New Folder Name Asymmetery Effects

→ Martin, FYI

LIGO PROJECT

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

FACSIMILE COVER SHEET

LIGO FAX (818) 304-9834 TELEPHONE CONFIRMATION (818) 395-2966

TO	David Shoemaker Joe Giaime	DATE	July 14, 1993
		FAX NUMBER	(617) 253–7014
		OFFICE NUMBER	(617) 253–6411

NUMBER OF PAGES (including this cover sheet): 7

FROM	Fred Raab California Institute of Technology 102–33 Bridge Laboratory Pasadena, CA 91125	OFFICE NUMBER	(818) 395-4053

Attached are notes on the calculation of asymmetry effects on the frequency, beam perturbation, and intensity noise sensitivity of a 40–meter interferometer. I expect LIGO interferometers to be less sensitive to such effects, and I was not sufficiently familiar with the parameters for the 5–meter interferometer to put in numbers for that case. Anyway, if we can all bless the formulae we can put numbers in later. Martin has checked me on two of the calculations, but not yet on the intensity noise estimate, which I revised again only last night. Expect an estimate for contrast and mode matching degradations tomorrow.

LIGO-T930032-00-D

Effect of Asymmetry on Sensitivity to Freq. Noise

FJR 6/21/93
MR 7/13/93

$$\phi_{FP}(f) = \frac{8\pi c \tau_s}{\lambda L} \left[\frac{1}{1+(2\omega\tau_s)^2}\right]^{1/2} \Delta x(f)$$

where $\phi_{FP}(f)$ is phase of reflected light from FP cavity

τ_s is energy storage time

$$\phi_A(f) = \frac{4\pi\delta}{\lambda} \frac{\Delta\nu(f)}{\nu_0}$$

where δ is asymmetry $\begin{pmatrix} +\delta \text{ in one arm} \\ -\delta \text{ in other} \end{pmatrix}$

$\frac{\Delta\nu(f)}{\nu_0}$ = fractional freq noise = $\frac{\Delta x(f)}{L}$

(equiv displaceme

$$\left.\begin{matrix}\text{Ratio of} \\ \text{asymmetry} \\ \text{phase to} \\ \text{FP phase} \\ \text{w/ } \Delta\nu(f) \\ \text{present}\end{matrix}\right\} = \frac{\phi_A(f)}{\phi_{FP}(f)} = \frac{\delta}{2c\tau_s}\left[1+(4\pi f \tau_s)^2\right]^{1/2}$$

(Note for $\tau_s \simeq 2\times10^{-3}$ s, we have $c\tau_s = 600$ km)

Effect of Asymmetry on Noise from Spatial Beam Perturbations

FJR 7/4/93
MR 7/13/93

The amplitudes of higher order modes for mode-matching errors due to small spatial misalignments in angle α, lateral position a, waist size error Δw_0, and waist position error Δz, are given by

$$\epsilon_\alpha \simeq \frac{\alpha}{\theta_D} \tag{1}$$

$$\epsilon_a \simeq \frac{a}{w_0} \tag{2}$$

$$\epsilon_{\Delta w_0} \simeq \frac{\Delta w_0}{w_0} \tag{3}$$

$$\epsilon_{\Delta z} \simeq \frac{\Delta z}{2 z_0} \tag{4}$$

respectively, where $\theta_D = \frac{\lambda}{\pi w_0}$ is the far-field diffraction angle of the cavity, w_0 is the waist size, and z_0 is the confocal parameter. Consider the interferometer shown in Figure 1 with asymmetry parameter δ (i.e., the distance between the beam splitter and the two arm-cavity input mirrors is 2δ). The only term involving δ directly is

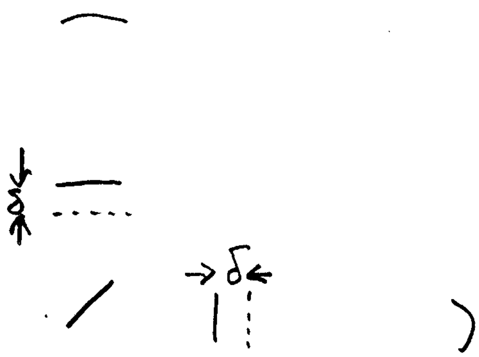


Figure 1.

