New Folder Name Shot Noise Estimate

Shot Noise Estimate for Initial LIGO Interferometer

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1. BACKGROUND AND REFERENCES

The shot noise formula we have been carrying to date—most recently used to make Figures 9 and 10 in [Science 92]—is from [KST, 300]; the formula is referred to here as "[1]". This formula was the basis for the comparison of noise sources (Fig. III–2) and for comparison with source strengths (Fig. II–2) in Volume I of the December 1989 proposal. Owing to an error in the proposal assembly, different laser power levels were assumed for the two figures; the "experimental" curve assumed $\eta P = 1$ W, and the "source" curve assumed $\eta P = 3$ W. Here ηP is the power incident on the beamsplitter, corrected for photodetector quantum efficiency and for optical losses between the beamsplitter and cavity input mirrors.

This error came to light in the preparation of [Science 92], wherein $\eta P = 2$ W was chosen as a compromise: a power level twice the original estimate, but probably still achievable with a single argon ion laser.

Separate from the question of achievable power is the question of whether [1] is appropriate for the foreseen optical configurations. It was checked by A. Abramovici [AA, May 92] by independent derivation. These calculations, however, do not account for the details of phase modulation. More specialized calculations—see the summary [RES, Feb 92] and references therein—indicate that, even in the limit of small and optimal modulation, the noise in LIGO interferometers will be larger than [1] by a factor of 2 to 2.5.

2. SENSITIVITY COMPARISON

Figure 1 Shows the displacement sensitivity corresponding to the three shot noise formulae, along with other noise sources and the "most optimistic" neutron star inspiral signal strength for 3 events per year. It can be seen that if [1] is correct, there is some chance of detecting NS inspiral signals with the parameters of Table 1, but the noise predicted by formulae [2] and [3] is too high for detection. There is still some question as to the accuracy of [2] and [3]; they have not yet been checked, and [3], for example, carries the counterintuitive implication that sensitivity is not improved by recombination.

Symbol	Description	Value	Notes
λ	Laser wavelength	514 nm	
L	Arm Length	4 km	
m	Cavity mirror mass	7 kg	Sets radiation pressure noise
T_1	Input mirror pwr xmsn	3%	T_1 assumed to be dominant loss. Noise insensitive to T_1 for $f > f_k$.
f_k	Knee frequency	90 Hz	$2\pi f_k = cT_1/4L$
ηP	Pwr on beamsplitter	2 W	Corrected for inefficiencies. May be achieved, e.g., with 3 W out of input mode cleaner, 75% efficiency of remaining optics, and 90% photodetector quantum efficiency.
$B_{\rm rec}$	Recycling factor	30	Defined as power buildup in arm cavities, compared to non-recycled cavities. Scattering and absorption losses alone allow larger $B_{\rm rec}$; assumed limited by recombined beam contrast defect.

Table 1: Parameters that that affect shot noise (high frequency) and radiation pressure noise (low frequency).

The uncertainty in the calculation needs to be resolved. At the same time, the prospects for increasing the power and recycling factor beyond the values shown in Table 1 should be investigated.

References

[Science 92] The just-published Science article.

[RES, Feb 92] "Comparison of Shot Noise Formulas," R. E. Spero, 10 February 1992, `~robert/memos/shotloss/shot-comp.tex`

[AA, May 92] "Phase, Displacement, h Sensitivity for Recombined Fabry-Perot Interferometer," notes from A. Abramovici, 3 May, 1992.

[KST, 300] K. S. Thorne, in *300 Years of Gravitation*, S. W. Hawking and W. Israel, Eds., (Cambridge University Press, Cambridge, 1987), pp. 330-458. Equation is 115 is "[1]".

