New Folder Name RMS MIRROR Motion

LIGO PROJECT

CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

109

List of 4/10/92

TO Science Team

DATE 12 February, 1991; revised 20 Feb 92

FROM DHS

E-MAIL dhs

SUBJECT RMS mirror motion

FILE ~ dhs/ifoconcept/rms-new.tex

Considerations of the RMS motion of the LIGO cavity mirrors

Summary

- Seismic noise model changed from 89 proposal by adding rising noise from 1.0 to 0.1 Hz
- when possible, data from the MIT prototype for the LIGO stack is used as the stack model
- a simple single wire sling pendulum suspension is assumed
- the net horizontal RMS single mirror motion is found to be 3.5×10^{-7} m RMS, dominated by 0.1 to 1 Hz ground motion
- the net vertical RMS single mirror motion is found to be 5.3×10^{-7} m RMS, assuming a vertical undamped Q of 10000; it is dominated by the motion at the 10 Hz vertical motion
- the net angular RMS single mirror motion around the vertical axis is found to be 3.5×10^{-7} radians RMS, dominated by the 0.1 to 1 Hz ground motion
- the net angular RMS single mirror motion around the horizontal axis is found to be $\approx 8.6 \times 10^{-9}$ radians RMS, assuming a 0.3 Hz tilt resonant frequency (it grows rapidly for higher tilt frequencies).

Requirements:

- the length of the LIGO cavities must be held to within 6×10^{-11} m of resonance, to keep the matching of the storage times better than 10^{-2}
- the alignment of the LIGO cavities must be held to within 2×10^{-7} rad, to maintain a contrast in the interferometer of $C < (1 - 1 \times 10^{-3})$ and thus a reasonable recycling gain

LIGO-T920005-00-R

Introduction

We will principally address the question of the LIGO cavities. The physics stays the same for prototypes, but the ground motion and the present stack in the 40m prototype are considerably different than the LIGO equivalents. The seismic driving spectrum is between 10 and 100 times greater at the Caltech and MIT laboratories than the probable LIGO site seismic noise spectrum (as detailed below), on average, and the noise is non-stationary (like the trucks and other objects which produce it): there are peaks of motion which are much greater than the average. For this reason, some of the qualitative conclusions about LIGO RMS motions do not hold for the prototypes, and some of the solutions for mass control must be different in the prototypes. It would be worthwhile to compare the results given here with the 40 m experience.

Recent (Fall '91) re-examination of the available information about seismic noise indicates that the excitation probably grows at frequencies below 1 Hz (based on measurements in the 40m lab, JILA, german site measurements, and many papers; see e.g., Brune and Oliver, Bull. Seism. Soc. Am., 49, 349-353). This change in the parent spectrum will determined the expected RMS motion. For the following, we take the following seismic input spectrum, shown in Figure 1:

- $x(f) = 10^{-9}/f^3$ m/$\sqrt{\text{Hz}}$ from 0.1 to 1.0 Hz
- $x(f) = 10^{-9}$ m/$\sqrt{\text{Hz}}$ from 1.0 to 10 Hz
- $x(f) = 10^{-7}/f^2$ m/$\sqrt{\text{Hz}}$ from 10 Hz and higher frequencies

We assume here that the noise is isotropic, incoherent, and stationary. This will certainly not be strictly true. At yet lower frequencies (longer periods), temperature variation and tidal forces cause motions of the mirror. Internal to the LIGO, the drifts in the stack and support are important. The RMS motion of the ground in this model, over this frequency range, is 3.5×10^{-7} m RMS.

For the stack model, we use the MIT prototype for the LIGO stack as characterized by J. Giaime, Nov '91. It has several low frequency resonances: In the horizontal-horizontal transfer function, there are two broad resonances: one at 2 Hz (modeled as a pair of resonances at 1.8 and 2.1 Hz, each of Q=5.6), and one at 6.5 Hz with a Q of 2.3; the transfer function crosses 0 dB at 9.3 Hz and falls as about $(10/f)^5$ from 10 to 100 Hz. The vertical transfer function is similar, with somewhat higher frequency resonances. Because of the large seismic noise at frequencies below 1 Hz, the Q of the stack resonances does not influence the RMS noise appreciably (although it does change the control forces which are required). The model used for the horizontal-horizontal transfer function is shown in Figure 2, and for the vertical-vertical in Figure 3. The transfer function for vertical excitation to horizontal motion is roughly 40 dB below the other transfer functions in the low-frequency regime of importance for the RMS motion.

The transfer function of the stack for horizontal excitation to rotations of the top plate $T_{\phi,x}$ (in radians/meter) is similar in form to that for the horizontal to vertical transfer function

& coupled in because of down tube?

length of down tube?

T_{yx} (in meters/meter) described above for frequencies above 20 Hz; for lower frequencies, the transfer function has not been measured, but should quickly fall (as f^n with $n \geq 2$). In fact, the dominant coupling from horizontal to vertical comes from rotations of the top plate, and the numerical value of the transfer function from horizontal to rotations is simply that for the horizontal to vertical divided by the top plate radius of 0.35 m: $T_{\phi_x x} = T_{yx}/0.35$. The model we use assumes that the tilt for low frequencies (below the first stack resonance f_0) is $(f^2/f_0^2) \times 1/0.35$ and then folds into the measured tilt transfer function for higher frequencies (as shown in Figure 4). The product of this curve with the ground noise is shown in Figure 5; the integral under the curve is 1.9×10^{-8} rad RMS.

The horizontal pendulum transfer function is taken to be slightly less than (electronically) critically damped at its resonant frequency of 1 Hz (length of 0.25 m, Q=1.5; pendulum mass of 10 kg), with the damping gain rolling rapidly off toward higher frequencies; the transfer function starts to fall as $1/f^2$ at about 10 Hz. This is shown in Figure 7. The angular motion of the mirror about the vertical axis is assumed to have a resonant frequency of 1 Hz; about the horizontal axis, the frequency is taken to be 0.3 Hz. Both are assumed to be almost critically damped. The vertical wire-stretching resonance is assumed to be at 15 Hz, and different cases (damped, not damped) are examined.

I) Excitation of the masses

a) along the optic axis

frequencies of 0.1 Hz and higher: The product of the ground motion, stack transfer function, and pendulum transfer function give the expected mirror motion due to simple horizontal motion of the stack top plate. The spectrum of motion for a single mirror is shown in Figure 8. To calculate the RMS motion, the linear spectral density is squared and integrated over the frequency band from 0.1 to 100 Hz. The RMS motion due to this excitation is 3.5×10^{-7} m, which is completely dominated by the contributions from the 0.1 to 1 Hz range where the stack behaves principally as a rigid body. The stack resonances do not dominate because they are so well damped.

The tilting motion of the ground is another source of seismic noise. For the low frequencies (<1 Hz)which are important for the RMS motion, the wavelength of the surface waves (which travel at roughly 3×10^2 m/sec) is large compared with the structure size. Assuming that the surface waves have an amplitude equal to the LIGO standard seismic noise spectrum N_{seismic}, we can calculate the maximum slope of the ground; this gives $\theta_{\mathrm{seismic}} \approx (2\pi f/v_{\mathrm{seismic}})N_{\mathrm{seismic}} = 2\times10^{-11}/f^2$ rad$\cdot$Hz$^{-\frac{1}{2}}$ for $0.1 \leq f \leq 1$ Hz. With a lever arm of 2 m (the approximate height of the suspension point) this gives a contribution of $4\times10^{-11}/f^2$ m/$\sqrt{\mathrm{Hz}}$ at the suspension point. This is smaller than the direct effect for the RMS motion (note that this does not necessarily hold true for GW band frequencies).

In LIGO, the pendulum will be suspended from a cage (named the 'down-tube') that is mounted below the stack. At low frequencies, the down-tube can be considered to be rigid, and tilts of the stack top plate will be converted into horizontal motions of the pendulum suspension point by this down-tube. If the length of the down-tube is roughly 1 meter, it will make a contribution smaller than the direct horizontal-horizontal excitation; we calculate 7.9×10^{-9} m RMS for a 1-meter down-tube. The spectrum of tilt stack motion is shown in Figure 5, and the net mirror motion due to this effect is shown in Figure 6. An important difference from the horizontal-horizontal coupling is that the bulk of the contribution to the motion comes from the stack resonances, and not from the very low frequency (0.1 to 1 Hz) regime, and so changes in the stack Q would affect this contribution. Presently, its contribution is small.

In summary, we will take the incoherent sum of the tilt-induced and the direct horizontal motion to predict the motion of one mirror. The spectrum of the motion is shown in Figure 9, and the RMS motion is about 3.5×10^{-7} m RMS, or about 1 μm peak-to-peak. For two widely-separated mirrors forming a cavity, the statistically independent motions of the mirrors increases the expected RMS noise by a factor of $\sqrt{2}$; thus we expect about 5×10^{-7} m RMS (or 1.5 μm peak-to-peak). The RMS motion of the test mass is effectively determined by the ground noise for frequencies less than 1 Hz. If one of the stack resonances near 2 Hz had a Q of 30 (rather than the present 5.6), the RMS motion does not change from the above value.

very low frequency drift: On a longer time scale, a number of sources of drift are apparent. The important effect will be on the optic axis length, although there will be motions perpendicular and in the vertical. The biggest effect is the Earth tide, which will cause a differential motion Δx of as much as $\Delta x = 2\times10^{-4}$ m with a 12 hour period (Berger et al., Science vol. 170 pg 296). Calculations of of two 4 km-long cavities (Weiss, Thermal considerations for LIGO tubes) indicate that temperature fluctuations (sun, clouds, rain) can induce motions of $\Delta x = 40\times10^{-4}$ m on a time scale of tens of minutes.

The MIT prototype stack shows a temperature drift in height of 32 μm/°C; the ambient temperature around the stack will vary by 3 °C, and the stack might have as much as 10% differential thermal expansion coefficient from one side to the other. This would result in tilts at the top of the stack of 10 μrad, or 10 μm of translation at the end of a one meter 'down-tube' with a time scale of the temperature fluctuations (probably 24 hours). The long term vertical drift of the MIT prototype stack is between 1.6 and 4.0 mm/year (depending on the drift model assumed); the daily drift after 80 days is 11 μm/day. This vertical drift could also have a horizontal component; assuming a 10% effect, one year of drift results in 0.5 mm/year.

