New Folder Name Shot Noise Formulas

Comparison of Shot Noise Formulas

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ASSUMPTIONS AND DEFINITIONS

1. Sensitivity is relative to calibration obtained by moving one of the test masses; $\tilde{x}(f) = L\tilde{h}(f)$, $L =$ length of one arm.
2. $\mathcal{L} \equiv$ total of losses in one arm (sum of scattering, transmission, absorption for both mirrors).
3. $\tilde{x}(f) = \tilde{x}(0)\sqrt{1+(f/f_k)^2}, 2\pi f_k = \omega_k = c\mathcal{L}/4L.$
4. Formulas below do not include recycling, which would improve sensitivity by factor $R^{-1/2}$.
5. $\lambda =$ optical wavelength; $\lambda\!\!\!^{-} = \lambda/2\pi$; $\dot{N} \equiv P/h\nu$; $P =$ Laser power incident on beamsplitter, corrected for photodiode quantum efficiency.

NOT RECOMBINED (40 m Mark I; included for reference)

The displacement sensitivity of the present (Mark I, not recombined) interferometer is[1]

$$\tilde{x}_{\mathrm{Mk\ I}}(f) = \frac{L}{\pi\tau_e}\sqrt{\frac{3}{8}}\sqrt{\mathcal{M}}\left[\frac{\lambda h}{cP}\left(1+[f/f_k]^2\right)\right]^{1/2}\left(\frac{1}{\sqrt{2}}\right).$$

The cavity energy storage time is $\tau_e = 2L/c\mathcal{L}$, and the modulation function $\mathcal{M}$ has the minimum value of 1. Define

$$\tilde{x}_0 = \lambda\!\!\!^{-}\mathcal{L}\frac{1}{\sqrt{\dot{N}}}\frac{\sqrt{3}}{4}$$

Then in the limit of low frequency f, optimum mirror parameters, and optimum (that is vanishingly small) modulation, $\tilde{x}_{\mathrm{Mk\ I}}(f) = x_0$. For $P = 1$ W and $\mathcal{L} =$100 ppm, $\tilde{x}_0 = 2.2 \cdot 10^{-21}$ m/$\sqrt{\mathrm{Hz}}$.

1. PROPOSAL ASSUMPTION

The December '89 proposal curves were based on the formula in *300 Years of Gravitation*, Kip's equation 115:

$$S_{h_{\mathrm{FR}}}(f) = \frac{2\hbar c\lambda\!\!\!^{-}}{P}\left(\frac{1}{2BL}\right)^2\left[1+(2\pi BLf/c)^2\right]$$

For the Fabry-Perot cavity $B = 4/\mathcal{L}$, and the low-frequency limit is

$$x_{300} = x_0/\sqrt{6} = 0.4x_0 \qquad (1)$$

For knee-frequency much less than 1 kHz, 4 km arms, and $P = 2$W, this gives $x_{300}(f = 1\mathrm{kHz}) = 2.13 \cdot 10^{-18}$ m/$\sqrt{\mathrm{Hz}}$ (independent of $\mathcal{L}$).

[1]This includes the $1/\sqrt{2}$ correction to Stan's formula proposed by Harry and checked by Stan.

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2. APPLIED OPTICS, 1991

The non-recycled fixed mass interferometer at MIT was described by David et. al. in *Applied Optics* **30**, No. 22, August 1991, p. 3133. The low-frequency sensitivity is given as

$$\tilde{x} = \left(\frac{\lambda}{8F}\right)\left(\frac{2e}{I_{\text{Max+?}}}\right)^{1/2} \times \mathcal{M}$$

The text states that the minimum value of $\mathcal{M}$ is 2. $F = 2\pi/\mathcal{L}$, and the second term is $\sqrt{2/\dot{N}}$; Therefore

$$x_{\text{FM1}} = \sqrt{\frac{2}{3}}x_0 = 0.8x_0 \tag{2}$$

3. MARTIN, JANUARY, 1992

Martin's preliminary calculation of the shot noise in the fixed-mass interferometer he is setting up:

$$S_x^{\frac{1}{2}}(\omega) = \lambda \frac{\sqrt{|E_{\text{DC}}|^2 + 3\,|E_+|^2}}{|E_2||E_+|} \frac{(1 - r_3r_4)^2}{t_3^2 r_4} \sqrt{1 + (\omega/\omega_c)^2}$$

where E_{DC} = amplitude of extraneous light at photodiode (e.g. due to contrast efect), in $\sqrt{\text{photoelectrons/sec}}$, $E_+ = J_1(\Gamma)DE_L$ amplitude of field due to one modulation sideband at photodiode, D = transmission factor, typically 0.85, E_L = laser field, $E_2 = \sqrt{R}J_0(\Gamma)E_L$ = field due to light circulating inside recycling cavity, travelling from recycling mirror to beamsplitter, r_3, t_3 = cavity input mirror amplitude reflectivity, transmission, and r_4 is for end mirror. The term with all the E's has optimum value $\sqrt{3}/\sqrt{R}E_{\text{L}}$, and the term with r's and t's is $\mathcal{L}/4$, so

$$x_{\text{FM2}} = x_0 \tag{3}$$

4. VINET et. al.

In *Phys. Rev.* **D38**(2), p. 433 (1988), Vinet, Meers, Man and Brillet calculated the shot noise for lots of interferometer configurations (all without modulation). Their equation (12) implies (with $\tau'' = 1/(4\pi f_k)$)

$$\tilde{h}(0) = \frac{1}{\sqrt{\dot{N}}}\sqrt{2f_k{}^2}\frac{1}{c/\lambda}$$

so that

$$x_{\text{VMMB}} = \sqrt{\frac{2}{3}}x_0 = 0.8x_0 \tag{4}$$