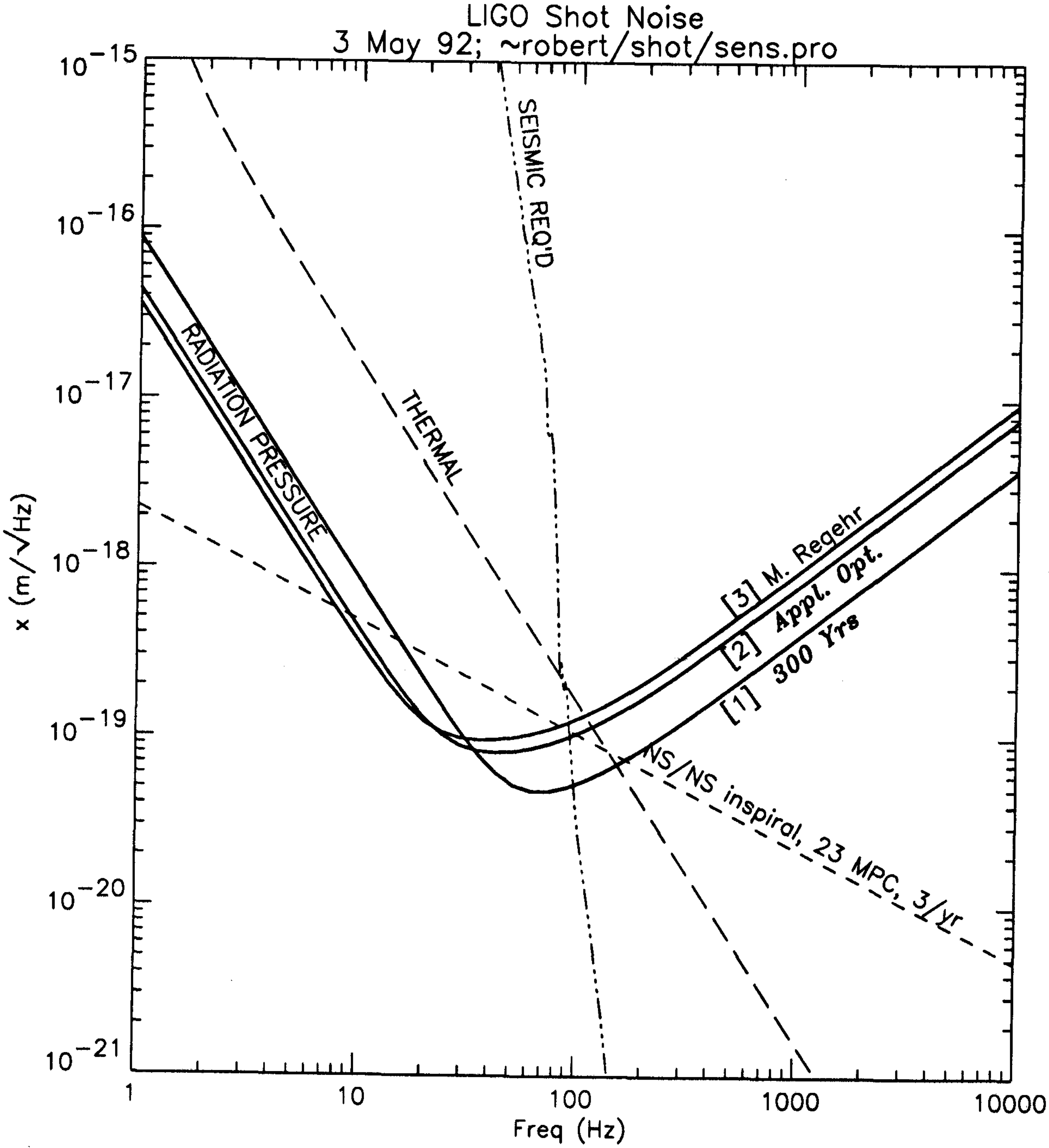


Figure 1: Three different calculations of shot noise (*solid lines*) for the parameters of Table 1. See [RES, Feb 92] for formulae and references. The radiation pressure lines intersect the photon-counting curves and the standard quantum limit (not shown) at 50, 25, and 20 Hz, for curves [1], [2], and [3], respectively. Curve [1] is the same calculation (and using the same parameters) as in [Science 92]. The NS/NS inspiral line is derived from Figure 10 of [Science 92].

$\phi = \phi_0 + \varphi_s + \phi_m \sin \omega_m t$

$\cos \phi_0 + \phi_m$

$\cos\{\phi_m \sin \omega_m t\}$

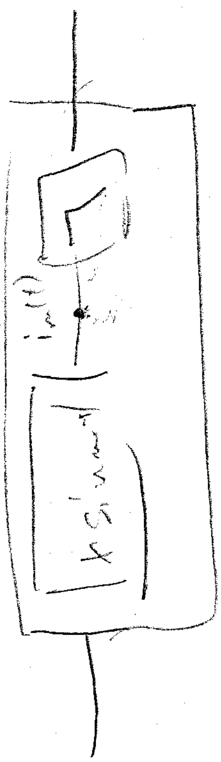
$\cos x = 1 - \frac{1}{2}x^2$

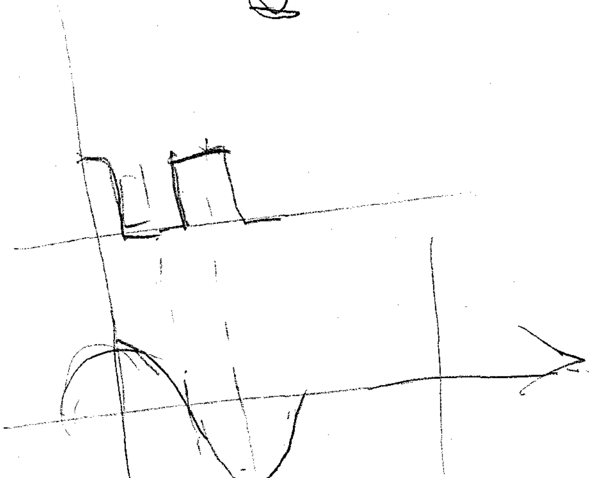
$= 1 \mp \frac{1}{2}\phi_m^2 \sin^2 \omega_m t =$

$= \frac{\phi_m^2}{4}\left(1 - \cos 2\omega_m t\right) = E\{i_p(t)\}$

$\sqrt{1.5}$ 1.25

$R_{xx}(t)$





$\hat{i}_d(t) = i_p(t) \sin \omega t$

$R_{i_d i_d}(t, t+\tau) = E\{i_d(t)\, i_d(t+\tau)\}$