To summarize, we expect

- 5×10^{-4} m peak-peak, period 1 year (stack drift)
- 2×10^{-4} m peak-peak, period 12 hours (earth tides)
- 4×10^{-5} m peak-peak, period 10 minutes (temperature fluctuations)

b) along the perpendicular axis: The pendulum will filter the horizontal-perpendicular motion as it will the horizontal-parallel motion, and so we expect the net horizontal mirror motion to be the same as along the optic axis.

The vertical motion will be filtered by the vertical resonance of the pendulum, which (for a reasonable safety factor in the pendulum wire breaking strength) will be about 15 Hz. This motion will be more difficult to critically damp due to geometry and stiffness. If damped to a Q of 10, the net vertical RMS motion of the test mass is about 3.5×10^{-7} m RMS; the spectrum of motion is shown in Figure 10. For a Q of 10000 and $f_{res} = 15$ Hz, the RMS motion is now dominated by the motion from the pendulum vertical resonance, giving about 5×10^{-7} m RMS. This case is shown in Figure 11, where the peak is not resolved. The peak value at 15 Hz is roughly 5×10^{-6} m/$\sqrt{\text{Hz}}$.

c) tilts and twists

1) rotations around the vertical axis: Direct rotational excitation will be small (comparable to the direct tilt excitation). Cross-coupling of horizontal or vertical translations will, however, lead to rotations. The importance of this effect is dependent on the stack design. The MIT LIGO prototype stack has not been characterized for this cross-coupling at frequencies lower than 20 Hz. A conservative prediction for the rotation can be made: Assume that the stack top-plate horizontal motion is due to rotations about an edge of the stack top plate. Then the transfer function for horizontal excitation causing radial motion will have the form of the horizontal-horizontal transfer function, with a scaling factor of the top plate radius (conservatively estimated at about 1 m). We can take the numbers derived above for the horizontal translations: Figure 9 is correct, but with $\text{rad}\cdot\text{Hz}^{-\frac{1}{2}}$ as units. Thus the RMS motion due to this excitation will be about 3.5×10^{-7} rad RMS or about 1 μm peak-to-peak.

2) rotations around the horizontal axis: If the suspension is a simple single wire loop, rotations of the pendulum suspension point only couple through the stiffness of the suspension wire to the pendulum. The more important effect is probably due to the horizontal translations of the pendulum suspension point. The low frequencies where the contribution to the RMS translational motion is important are below the tilt and pendulum mode frequencies ($\approx$1 Hz); in this regime, the pendulum simply translates with the suspension point, making very small angles. It is only near the pendulum and stack resonance frequencies that the angular motion of the pendulum (as shown in Figure 12) is important, so the influence of the down-tube structure is large (turning stack resonances into horizontal translations). The RMS angle of the pendulum wire is $\theta_{\text{pend}} = 1\times10^{-7}$ rad RMS.

The mirror rotation around its center-of rotation horizontal axis will be excited (only) because the wire departure point is not at the center of mass of the mirror. A simple model for the coupling with possible values gives $\theta_{\text{mirror}} \approx (3bg\theta_{\text{pend}}/r^2M) \times H_{\text{mirror}} = 1.5 \times \theta_{\text{pend}}H_{\text{mirror}}$, where $b = 5\times10^{-3}$ m is the distance of the wire attachment above the center of mass of the mirror, g is the gravitational constant, $r = 0.1$ m and $M = 10$ kg are the radius and the mass of the mirror respectively, and $H_{\text{mirror}} = 1/((\omega^2-\omega_0^2)^2+(\omega\omega_0/Q)^2)$ is the transfer function of the mirror resonance about the horizontal axis. Taking $\omega_0 = (2\pi) \times 0.3$, we predict a spectrum of mirror motion shown in Figure 13. The RMS motion is 8.6×10^{-9} rad RMS.

II) Sensitivity of the interferometer to RMS motion

In the following it is assumed that the instrument is a recombined-beam broad-band recycled interferometer. Thus the interferometer path length difference (dark fringe) and the two cavity lengths (resonance condition) can be, and must be, separately controlled.

a) along the optic axis

A number of effects require that the cavities be held close to resonance: sensitivity to power fluctuations in the laser, the loss of power to the recycling cavity, and the change in slope of $d\phi/dx$ introducing a sensitivity to frequency noise. The latter is the most important (See DHS: Modulation and topolgy: Deviations from resonance). For the sample mirror parameters $T_1 = 0.03$, $A_1{=}A_2{=}100$ ppm, the slope (normalized to the slope on resonance) is approximately $d\phi/dx \approx= -1 + 4.35\times10^3x^2 - 2\times10^7x^4$ where $x = 4\pi(v - v_0)l/c$ is the deviation from resonance. To put the numbers in context, if we have a cavity storage time matching of $\beta = 10^{-2}$ and do not want the deviation from resonance of the cavities to change the effective storage time by more than $\beta\tau$, the difference of the deviations of the two cavities from resonance should not exceed 2×10^{-5} rad $\hat{=}\,0.05$ linewidths $\hat{=}\,6\times10^{-11}$ m, or about $\phi = 6° \hat{=}$ 0.1 rad of phase shift on reflection from the cavity.

b) cross-coupling to longitudinal motion:

All of these 'indirect' effects (those not directly affecting the length of the cavities), and any servo corrections applied to correct for them, will be seen with some reduced effect in the length of the cavities. Some examples, probably not exhaustive:

1) angular changes: If the beam is not centered on the axis of rotation of the mirror, then there will be a linear coupling from mass rotations to cavity length. Given the expected ratio of angular to translational motion, and probably offsets from the axis of rotation, this should be small: $x = \theta y_{\rm offset} \approx 4\times10^{-8}$ rad $\times$ 0.01 m $= 4\times10^{-10}$ m, a factor of 100 less than the expected direct translations.

2) horizontal or vertical translations: The cavity length is a quadratic function of the displacement perpendicular to the beam of a curved mirror. If this motion is locally damped (OSEMS) it should be a negligible effect.

3) up-conversion: If there are surface irregularities on the mirror, motions of the mirror perpendicular to the beam may cause modulation of the length of the cavity with characteristic frequencies much higher than the spectrum of the mirror motion. No attempt to model this analytically is made here; measurements should be made, with numerical simulation to back up the numbers.

c) tilts and twists: Changes in the pointing of the test mass due to excitation by seismic noise have (at least) the following effects:

1) TEM_{00} *mode coupling:* The amount of power coupled into the fundamental mode of the cavity varies as a function of mirror angle. As one example, assume a flat-concave cavity and take motions of the concave mirror of radius R. Rotations θ of that mirror make translations in the optic axis of the cavity of $y = \theta R$ (for small angles). The overlap integral of the optic axis of the cavity and the optic axis of the light is quadratic with a scale of the $1/e$ diameter of the beam: $M \approx (1-(y/w)^2)$ for $y << w$. The mode matching M as a function of rotations of the far concave mirror will be $M \approx 1-(\theta R/w)^2$. For R=4 km, w=5 cm, and M >0.99, θ must remain less than 1μrad.

2) fluctuations of the contrast: The contrast of the recombined beam interferometer varies with angle variations. Taking as an example the motion of a near, flat, mirror, rotations θ of that mirror make rotations in the optic axis of the cavity of θ. The contrast C of a simple non-recycled interferometer varies as $C = 1-(k\theta w/2)^2$ (for small angles); we take this model for simplicity. Choosing to maintain the contrast as limited by this effect to be greater than $C < (1 - 1\times10^{-3})$ (this makes the power lost to a recycling gain of 30 negligible, and keeps the power on the photodetectors at a reasonable level) requires that the angular motion in both axis be less than 0.2 μrad.

III) Requirements and Servo systems

From the above estimates for the motion of the test mass and the sensitivity of the interferometer to those motions, the transfer functions of the servo systems needed to bring the motions to an acceptable level can be described. The requirements and servo systems below are approximate, and are designed to help choose the style of control rather than to be specific designs.

a) along the optic axis: The primary requirement of this servo system is to bring the deviations of the main cavities from resonance to an acceptable level. A further question is whether the servo loop should have a unity-gain frequency greater than, or less than, the GW band of frequencies. Without entering into this discussion, we calculate the servo system needed for the low unity-gain case, because this is the minimum condition to address the RMS motion problem.

1) required gain as a function of frequency: In **I a** and **II a** above, we found that the motion of the test masses must be reduced from 1×10^{-6} m to roughly 6×10^{-11} m RMS. Most of the motion is in the frequency range from 0.1 to 1 Hz. To obtain 6×10^{-11} m RMS, we need a flat power spectral density of motion of 2×10^{-11} m/$\sqrt{\text{Hz}}$ from 0 to 10 Hz, then falling rapidly with increasing frequency (due to the stack and seismic noise spectrum). This requires a unity-gain frequency of (at least) 10 Hz, a gain of 200 at 1 Hz, and a gain of 1×10^5 at 0.1 Hz.

2) 'locking': The expected damped motion of one test mass is 3.5×10^{-7} m RMS or of two independent masses 1.5μm peak-to-peak. This is about 750 cavity linewidths (the FWHM of the cavity is 2×10^{-9} m), and about 1/2 FSR. The principal frequency of these motions is 0 to .5 Hz, which means that the cavity passes through resonance moderately slowly compared to its storage time of 0.5 msec. This means that the during 'locking' period, the servo loop receives a more-or-less continuous error signal with the correct sign to drive the system into resonance ('lock').

3) required forces: There are two regimes: initial damping, and then damped or locked. The first regime forces will be determined by the time that we wish to wait before the mass is damped. In the second regime, forces must be exerted to hold the pendulum in position against forces exerted by changes in the pendulum suspension point. Below the pendulum resonance frequency, this will be dominated by the gravitational restoring force xg/l, and above by the inertial force ma. The total force is shown in Figure 14 as a function of frequency; the integrated RMS force required is 1.4×10^{-4} N. This does not take into account static forces which may be required to offset drifts in the pendulum suspension point position. To set the scale, a 0.1 mm static deflection requires 4×10^{-2} N.

