New Folder Name Wedges

CIRCA 7/92

The Use of Wedges in the LIGO Interferometer

Yaron Hefetz
Nergis Mavalvala

LIGO-T920034-00-D

I. INTRODUCTION

Wedges are an essential part of the LIGO interferometer. Wedges are optical components with surfaces that are not parallel but are at a slight angle. Such components are used to ensure that parasitic cavities do not contaminate the interferometer. The interferometer is currently envisioned to have a pick-off between the beamsplitter and the near test mass in each arm, as shown in Figure 1.

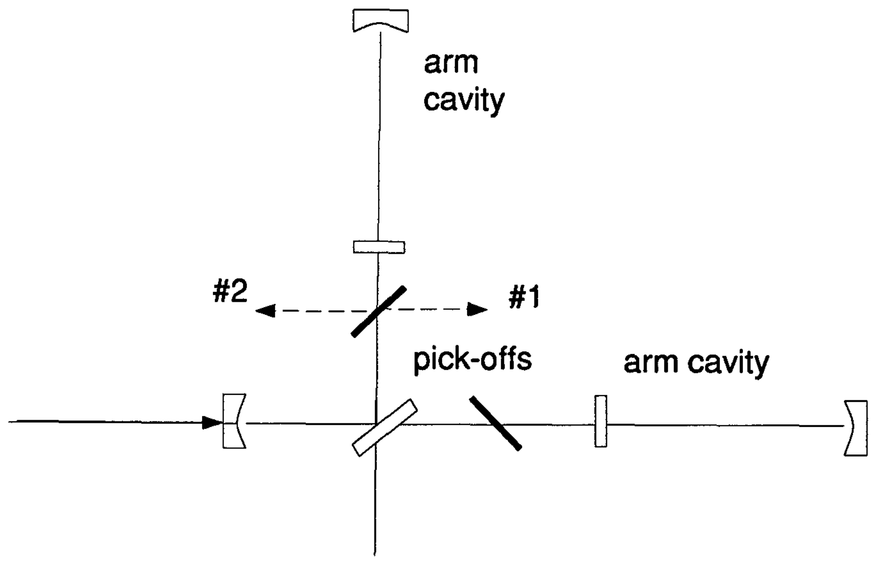

Figure 1

We are proposing that the plane (back) surfaces of each of the near test mass mirrors be angled.

This not only allows us to dispense with the pick-off in the each arm, but it can also act as a sensor for detecting global angular misalignment of the near mirror even before the cavity is aligned or locked. As shown in Figure 2, the beam position with respect to the detector is an indicator of the angle of the mirror.

In this memo we will address the following questions:

1. What are the necessary wedge angles?
2. How such a scheme effects the beam diameter (astigmatism)?

3. How this, in turn, effects the coupling efficiency into the cavity?

We demonstrate that the coupling efficiency is reduced by less than a few parts in 10^{-11}.

II. CALCULATIONS FOR WEDGE ON NEAR TEST MASS

(i) *Condition of minimum for the angle of the surface*:

Figure 2 shows a test mass in which the AR coated face makes a wedge at angle α. By applying a coating with small (˜.1%) reflectance this face can be used as a pick-off. Beam #1 gives information on the incoming light, while #2 monitors the light reflected from the cavity (beam #3 is very weak ˜.00003 of the incoming light).

Applying Snell's law to the ray at the angled surface, we get

$$\sin(\alpha') = n \sin(\alpha)$$

which can be written for $\alpha << 1$ as

$$\alpha' \simeq n\,\alpha$$

For an incident beam of diameter ω_0, we require that the separation between the incident and specular reflected beam be at least three times ω_0 at some distance L

