $\sigma > \sigma_1$; with some threshold $\sigma_2 > \sigma_1$

   Then for a time τ_3, on only those regions that gave $\sigma > \sigma_2$, with some threshold $\sigma_3 > \sigma_2$.

2. Adjust (σ_1, τ_1), (σ_2, τ_2), (σ_3, τ_3) so:

   a. The total ~~computation time~~ number of flops to get to (σ_3, τ_3) is minimized and is equal to some pre-chosen number, say $N_{flops} \simeq 10^{20}$ corresponding to 3 yrs at one teraflop.

   [Presumably each step will end up using about the same number of flops.]

3. I did some rough calculations (attached) which suggested that for a 2-step algorithm, using 2 detectors, with $\sigma_2 = 5$ [for high confidence of any discovered signals], to keep $N_{flops} \lesssim 3 \times 10^{19}$ we must take $\tau_2 \lesssim 2 \times 10^6$ sec .... which means we lose a factor 2 in the strength of a detectable pulsar compared to the usual "canonical" 10^7 sec search.

C. Issues

1. I'm very suspicious of Schutz's estimates of the number of independent Doppler patches on the sky

   a. When $\hat{\tau} \gtrsim \pi/\Omega$, I think the size of a Doppler patch does not become constant as he claims, but rather keeps decreasing as $\Delta\theta \propto 1/\hat{\tau}$ [because the bandwidth of the Doppler corrections becomes constant, but the bandwidth to which we are sensitive keeps decreasing as $1/\hat{\tau}$].

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a. More specifically, I claim that Schutz's (16.39) should be:

$$\Delta\theta = \min_{\uparrow \atop \text{not max}} \left\{ \underbrace{\frac{c}{\Omega R}\frac{1}{f\hat{\tau}}}_{\uparrow \atop \text{he claims } \frac{4c}{fR}}, \frac{c}{\Omega^2 R f \hat{\tau}^2} \right\}$$

2. For times $\hat{\tau} \gg 12$ hrs & $\hat{\tau} \ll 6$ months, I think the Earth's orbital motion will constrain the Doppler correction box in only one dimension; the other dimension will be constrained (less severely) by the Earth's rotation.

   a. This reduces the number of patches on the sky from Schutz's $N_{patch} = \dfrac{4\pi}{(\Delta\theta_{orbit})^2} = 1\times10^{13}\left(\dfrac{f}{1\text{kHz}}\right)^2\left(\dfrac{\hat{\tau}}{10^7\text{s}}\right)^4$ [his Eq. (16.42)] to

$$N_{patch} = \frac{4\pi}{(\Delta\theta_{orbit})(\Delta\theta_{rot})} \simeq 3\times10^{11}\left(\frac{f}{1\text{kHz}}\right)^2\left(\frac{\hat{\tau}}{10^7\text{s}}\right)^3$$

   — a reduction that can be ~ 3 to 10.

   b. We need to look much more carefully at the number of patches.

3. Somebody should evaluate the size of the computation task along the lines of B. above, for one detector & for two — and determine thereby, as a function of computing power, just how deep we can go in an all sky ~~search~~ & all $f \lesssim 2$ kHz search.

4. We should also look at whether anything significant can be gained by using dual recycled detectors in the search.

Pulsar Searches

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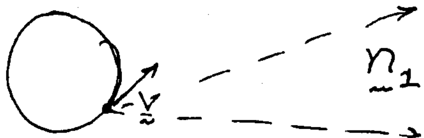
A. Doppler Boxes

1. Differential Doppler shift for two points on the sky, separated by $\Delta\theta$

a. Detector orbits @ radius R, ang vel Ω



b. Suppose we integrate over $\hat{\tau} \gtrsim 2\pi/\Omega$

$$\frac{\Delta f}{f} = \frac{\delta \underset{\sim}{v} \cdot (\underset{\sim}{n}_2 - \underset{\sim}{n}_1)}{c} \qquad \delta \underset{\sim}{v} = 2\Omega R \underset{\sim}{n} \;;\; \underset{\sim}{n} \cdot (\underset{\sim}{n}_1 - \underset{\sim}{n}_2) = \Delta\theta$$

$$= \frac{2\Omega R \Delta\theta}{c}$$

$$\Rightarrow \Delta f = \frac{2\Omega R \Delta\theta}{c} \cdot f < \underbrace{\Delta f_{BW}}_{\text{bandwidth}} = \frac{2}{\hat{\tau}}$$

$$\Rightarrow \Delta\theta < \frac{c}{\Omega R} \frac{1}{f\hat{\tau}}$$

c. So the size of the region that can be treated as having the same Doppler shift is

$$\boxed{\Delta\theta \simeq \frac{c}{\Omega R} \frac{1}{f\hat{\tau}}} \quad \text{for } \Omega\hat{\tau} \gtrsim 2\pi$$

d. For $\Omega\hat{\tau} \ll 2\pi$:

$$\delta \underset{\sim}{v} = \cancel{2}\Omega^2 R \hat{\tau} \underset{\sim}{n}$$

$$\frac{\Delta f}{f} = \frac{\cancel{2}\Omega^2 R \hat{\tau} \cdot \Delta\theta}{c} < \frac{2}{f\hat{\tau}} \Rightarrow \boxed{\Delta\theta \simeq \frac{c}{\Omega^2 R f \hat{\tau}^2}} \text{ for } \Omega\hat{\tau} \ll$$