4) very low frequency drifts: To be written. A temperature servo on Viton stack elements (using their temperature coefficient of expansion as a control element) may be possible. Motors for longer time periods, mounted on the outside of the vacuum system or exerting pressure through elastomer pieces, can be used.

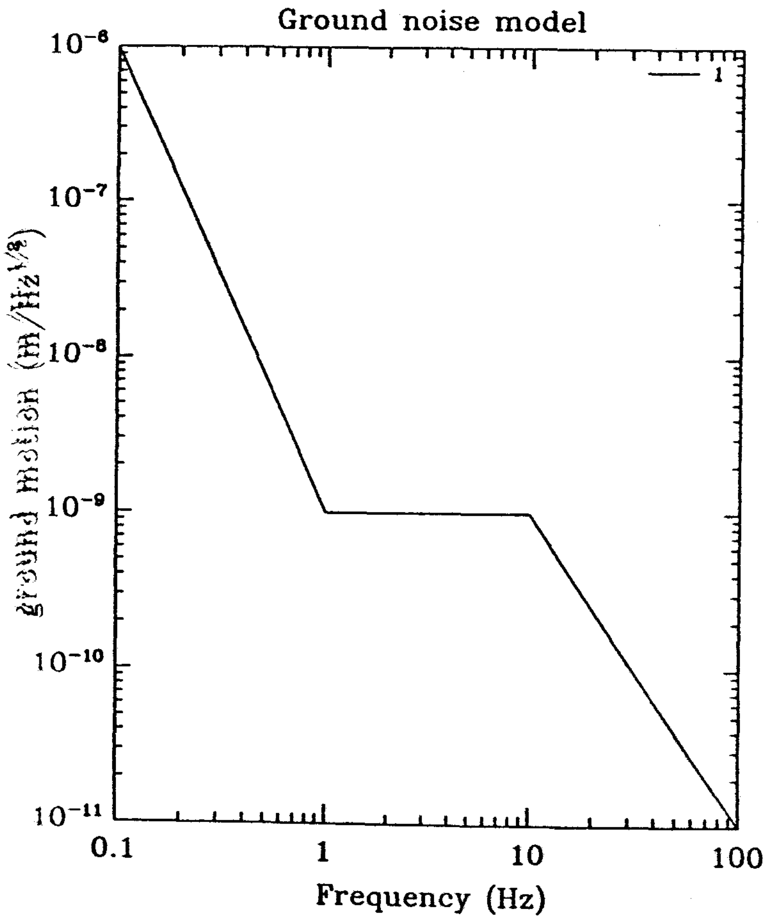
5) length of time lock can be held: The length of a contiguous time record and the RMS fluctuation in the recorded strain signal affect the signal-to-noise ratio for detection of periodic and stochastic sources. The difficulty is in determining the average 'dc' level of a segment and melding it with the proceeding and following segments (see Weiss, ?document). This requirement has not yet been established, but will influence the strategy for compensating for any long-term motions of the mirrors, whether due to the stack, temperature variations, or whatever.

b) transverse motion: To be determined. Up-conversion mechanisms must be studied experimentally.

c) rotations: The primary requirement of these servo systems is to bring the deviations of the main cavities from alignment to an acceptable level.

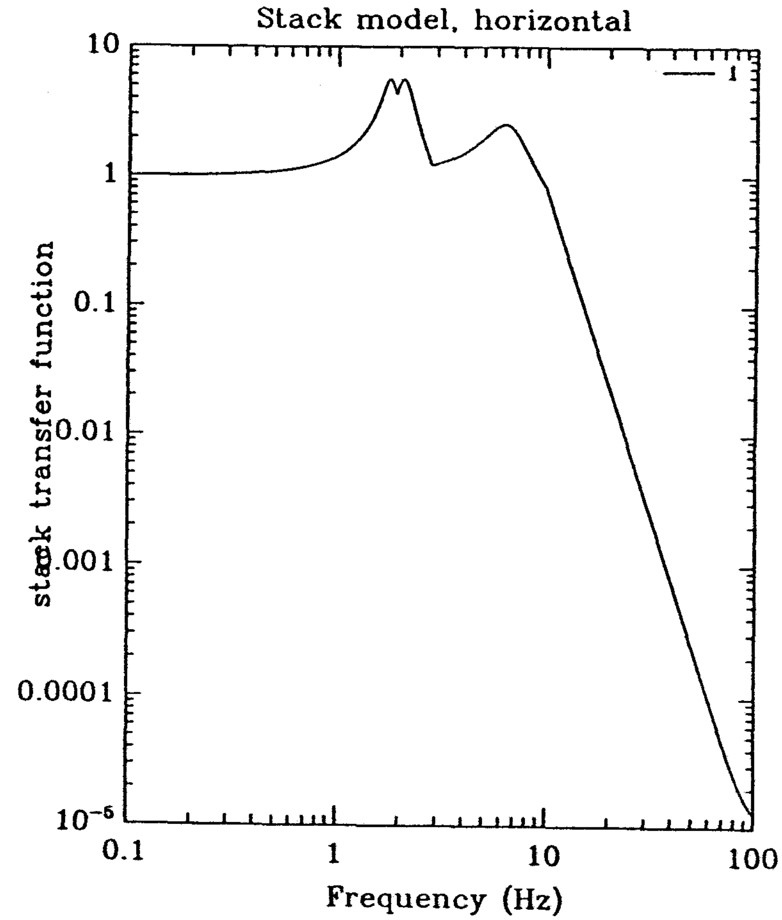
1) required gain as a function of frequency: About the vertical axis, the expected RMS motion is 3.5×10^{-7} rad, and about the horizontal axis is 9×10^{-9} rad (for the models above). The horizontal angular motions are already much less than the requirement of $< 0.2\ \mu$rad. The vertical angle will need a servoloop with a gain of several at 0.1 Hz (the main contribution to the motion). The alignment servos must provide the near-critical damping of the masses in angle, as discussed below.

2) signal to noise: The alignment servo must provide the angular damping for running conditions, and so must have a gain of roughly 1 at 1 Hz, and have a noise level compatible with the usual criterion: the influence on the test masses at GW frequencies (>100 Hz) must be negligible. The sensing noise must be less than 0.2μrad for frequencies below 1 Hz, or roughly 2×10^{-8} rad$\cdot$Hz$^{-\frac{1}{2}}$. This corresponds to motions of the beam at the other mirror (4 km distant) of 0.1 mm. The gain of the servo must be much less than unity in the GW band, but this is made easier by the low unity-gain frequency needed in this servo system.