$$2\alpha' L \geq 3\omega_0$$

then we get a condition on α,

$$\alpha \geq \frac{3\omega_0}{2nL}$$

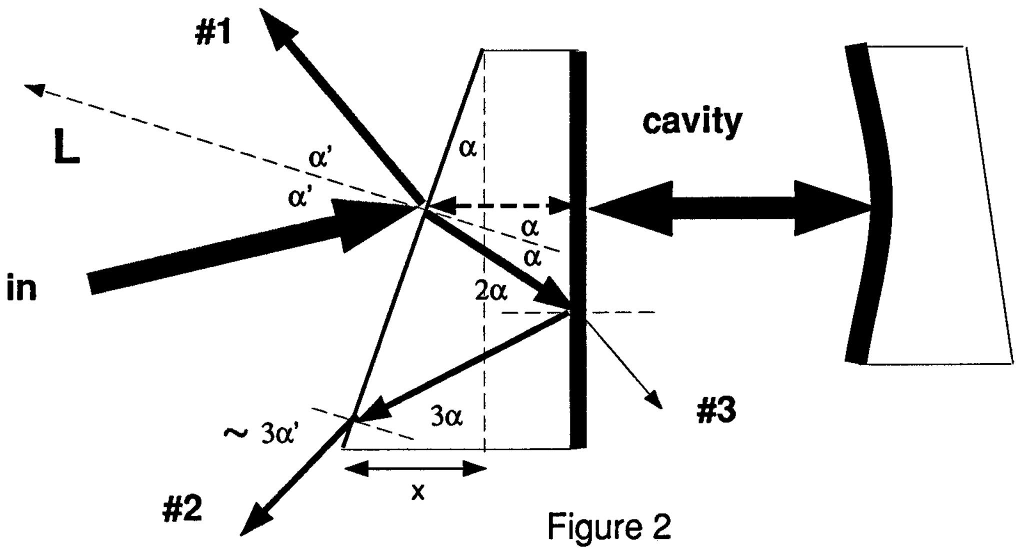

Figure 2

To estimate of the minimum value of α for LIGO, we choose L=6.5 m as convenient distance to require the separation of $3\omega_0$ between the incident and reflected beam. Using ω_0=2.6 cm and n=1.5, we get

$$\alpha = 4\,\text{mrad} \;\approx 14\,\text{min}$$

If we take the mirror diameter to be 20 cm, then the plane surface has to be angled such that

$$x = 0.08\,\text{cm} \approx 1\,\text{mm}$$

which is remarkably small. In other words, introducing a slight angle on the plane face would be sufficient to use the face as a pick-off.

(ii) *Astigmatism due to change in beam diameter:*

Figure 3 illustrates how the beam diameter is altered due the wedge. The ratio of the beam diameters, using the same parameters as above is

$$\frac{\omega'}{\omega_0} = \frac{\cos(\alpha')}{\cos(\alpha)} = 0.99999$$

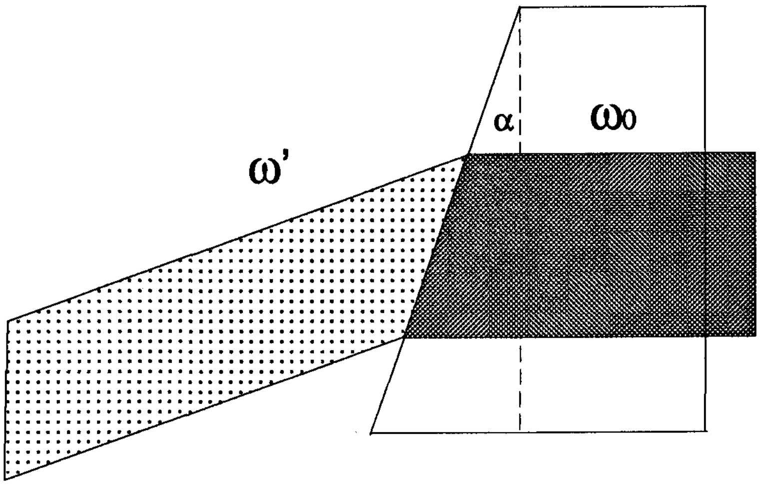

Figure 3

(iii) *Coupling efficiency:*

To calculate the coupling efficiency, we define the normalized functions

$$f(x,y) = \sqrt{\frac{2}{\pi}}\frac{1}{\omega_0}e^{-\frac{(x^2+y^2)}{\omega_0^2}}$$

for the original beam. The beam that is distorted by astigmatism in one dimension only, the y - direction, for example, is given by

$$f'(x,y) = \sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{\omega_0}}\frac{1}{\sqrt{\omega'}}e^{-\left(\frac{x^2}{\omega_0^2}+\frac{y^2}{\omega'^2}\right)}$$

The coupling efficiency is given by the overlap integral for these two functions

$$C = \int dx \int dy\; f(x,y)f'(x,y) = \sqrt{\frac{2\,\omega_0\omega'}{(\omega_0^2+\omega'^2)}}$$

and if we insert

$$\omega' = 0.99999\;\omega_0$$

we get

$$C = 1 - 2.5 \times 10^{-11}$$

which is a few parts in 10^{-11}, as promised.

III. CALCULATION FOR WEDGED BEAMSPLITTER

Clearly the test mass is not the only feasible place for a wedge. Conceivably, we could have a wedge on the beamsplitter. We now explore this possibility.

(i) *Separation of the beams at distance L*:

As we did above, Snell's law can be applied to Figure 4 to get

$$\theta = \sin^{-1}\left(\frac{\sin\frac{\pi}{4}}{n}\right) = 0.49088\,\text{rad}$$

With α = 0.004 rad (the minimum value of α that we found in Section II) we get

$$\theta' = \theta - \alpha = 0.48688\,\text{rad}$$

$$\phi = \sin^{-1}\left(n\sin\theta'\right) = 0.77793\,\text{rad} = 44.569°$$

$$\theta'' = \theta + \alpha = 0.49488\,\text{rad}$$

$$\phi' = \sin^{-1}\left(n\sin\theta''\right) = 0.79290\,\text{rad} = 45.43°$$

Finally, we get the angle between the main reflected beam and the pick-off beam

$$\alpha' = \phi' - \phi = 0.014967\,\text{rad} = 51.5\ \text{min}$$

For L=6.5 m we get the beam separation to be 9.7 cm, which satisfies the required condition that separation between the two beams be greater than three times the beam diameter.