2. Earth rotation & $\hat{\tau} \gtrsim \frac{1}{2}$ day $\quad \Omega = \frac{2\pi}{24\,\mathrm{hr}} \quad R = R_\oplus, \; f = 1\,\mathrm{kHz}$

$$\Delta\theta \simeq \frac{3\times10^5\,\mathrm{km/s}}{(2\pi/8.6\times10^4\,\mathrm{s})(6.4\times10^3\,\mathrm{km})} \frac{1}{10^3\,\mathrm{s}^{-1} \cdot (8.6\times10^4\,\mathrm{s})\,\hat{\tau}/\mathrm{day}}$$

$$\boxed{\Delta\theta \simeq \frac{0.007\,\mathrm{rad}}{\hat{\tau}_{\mathrm{days}}} \simeq \frac{7\times10^{-5}\,\mathrm{rad}}{(\hat{\tau}/100\,\mathrm{days})} \quad \text{for earth rotation}, \; \hat{\tau} \gtrsim \tfrac{1}{2}\,\mathrm{day}}$$

3. Earth-Moon Motion & $\hat{\tau} \gtrsim 1$ month $\Omega = \frac{2\pi}{1\,\text{mo}}$, $R \simeq ?$

$$\boxed{\Delta\theta \simeq \frac{(2\times10^{-3}\,\text{rad})}{\hat{\tau}_{\text{months}}} \simeq \frac{5\times10^{-4}\,\text{rad}}{(\hat{\tau}/4\,\text{mos})} \quad \text{for earth-moon motion}}$$

4. Earth-Sun Motion:

$$\Delta\theta \simeq \frac{c}{\Omega^2 R f \hat{\tau}^2} \simeq \frac{3\times10^{10}\,\text{cm/s}}{(2\pi/3\times10^7\text{s})^2(1.5\times10^{13}\text{cm})(10^3\text{Hz})(10^7\text{s})^2}$$

$$\boxed{\Delta\theta \simeq \frac{5\times10^{-7}\,\text{rad}}{(\hat{\tau}/10^7\text{s})^2} \quad \begin{array}{l}\text{for earth-sun motion}\\ \hat{\tau} \lesssim 10^7\,\text{sec}\end{array}}$$

$\leftarrow 0.''1$

~~3.~~

5. Size of Doppler Correction Boxes:

a. The above sizes are correct only along one dimension.

b. The earth-sun effect will dominate on one dimension; the Earth rotation along the other. This makes the number of boxes, for $10^7\,\text{sec} = \hat{\tau}$, 100 x smaller than Bernie estimates: $\sim(\Delta\theta_{\oplus\odot} \cdot \Delta\theta_{rot})^{-1}$

— No: only so for $\hat{\tau} \ll 10^7$s because then only then is $\underline{V}$ nearly fixed in direction!

~~6.~~

6. Number of Doppler boxes:

a. I claim: ~~For 10^7s independent~~

$$N_{\text{patches}} \simeq 3\times10^{11}\left(\frac{f}{1\,\text{kHz}}\right)^2\left(\frac{T_{obs}}{10^7\text{s}}\right)^3$$

if $T_{obs} \gg 1$ day, $T_{obs} \ll 10^7$ sec

b. Bernie gives, instead

$$N_{\text{patches}} = 1\times10^{13}\left(\frac{f}{1\,\text{kHz}}\right)^2\left(\frac{T_{obs}}{10^7\text{s}}\right)^4$$

~~7.~~

3. Use Bernie's Method

1. Heterodyne the signal down to the Doppler shift band width:

$$\Delta f = \frac{\Delta V}{c} f \simeq \frac{2\Omega R_\oplus}{c} f = 2 \times \frac{2\pi}{3\times10^7 s} \times \frac{1.5\times10^8 km}{3\times10^5 km/s} \times 10^3 Hz$$

$$= \underbrace{\frac{60 \, km/s}{3\times10^5 km/s}}_{2\times10^{-4}} \times 10^3 Hz = \underline{\underline{0.2 \, Hz}} = \Delta$$

$$\Rightarrow \Delta t = \frac{1}{2\Delta f} = \frac{1}{.4} = 2.5 \sec$$

2. Then in $\hat{\tau} \simeq 10^7 \sec$ we have

$$N_{pt} \simeq \frac{\hat{\tau}}{\Delta t} = 4\times10^6 \text{ data points.}$$

However – such heterodyning restricts us to look for signals in only a band Δf ?! – We want to search over 1000 Hz band, so we should not heterodyne down !!