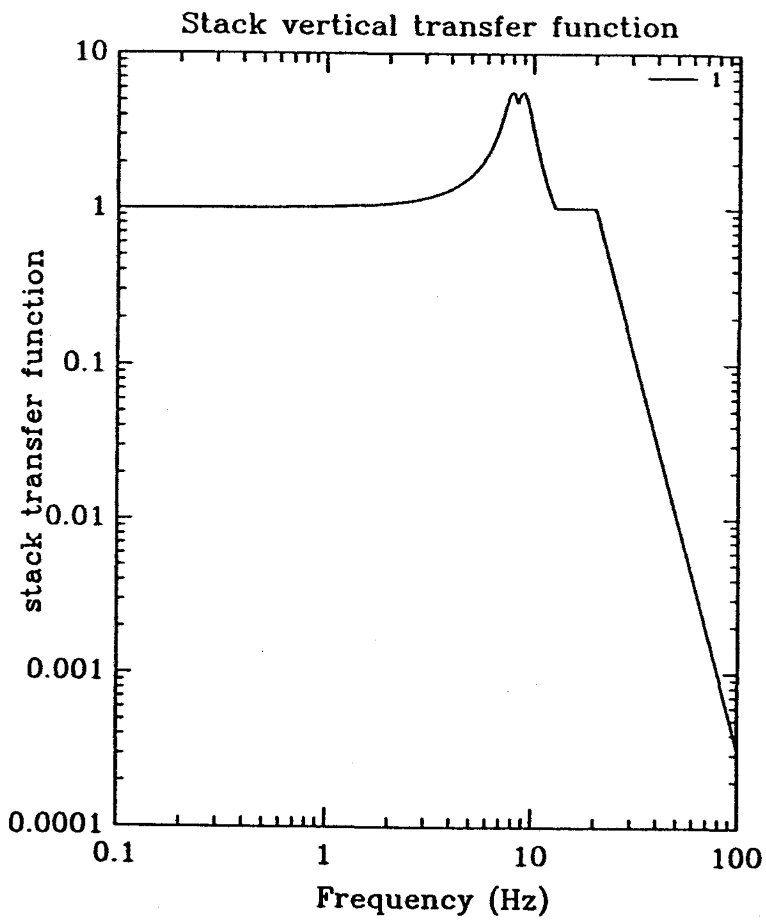
Ground noise model. 3.5×10^{-7} m rms

Figure 1



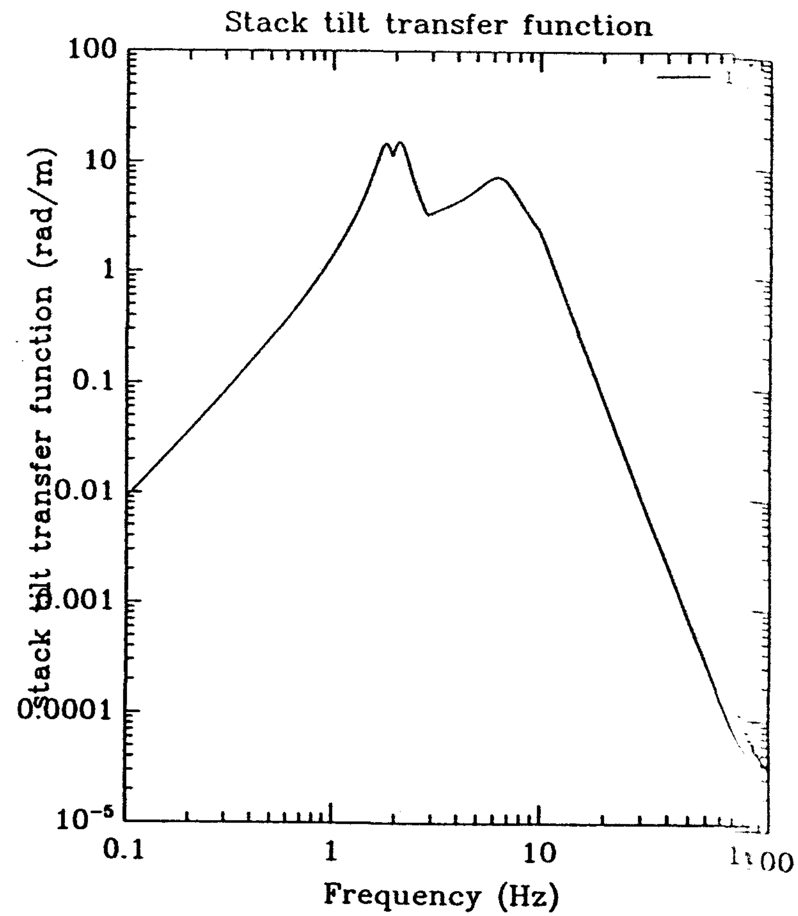
stack model, horizontal–horizontal

Figure 2



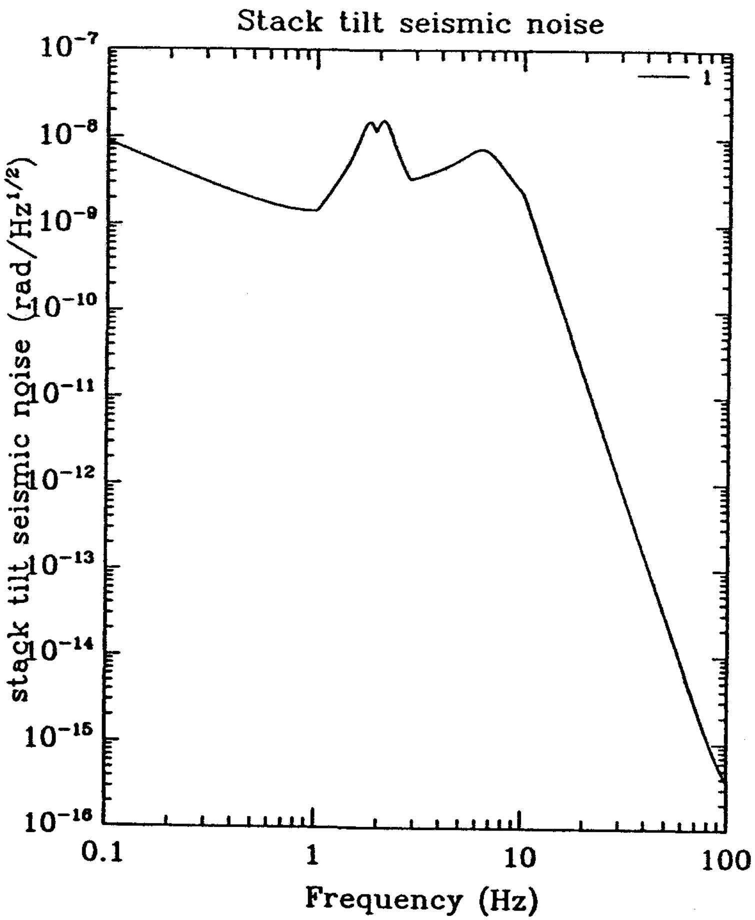
vertical stack transfer function

Figure 3



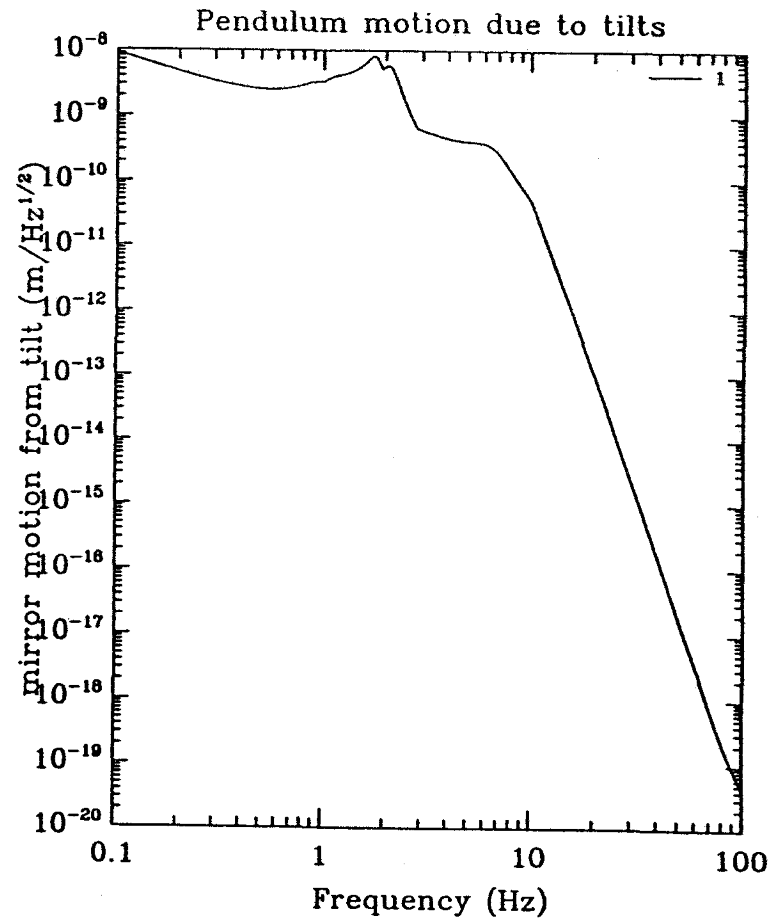
stack tilt transfer function

Figure 4



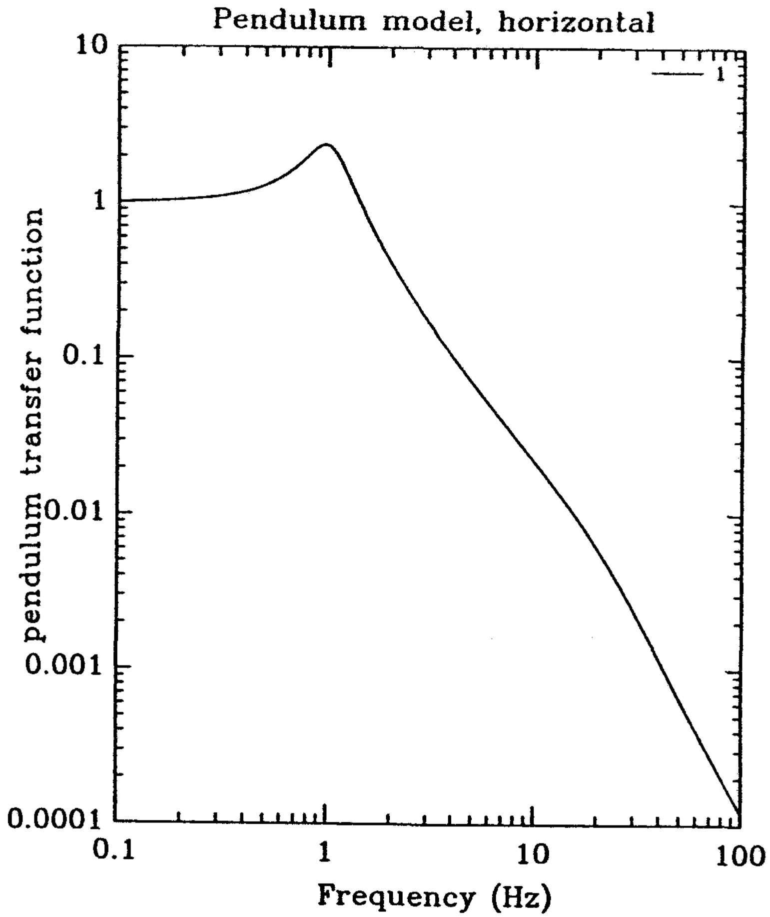
stack tilt seismic noise: 1.9×10^{-8} rad rms

Figure 5



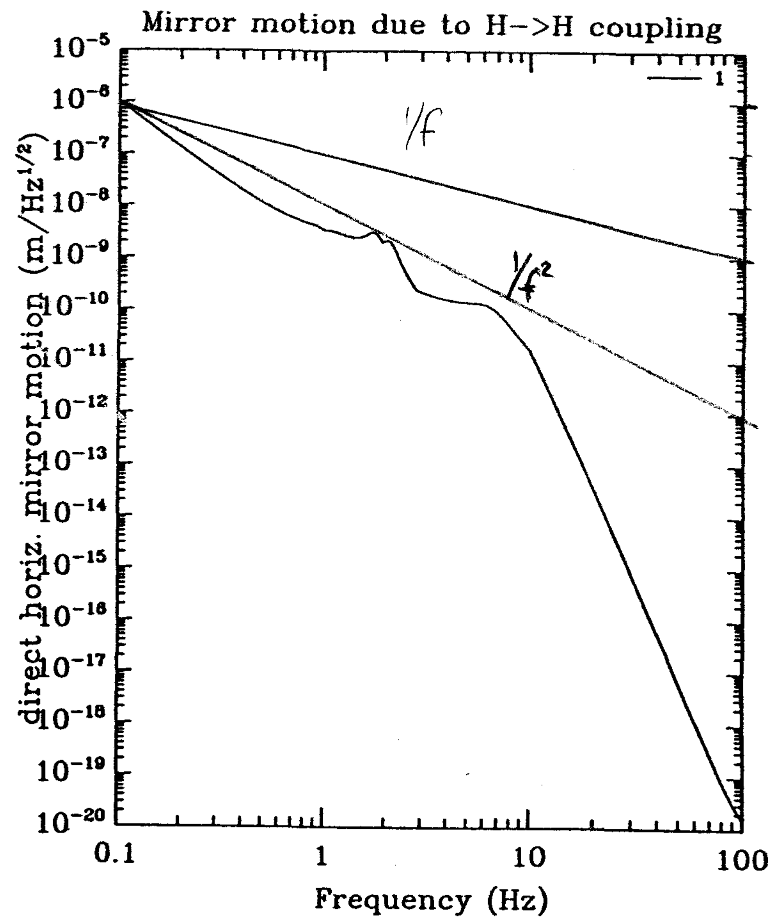
motion due to tilts and down-tube: 7.9×10^{-9} m rms