given by

$$\epsilon_{\Delta z} \simeq \frac{\delta}{2z_0} \qquad (5)$$

The asymmetry can also cause a differential response of the two arm cavities due to the second order response to a transverse angular misalignment α. Such an error, coupling to the asymmetry, gives different lateral displacements of the light at the input mirrors. This gives rise to a higher order mode amplitude

$$\epsilon_{\alpha,\delta} \simeq \frac{\alpha\delta}{w_0} \qquad (6)$$

The effect of any static higher-order mode amplitude ϵ_s is to allow any fluctuating amplitude $\epsilon(t)$ in the same mode to produce phase noise $\phi_N(t)$ in light reflected from the cavity such that

$$\phi_N(t) \propto \frac{\epsilon_s \, \epsilon(t)}{\mathcal{F}^2} \qquad (7)$$

where $\mathcal{F}$ is the cavity finesse.

The important higher order mode amplitudes will be more important in the 40-meter prototype than in LIGO. Using (equations (5) and (6) ~~and~~) the parameters of the current 40-meter prototype ($w_0 = 2.15$ mm, $z_0 = 28$ m, $\theta_D = 7.6 \times 10^{-5}$) gives

$$\epsilon_{\Delta z} \simeq 5 \times 10^{-3} \left[\frac{\delta}{30\,\mathrm{cm}} \right] \qquad (8)$$

$$\epsilon_{\alpha,\delta} \leq 10^{-2} \left[\frac{\delta}{30\,\mathrm{cm}} \right] \qquad (9)$$

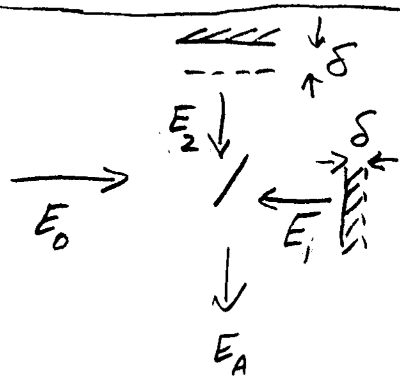
For comparison, a mode-matching error of 1% in power corresponds to higher mode amplitudes of order $\epsilon_s \simeq 0.1$. Also note that equation (9) is a worst case estimate corresponding to a misalignment angle $\alpha \simeq \theta_D$. Thus we may conclude that asymmetry-related sensitivity to spatial beam perturbations is small.

Feedthrough of Intensity Noise Due to Asymmetry

FJR 7/7/93
rev'd 7/13/93

Let $E_0 = \left(\frac{a}{4}e^{i\Omega t} + 1 + \frac{a}{4}e^{-i\Omega t}\right)e^{i\omega t}$

$\Rightarrow P_0 = 1 + a\cos\Omega t + \xrightarrow{\text{neglect}} \cancel{\Theta(a^2)}$; $a = \Delta P_0$



$$E_A = \frac{1}{\sqrt{2}}(E_1 - E_2) = \frac{1}{2}\Big[\frac{a}{4}e^{i\Omega(t-\tau)} + 1 + \frac{a}{4}e^{-i\Omega(t-\tau)}$$
$$- \frac{a}{4}e^{i\Omega(t+\tau)} - 1 - \frac{a}{4}e^{-i\Omega(t+\tau)}\Big]$$

where $\tau = \frac{2\delta}{c}$; $\omega\tau = 2\pi\cdot(\text{integer})$

$$P_A = |E_A|^2 = \frac{1}{4}\left[\frac{a}{2}\left(\cos\Omega(t-\tau) - \cos\Omega(t+\tau)\right)\right]^2$$
$$= \frac{a^2}{16}\Big[\cos\Omega t\cos\Omega\tau + \sin\Omega t\sin\Omega\tau$$
$$-\cos\Omega t\cos\Omega\tau + \sin\Omega t\sin\Omega\tau\Big]^2$$
$$= \frac{a^2}{16}\cdot 4\sin^2\Omega t\sin^2\Omega\tau$$

$$P_A \simeq \frac{1}{4}\cdot\Delta P_0\cdot(\Omega\tau)^2\cdot\sin^2\Omega t$$

$$P_A \simeq 4\times 10^{-13}\cdot\left[\frac{f}{100\,Hz}\right]^2\cdot\left[\frac{\delta}{30\,cm}\right]\cdot\Delta P_0\cdot\sin^2\Omega t$$

Compare to contrast-limited intensity feedthrough:

$$P_A = \underbrace{(1-c)}_{\sim 10^{-3}} \cdot P_0$$