3. Thus, we must keep $N_{pts} = 2 f_u \hat{\tau}$ where f_u is the upper limit on the frequency to which we search.

~~4. Suppose we~~

4. What S/N do we require to believe a signal? [@ two detectors]

a. With $f/\Delta f = 2f\hat{\tau}$ frequency bands $= 2\cdot10^{10} \frac{f}{10^3 Hz} \frac{\hat{\tau}}{10^7 sec} = N_{pts}$

we need $\underbrace{\left[\frac{2}{\sqrt{2\pi}}\int_\sigma^\infty e^{-x^2/2} dx\right]^2}_{} << \frac{1}{N_{pts}} = \frac{1}{2\times10^{10}}$

$$\frac{2}{\pi}\left[\sigma e^{-\sigma^2/2}\right]^2 \ \ldots \ \underline{\underline{\sigma \geq 5}} \text{ is needed.}$$

b.

5. $(S/N)^2$ grows linearly in time; we need $(S/N)^2 \geq 25$.

6. Suppose we integrate for time $\tau_* < \hat{\tau}$. Consider a pulsar that has $(S/N)^2 = 25 \frac{\tau_*}{\hat{\tau}} = \sigma_*^2$

~~a. What are the odds that it gives~~

a. Probability of a false alarm is $\frac{2}{\pi}\left[\sigma_*^2 e^{-\sigma_*^2}\right] = P_F(\sigma_*)$.

This is the amount by which we reduce the number of sky points we subsequently must search, if we threshold @ σ_*.

b. But we don't want to threshold [illegible] because [illegible] we would lose some of our signals: We want to keep the probability of missing a true signal below, say, $q \simeq 0.1$.

~~P_{m}~~ =

c. The true signal will be S_*. The actual noise is N; so we see an actual signal $S = S_* + N$; and
$$\frac{S}{N_{rms}} = \frac{S_*}{N_{rms}} + \frac{N}{N_{rms}} = \sigma_* + \frac{N}{N_{rms}}$$

d. The probability that $\frac{N}{N_{rms}} \geq n$ ~~[illegible]~~ $- n_0$ in both detectors is < 0.1 if $n_0 \simeq 1$, so —

For a rough estimate put $\frac{S}{N_{rms}} = \sigma_* - 1$ as the threshold.

7. Then $\boxed{P_F(\sigma_* - 1) = \frac{2}{\pi}\left[(\sigma_* - 1)^2 e^{-(\sigma_* - 1)^2}\right]}$ is the amount by which we reduce the number of sky points for the next pass.

8. Try to make the computational task equal on the two passes.

a. First pass, if I am right about $N_{patches}$

$$N_{patches} = \underbrace{3\times10^{11}}_{\cancel{[illegible]}} f_3^{\,2}\, \tau_{*7}^{\,3}$$

$$N_{points} = 2\times10^{10} f_3\, \tau_{*7}$$

$$N_{flops\,1} = \underbrace{10 \times 3\times10^{11} \times 2\times10^{10} f_3^{\,3}}_{6\times10^{22} f_3^{3} = N_1} \tau_{*7}^{\,4} = N_1\, \tau_{*7}^{\,4}$$

b. 2nd pass $\hat{\tau} = 10^7$

$$N_{flops} = 1\times10^{13} f_3^{\,2}\; 2\times10^{10} f_3 \times 10 \cdot P_F(\sigma_* - 1)$$

$$N_{flops\,2} = \underbrace{2\times10^{24} f_3^{\,3}}_{N_2}\; P_F(\sigma_* - 1)$$

c. $P_F(\sigma_* - 1) = \frac{N_1}{N_2} = \frac{1}{30}\tau_{*7}^{\,4}$. $\tau_* = \frac{\sigma_*^{\,2}}{25}$

d. $\boxed{P_F(\sigma_* - 1) = \frac{2}{\pi}(\sigma_* - 1)^2 e^{-(\sigma_* - 1)^2} = \frac{1}{30}\left(\frac{\sigma_*}{5}\right)^8}$

e, same

σ_*	$\frac{1}{30}\left(\frac{\sigma_\alpha}{5}\right)^8$	$\frac{2}{\pi}(\sigma_\alpha - 1)^2 e^{-(\sigma_\alpha - 1)^2}$
2	2×10^{-5}	2×10^{-1}
3	6×10^{-4}	5×10^{-2}
4	3×10^{-2}	7×10^{-4}
3.5	2×10^{-3}	8×10^{-3}
3.7	3×10^{-3}	3×10^{-3}

f. Then

$$N_{flops} = N_{flops\,1} + N_{flops\,2}$$

$$= N_2 \times 3\times10^{-3} = 6\times10^{21} = 6\times10^{9} \times 10^{12}$$

$\Rightarrow$ still need 200 years on a teraflop machine !

9. To make it doable on a 1 teraflop machine, we need $P_F(\sigma_\alpha - 1) \cancel{\simeq 3\times10^{19}}$ to reduce N's by 200 $\Rightarrow$ $\hat{\tau}$ down by $200^{1/4} \simeq 4$

$\Rightarrow$ we can only integrate for $\simeq 3\times10^6$ sec $\simeq \hat{\tau}$

Since ~~Z~~ $S/N \propto \hat{\tau}^{1/2}$ we lose a factor 2 in the strength of a pulsar that we can detect.

10. We can do the same thing in 3 steps.