Figure 6



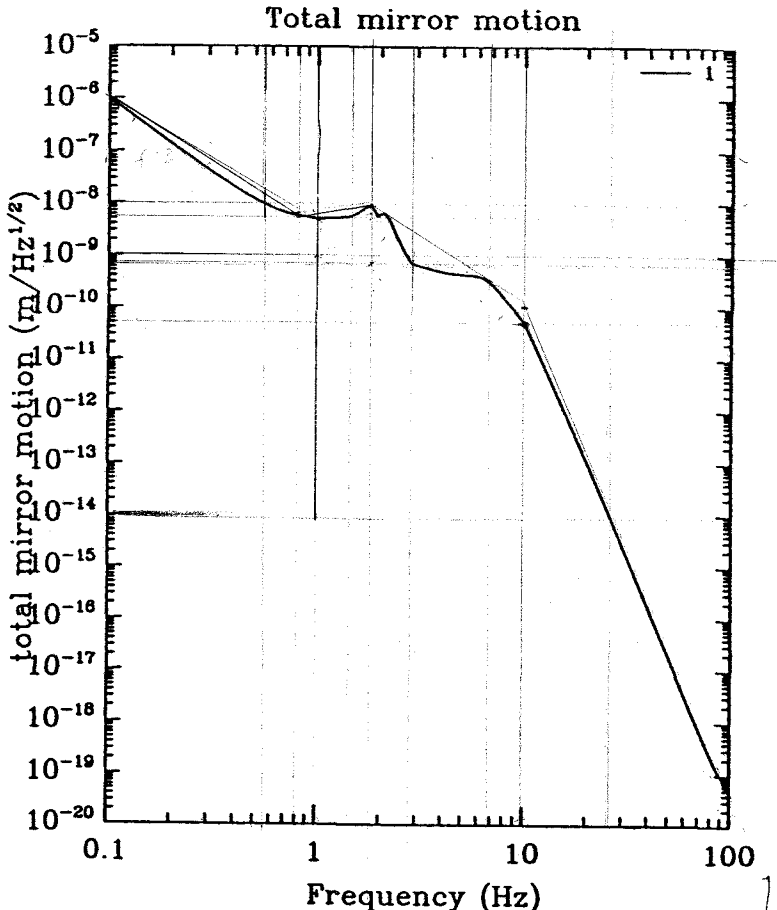
pendulum model, horizontal

Figure 7



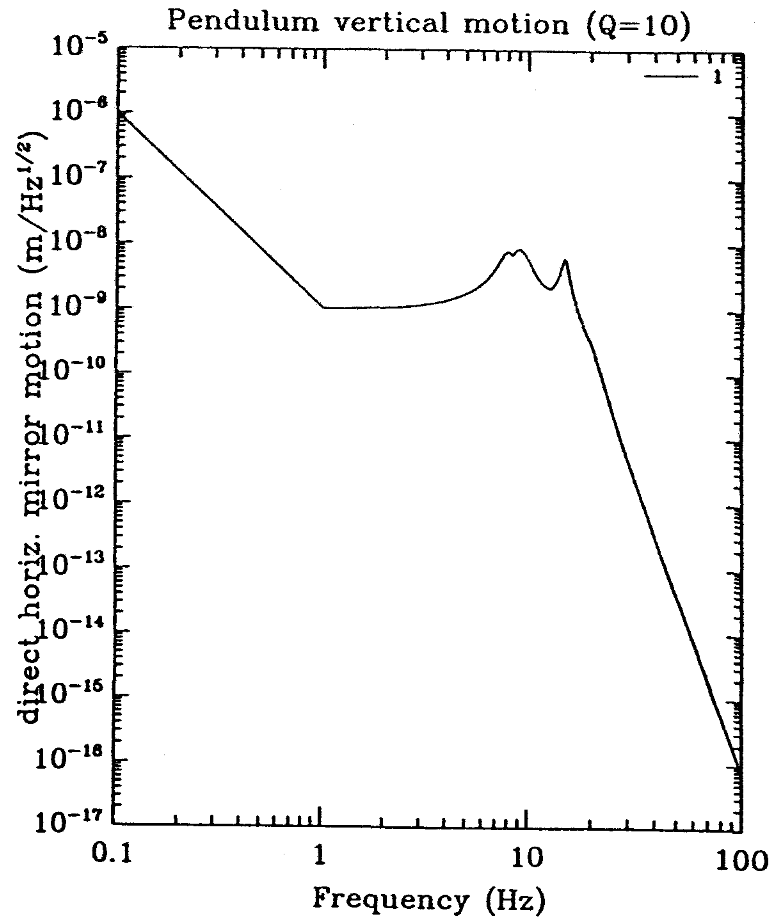
horizontal mirror motion from direct H->H coupling; 3.5×10^{-7} m rms

Figure 8



total mirror motion: 3.5×10^{-7} m rms

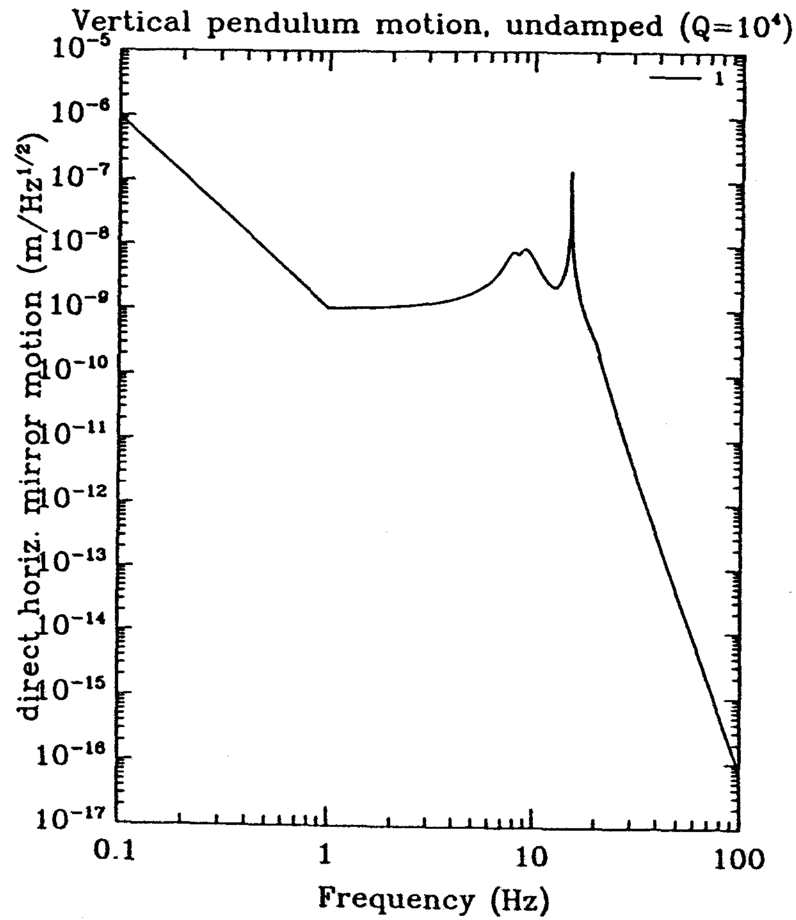
Figure 9



pendulum vertical motion: 3.5×10^{-7} m rms

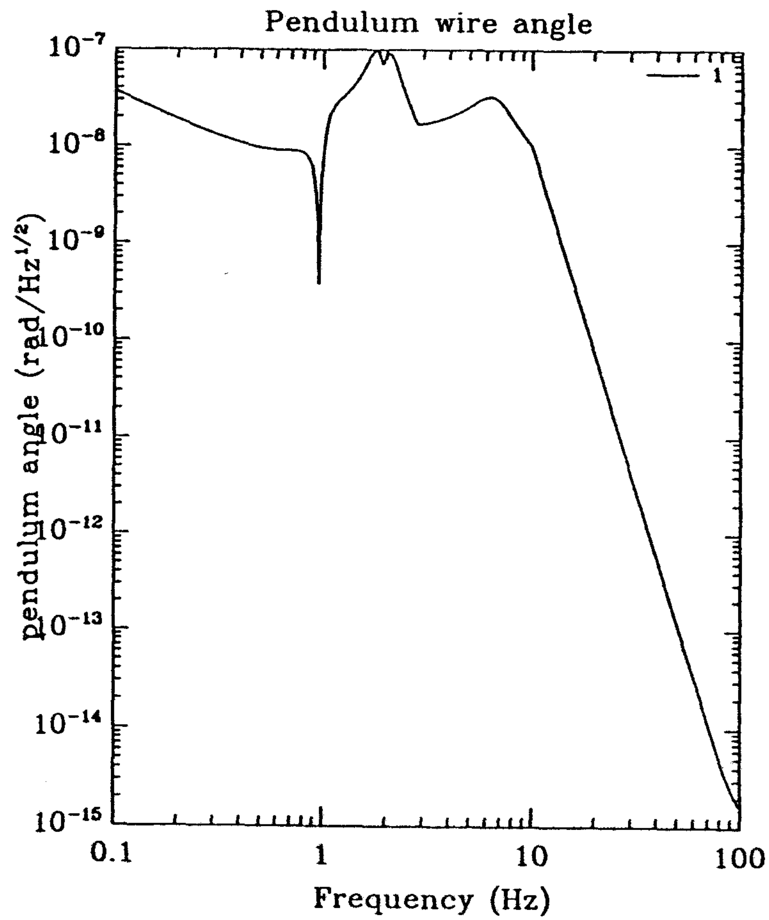
Figure 10

930621: DHS says Fig 9 would be ~~rms~~ motion of testmass without OSEM if one were to add a large spike at 1 Hz.