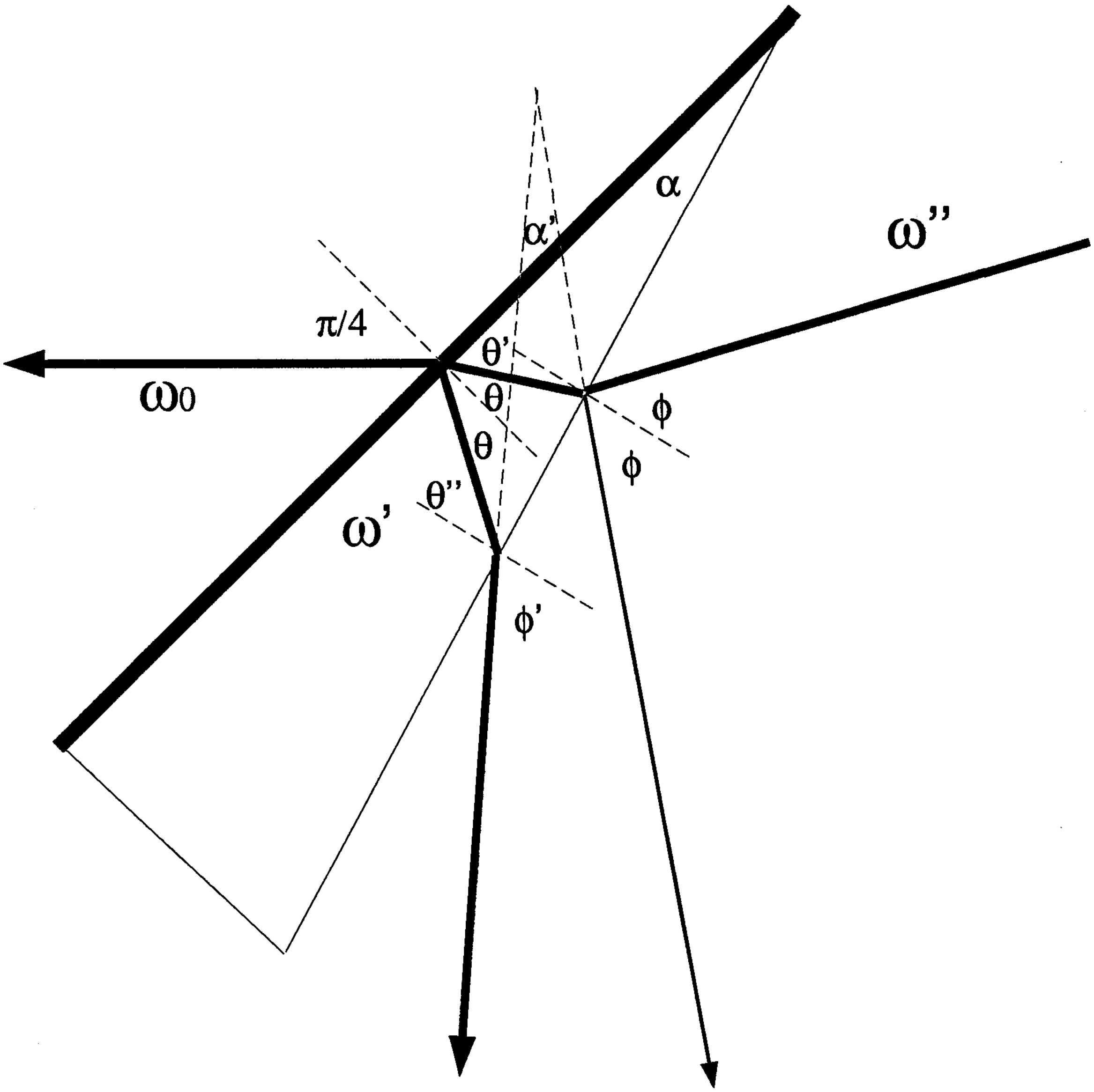

Figure 4

(ii) *Coupling efficiency*:

$$\frac{\omega'}{\omega_0} = \frac{\cos\theta}{\cos\frac{\pi}{4}}$$

and similarly,

$$\omega'' = \omega_0 \frac{\cos\phi}{\cos(\theta - \alpha)} \frac{\cos\theta}{\cos\frac{\pi}{4}} = 1.00529\,\omega_0$$

from which it follows that the coupling efficiency is

$$C = 1 - 6.96 \times 10^{-6}$$

Comparing this with the value of C in Section II, we see that for the same degree of angling, the coupling efficiency for a wedged beamsplitter is dramatically worse than that for a wedged test mass. This again supports our proposal that the plane face of the near test mass be a wedge.