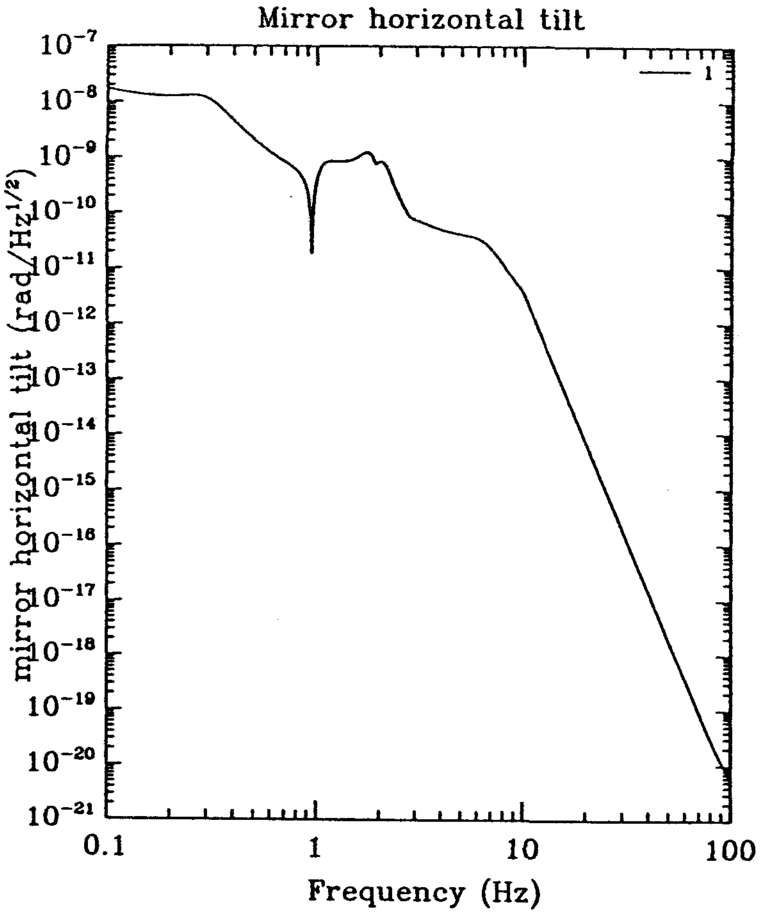
undamped vertical motion, peak unresolved; real total 5.3×10^{-7} m rms

Figure 11



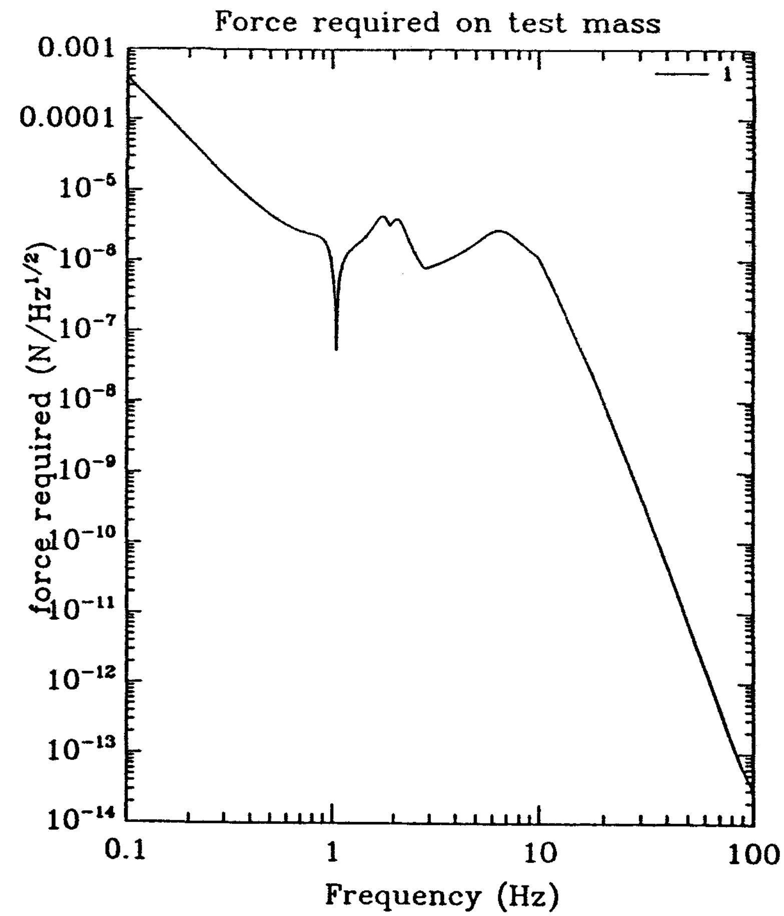
RMS angle 1.0×10^{-7} rad rms

Figure 12



mirror horizontal tilt: 8.6×10^{-9} rad rms

Figure 13



RMS force required 1.4×10^{-4} N rms

Figure 14

BATCH START

STAPLE
OR
DIVIDER

LIGO PROJECT

LIGO-T920005-00-R

CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

109

List of 4/10/92

TO Science Team

DATE 12 February, 1991; revised 20 Feb 92

FROM DHS

E-MAIL dhs

SUBJECT RMS mirror motion

FILE ~ dhs/ifoconcept/rms-new.tex

Considerations of the RMS motion of the LIGO cavity mirrors

Summary

• Seismic noise model changed from 89 proposal by adding rising noise from 1.0 to 0.1 Hz

• when possible, data from the MIT prototype for the LIGO stack is used as the stack model

• a simple single wire sling pendulum suspension is assumed

• the net horizontal RMS single mirror motion is found to be 3.5×10^{-7} m RMS, dominated by 0.1 to 1 Hz ground motion

• the net vertical RMS single mirror motion is found to be 5.3×10^{-7} m RMS, assuming a vertical undamped Q of 10000; it is dominated by the motion at the 10 Hz vertical motion

• the net angular RMS single mirror motion around the vertical axis is found to be 3.5×10^{-7} radians RMS, dominated by the 0.1 to 1 Hz ground motion

• the net angular RMS single mirror motion around the horizontal axis is found to be $\approx 8.6\times10^{-9}$ radians RMS, assuming a 0.3 Hz tilt resonant frequency (it grows rapidly for higher tilt frequencies).

Requirements:

• the length of the LIGO cavities must be held to within 6×10^{-11} m of resonance, to keep the matching of the storage times better than 10^{-2}

• the alignment of the LIGO cavities must be held to within 2×10^{-7} rad, to maintain a contrast in the interferometer of $C < (1 - 1\times10^{-3})$ and thus a reasonable recycling gain

Introduction

We will principally address the question of the LIGO cavities. The physics stays the same for prototypes, but the ground motion and the present stack in the 40m prototype are considerably different than the LIGO equivalents. The seismic driving spectrum is between 10 and 100 times greater at the Caltech and MIT laboratories than the probable LIGO site seismic noise spectrum (as detailed below), on average, and the noise is non-stationary (like the trucks and other objects which produce it): there are peaks of motion which are much greater than the average. For this reason, some of the qualitative conclusions about LIGO RMS motions do not hold for the prototypes, and some of the solutions for mass control must be different in the prototypes. It would be worthwhile to compare the results given here with the 40 m experience.

Recent (Fall '91) re-examination of the available information about seismic noise indicates that the excitation probably grows at frequencies below 1 Hz (based on measurements in the 40m lab, JILA, german site measurements, and many papers; see e.g., Brune and Oliver, Bull. Seism. Soc. Am., 49, 349-353). This change in the parent spectrum will determined the expected RMS motion. For the following, we take the following seismic input spectrum, shown in Figure 1:

- $x(f) = 10^{-9}/f^3$ m/$\sqrt{\text{Hz}}$ from 0.1 to 1.0 Hz
- $x(f) = 10^{-9}$ m/$\sqrt{\text{Hz}}$ from 1.0 to 10 Hz
- $x(f) = 10^{-7}/f^2$ m/$\sqrt{\text{Hz}}$ from 10 Hz and higher frequencies

We assume here that the noise is isotropic, incoherent, and stationary. This will certainly not be strictly true. At yet lower frequencies (longer periods), temperature variation and tidal forces cause motions of the mirror. Internal to the LIGO, the drifts in the stack and support are important. The RMS motion of the ground in this model, over this frequency range, is 3.5×10^{-7} m RMS.

For the stack model, we use the MIT prototype for the LIGO stack as characterized by J. Giaime, Nov '91. It has several low frequency resonances: In the horizontal-horizontal transfer function, there are two broad resonances: one at 2 Hz (modeled as a pair of resonances at 1.8 and 2.1 Hz, each of Q=5.6), and one at 6.5 Hz with a Q of 2.3; the transfer function crosses 0 dB at 9.3 Hz and falls as about $(10/f)^5$ from 10 to 100 Hz. The vertical transfer function is similar, with somewhat higher frequency resonances. Because of the large seismic noise at frequencies below 1 Hz, the Q of the stack resonances does not influence the RMS noise appreciably (although it does change the control forces which are required). The model used for the horizontal-horizontal transfer function is shown in Figure 2, and for the vertical-vertical in Figure 3. The transfer function for vertical excitation to horizontal motion is roughly 40 dB below the other transfer functions in the low-frequency regime of importance for the RMS motion.

The transfer function of the stack for horizontal excitation to rotations of the top plate $T_{\phi_x x}$ (in radians/meter) is similar in form to that for the horizontal to vertical transfer function

T_{yx} (in meters/meter) described above for frequencies above 20 Hz; for lower frequencies, the transfer function has not been measured, but should quickly fall (as f^n with $n \geq 2$). In fact, the dominant coupling from horizontal to vertical comes from rotations of the top plate, and the numerical value of the transfer function from horizontal to rotations is simply that for the horizontal to vertical divided by the top plate radius of 0.35 m: $T_{\phi_x x} = T_{yx}/0.35$. The model we use assumes that the tilt for low frequencies (below the first stack resonance f_0) is $(f^2/f_0^2) \times 1/0.35$ and then folds into the measured tilt transfer function for higher frequencies (as shown in Figure 4). The product of this curve with the ground noise is shown in Figure 5; the integral under the curve is 1.9×10^{-8} rad RMS.

The horizontal pendulum transfer function is taken to be slightly less than (electronically) critically damped at its resonant frequency of 1 Hz (length of 0.25 m, Q=1.5; pendulum mass of 10 kg), with the damping gain rolling rapidly off toward higher frequencies; the transfer function starts to fall as $1/f^2$ at about 10 Hz. This is shown in Figure 7. The angular motion of the mirror about the vertical axis is assumed to have a resonant frequency of 1 Hz; about the horizontal axis, the frequency is taken to be 0.3 Hz. Both are assumed to be almost critically damped. The vertical wire-stretching resonance is assumed to be at 15 Hz, and different cases (damped, not damped) are examined.

I) Excitation of the masses

a) along the optic axis

frequencies of 0.1 Hz and higher: The product of the ground motion, stack transfer function, and pendulum transfer function give the expected mirror motion due to simple horizontal motion of the stack top plate. The spectrum of motion for a single mirror is shown in Figure 8. To calculate the RMS motion, the linear spectral density is squared and integrated over the frequency band from 0.1 to 100 Hz. The RMS motion due to this excitation is 3.5×10^{-7} m, which is completely dominated by the contributions from the 0.1 to 1 Hz range where the stack behaves principally as a rigid body. The stack resonances do not dominate because they are so well damped.

The tilting motion of the ground is another source of seismic noise. For the low frequencies (<1 Hz)which are important for the RMS motion, the wavelength of the surface waves (which travel at roughly 3×10^2 m/sec) is large compared with the structure size. Assuming that the surface waves have an amplitude equal to the LIGO standard seismic noise spectrum $N_{\mathbf{seismic}}$, we can calculate the maximum slope of the ground; this gives $\theta_{\mathbf{seismic}} \approx (2\pi f/v_{\mathbf{seismic}})N_{\mathbf{seismic}} = 2 \times 10^{-11}/f^2$ rad$\cdot$Hz$^{-\frac{1}{2}}$ for $0.1 \leq f \leq 1$ Hz. With a lever arm of 2 m (the approximate height of the suspension point) this gives a contribution of $4 \times 10^{-11}/f^2$ m/$\sqrt{\text{Hz}}$ at the suspension point. This is smaller than the direct effect for the RMS motion (note that this does not necessarily hold true for GW band frequencies).

In LIGO, the pendulum will be suspended from a cage (named the 'down-tube') that is mounted below the stack. At low frequencies, the down-tube can be considered to be rigid, and tilts of the stack top plate will be converted into horizontal motions of the pendulum suspension point by this down-tube. If the length of the down-tube is roughly 1 meter, it will make a contribution smaller than the direct horizontal-horizontal excitation; we calculate 7.9×10^{-9} m RMS for a 1-meter down-tube. The spectrum of tilt stack motion is shown in Figure 5, and the net mirror motion due to this effect is shown in Figure 6. An important difference from the horizontal-horizontal coupling is that the bulk of the contribution to the motion comes from the stack resonances, and not from the very low frequency (0.1 to 1 Hz) regime, and so changes in the stack Q would affect this contribution. Presently, its contribution is small.

In summary, we will take the incoherent sum of the tilt-induced and the direct horizontal motion to predict the motion of one mirror. The spectrum of the motion is shown in Figure 9, and the RMS motion is about 3.5×10^{-7} m RMS, or about 1 μm peak-to-peak. For two widely-separated mirrors forming a cavity, the statistically independent motions of the mirrors increases the expected RMS noise by a factor of $\sqrt{2}$; thus we expect about 5×10^{-7} m RMS (or 1.5 μm peak-to-peak). The RMS motion of the test mass is effectively determined by the ground noise for frequencies less than 1 Hz. If one of the stack resonances near 2 Hz had a Q of 30 (rather than the present 5.6), the RMS motion does not change from the above value.

very low frequency drift: On a longer time scale, a number of sources of drift are apparent. The important effect will be on the optic axis length, although there will be motions perpendicular and in the vertical. The biggest effect is the Earth tide, which will cause a differential motion Δx of as much as $\Delta x = 2 \times 10^{-4}$ m with a 12 hour period (Berger et al., Science vol. 170 pg 296). Calculations of of two 4 km-long cavities (Weiss, Thermal considerations for LIGO tubes) indicate that temperature fluctuations (sun, clouds, rain) can induce motions of $\Delta x = 40 \times 10^{-4}$ m on a time scale of tens of minutes.

The MIT prototype stack shows a temperature drift in height of 32 μm/°C; the ambient temperature around the stack will vary by 3 °C, and the stack might have as much as 10% differential thermal expansion coefficient from one side to the other. This would result in tilts at the top of the stack of 10 μrad, or 10 μm of translation at the end of a one meter 'down-tube' with a time scale of the temperature fluctuations (probably 24 hours). The long term vertical drift of the MIT prototype stack is between 1.6 and 4.0 mm/year (depending on the drift model assumed); the daily drift after 80 days is 11 μm/day. This vertical drift could also have a horizontal component; assuming a 10% effect, one year of drift results in 0.5 mm/year.

To summarize, we expect

- 5×10^{-4} m peak-peak, period 1 year (stack drift)
- 2×10^{-4} m peak-peak, period 12 hours (earth tides)
- 4×10^{-5} m peak-peak, period 10 minutes (temperature fluctuations)

b) along the perpendicular axis: The pendulum will filter the horizontal-perpendicular motion as it will the horizontal-parallel motion, and so we expect the net horizontal mirror motion to be the same as along the optic axis.

The vertical motion will be filtered by the vertical resonance of the pendulum, which (for a reasonable safety factor in the pendulum wire breaking strength) will be about 15 Hz. This motion will be more difficult to critically damp due to geometry and stiffness. If damped to a Q of 10, the net vertical RMS motion of the test mass is about 3.5×10^{-7} m RMS; the spectrum of motion is shown in Figure 10. For a Q of 10000 and $f_{res} = 15$ Hz, the RMS motion is now dominated by the motion from the pendulum vertical resonance, giving about 5×10^{-7} m RMS. This case is shown in Figure 11, where the peak is not resolved. The peak value at 15 Hz is roughly 5×10^{-6} m/$\sqrt{\text{Hz}}$.

c) tilts and twists

1) rotations around the vertical axis: Direct rotational excitation will be small (comparable to the direct tilt excitation). Cross-coupling of horizontal or vertical translations will, however, lead to rotations. The importance of this effect is dependent on the stack design. The MIT LIGO prototype stack has not been characterized for this cross-coupling at frequencies lower than 20 Hz. A conservative prediction for the rotation can be made: Assume that the stack top-plate horizontal motion is due to rotations about an edge of the stack top plate. Then the transfer function for horizontal excitation causing radial motion will have the form of the horizontal-horizontal transfer function, with a scaling factor of the top plate radius (conservatively estimated at about 1 m). We can take the numbers derived above for the horizontal translations: Figure 9 is correct, but with $\text{rad}\cdot\text{Hz}^{-\frac{1}{2}}$ as units. Thus the RMS motion due to this excitation will be about 3.5×10^{-7} rad RMS or about 1 μm peak-to-peak.

2) rotations around the horizontal axis: If the suspension is a simple single wire loop, rotations of the pendulum suspension point only couple through the stiffness of the suspension wire to the pendulum. The more important effect is probably due to the horizontal translations of the pendulum suspension point. The low frequencies where the contribution to the RMS translational motion is important are below the tilt and pendulum mode frequencies ($\approx$1 Hz); in this regime, the pendulum simply translates with the suspension point, making very small angles. It is only near the pendulum and stack resonance frequencies that the angular motion of the pendulum (as shown in Figure 12) is important, so the influence of the down-tube structure is large (turning stack resonances into horizontal translations). The RMS angle of the pendulum wire is $\theta_{\mathbf{pend}} = 1\times10^{-7}$ rad RMS.

The mirror rotation around its center-of rotation horizontal axis will be excited (only) because the wire departure point is not at the center of mass of the mirror. A simple model for the coupling with possible values gives $\theta_{\mathbf{mirror}} \approx (3bg\theta_{\mathbf{pend}}/r^2M) \times H_{\mathbf{mirror}} = 1.5 \times \theta_{\mathbf{pend}}H_{\mathbf{mirror}}$, where $b = 5\times10^{-3}$ m is the distance of the wire attachment above the center of mass of the mirror, g is the gravitational constant, $r = 0.1$ m and $M = 10$ kg are the radius and the mass of the mirror respectively, and $H_{\mathbf{mirror}} = 1/((\omega^2-\omega_0^2)^2+(\omega\omega_0/Q)^2)$ is the transfer function of the mirror resonance about the horizontal axis. Taking $\omega_0 = (2\pi) \times 0.3$, we predict a spectrum of mirror motion shown in Figure 13. The RMS motion is 8.6×10^{-9} rad RMS.

II) Sensitivity of the interferometer to RMS motion

In the following it is assumed that the instrument is a recombined-beam broad-band recycled interferometer. Thus the interferometer path length difference (dark fringe) and the two cavity lengths (resonance condition) can be, and must be, separately controlled.

a) along the optic axis

A number of effects require that the cavities be held close to resonance: sensitivity to power fluctuations in the laser, the loss of power to the recycling cavity, and the change in slope of $d\phi/dx$ introducing a sensitivity to frequency noise. The latter is the most important (See DHS: Modulation and topolgy: Deviations from resonance). For the sample mirror parameters $T_1 = 0.03$, A_1=A_2=100 ppm, the slope (normalized to the slope on resonance) is approximately $d\phi/dx \approx= -1 + 4.35\times10^3 x^2 - 2\times10^7 x^4$ where $x = 4\pi(v - v_0)l/c$ is the deviation from resonance. To put the numbers in context, if we have a cavity storage time matching of $\beta = 10^{-2}$ and do not want the deviation from resonance of the cavities to change the effective storage time by more than $\beta\tau$, the difference of the deviations of the two cavities from resonance should not exceed 2×10^{-5} rad $\hat{=}$0.05 linewidths $\hat{=}6\times10^{-11}$ m, or about $\phi = 6° \hat{=}$ 0.1 rad of phase shift on reflection from the cavity.

b) cross-coupling to longitudinal motion:

All of these 'indirect' effects (those not directly affecting the length of the cavities), and any servo corrections applied to correct for them, will be seen with some reduced effect in the length of the cavities. Some examples, probably not exhaustive:

1) angular changes: If the beam is not centered on the axis of rotation of the mirror, then there will be a linear coupling from mass rotations to cavity length. Given the expected ratio of angular to translational motion, and probably offsets from the axis of rotation, this should be small: $x = \theta y_{\text{offset}} \approx 4\times10^{-8}$ rad $\times$ 0.01 m $= 4\times10^{-10}$ m, a factor of 100 less than the expected direct translations.

2) horizontal or vertical translations: The cavity length is a quadratic function of the displacement perpendicular to the beam of a curved mirror. If this motion is locally damped (OSEMS) it should be a negligible effect.

3) up-conversion: If there are surface irregularities on the mirror, motions of the mirror perpendicular to the beam may cause modulation of the length of the cavity with characteristic frequencies much higher than the spectrum of the mirror motion. No attempt to model this analytically is made here; measurements should be made, with numerical simulation to back up the numbers.

c) tilts and twists: Changes in the pointing of the test mass due to excitation by seismic noise have (at least) the following effects:

1) TEM_{00} *mode coupling:* The amount of power coupled into the fundamental mode of the cavity varies as a function of mirror angle. As one example, assume a flat-concave cavity and take motions of the concave mirror of radius R. Rotations θ of that mirror make translations in the optic axis of the cavity of $y = \theta R$ (for small angles). The overlap integral of the optic axis of the cavity and the optic axis of the light is quadratic with a scale of the $1/e$ diameter of the beam: $M \approx (1-(y/w)^2)$ for $y << w$. The mode matching M as a function of rotations of the far concave mirror will be $M \approx 1-(\theta R/w)^2$. For R=4 km, w=5 cm, and M >0.99, θ must remain less than 1μrad.

2) fluctuations of the contrast: The contrast of the recombined beam interferometer varies with angle variations. Taking as an example the motion of a near, flat, mirror, rotations θ of that mirror make rotations in the optic axis of the cavity of θ. The contrast C of a simple non-recycled interferometer varies as $C = 1-(k\theta w/2)^2$ (for small angles); we take this model for simplicity. Choosing to maintain the contrast as limited by this effect to be greater than $C < (1 - 1\times10^{-3})$ (this makes the power lost to a recycling gain of 30 negligible, and keeps the power on the photodetectors at a reasonable level) requires that the angular motion in both axis be less than 0.2 μrad.

III) Requirements and Servo systems

From the above estimates for the motion of the test mass and the sensitivity of the interferometer to those motions, the transfer functions of the servo systems needed to bring the motions to an acceptable level can be described. The requirements and servo systems below are approximate, and are designed to help choose the style of control rather than to be specific designs.

a) along the optic axis: The primary requirement of this servo system is to bring the deviations of the main cavities from resonance to an acceptable level. A further question is whether the servo loop should have a unity-gain frequency greater than, or less than, the GW band of frequencies. Without entering into this discussion, we calculate the servo system needed for the low unity-gain case, because this is the minimum condition to address the RMS motion problem.

1) required gain as a function of frequency: In **I a** and **II a** above, we found that the motion of the test masses must be reduced from 1×10^{-6} m to roughly 6×10^{-11} m RMS. Most of the motion is in the frequency range from 0.1 to 1 Hz. To obtain 6×10^{-11} m RMS, we need a flat power spectral density of motion of 2×10^{-11} m/$\sqrt{\text{Hz}}$ from 0 to 10 Hz, then falling rapidly with increasing frequency (due to the stack and seismic noise spectrum). This requires a unity-gain frequency of (at least) 10 Hz, a gain of 200 at 1 Hz, and a gain of 1×10^5 at 0.1 Hz.

2) 'locking': The expected damped motion of one test mass is 3.5×10^{-7} m RMS or of two independent masses 1.5μm peak-to-peak. This is about 750 cavity linewidths (the FWHM of the cavity is 2×10^{-9} m), and about 1/2 FSR. The principal frequency of these motions is 0 to .5 Hz, which means that the cavity passes through resonance moderately slowly compared to its storage time of 0.5 msec. This means that the during 'locking' period, the servo loop receives a more-or-less continuous error signal with the correct sign to drive the system into resonance ('lock').

3) required forces: There are two regimes: initial damping, and then damped or locked. The first regime forces will be determined by the time that we wish to wait before the mass is damped. In the second regime, forces must be exerted to hold the pendulum in position against forces exerted by changes in the pendulum suspension point. Below the pendulum resonance frequency, this will be dominated by the gravitational restoring force xg/l, and above by the inertial force ma. The total force is shown in Figure 14 as a function of frequency; the integrated RMS force required is 1.4×10^{-4} N. This does not take into account static forces which may be required to offset drifts in the pendulum suspension point position. To set the scale, a 0.1 mm static deflection requires 4×10^{-2} N.

4) very low frequency drifts: To be written. A temperature servo on Viton stack elements (using their temperature coefficient of expansion as a control element) may be possible. Motors for longer time periods, mounted on the outside of the vacuum system or exerting pressure through elastomer pieces, can be used.

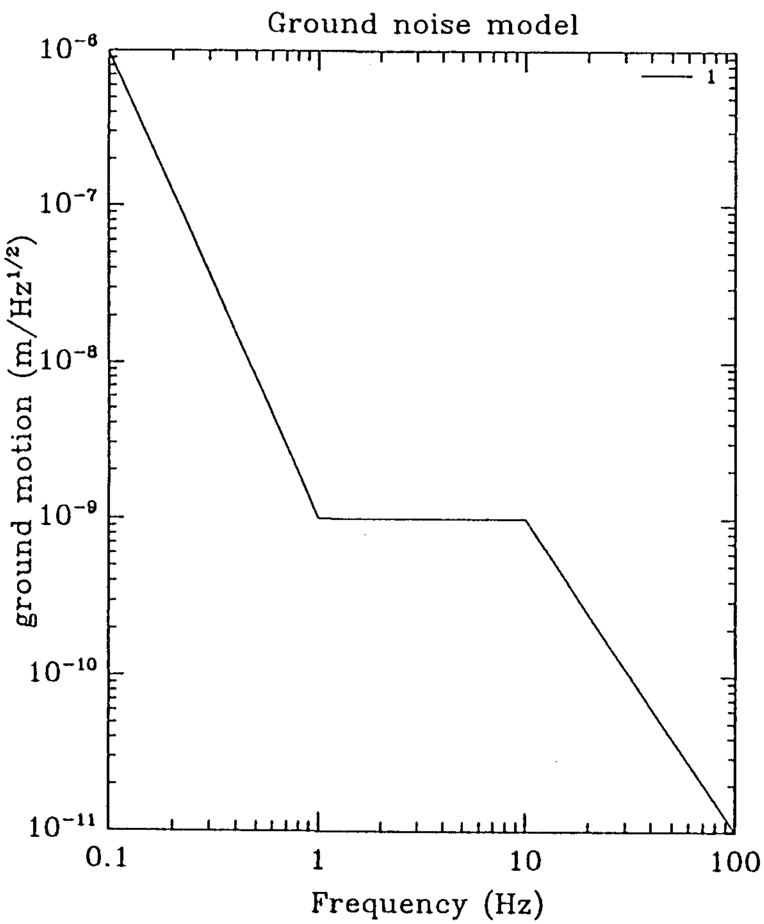
5) length of time lock can be held: The length of a contiguous time record and the RMS fluctuation in the recorded strain signal affect the signal-to-noise ratio for detection of periodic and stochastic sources. The difficulty is in determining the average 'dc' level of a segment and melding it with the proceeding and following segments (see Weiss, ?document). This requirement has not yet been established, but will influence the strategy for compensating for any long-term motions of the mirrors, whether due to the stack, temperature variations, or whatever.

b) transverse motion: To be determined. Up-conversion mechanisms must be studied experimentally.

c) rotations: The primary requirement of these servo systems is to bring the deviations of the main cavities from alignment to an acceptable level.

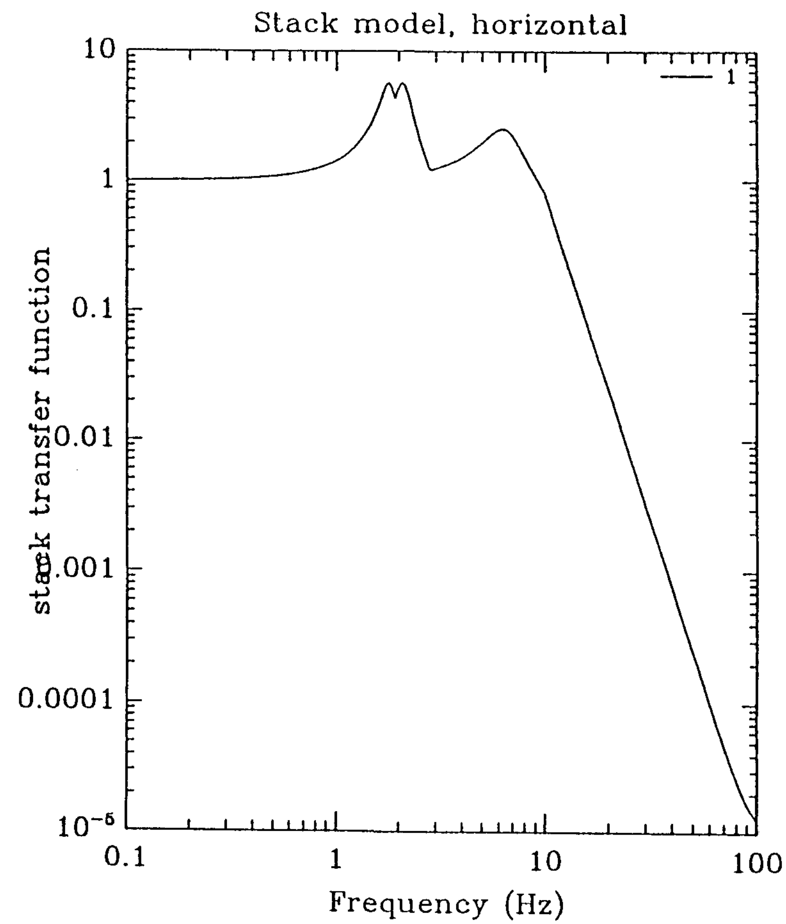
1) required gain as a function of frequency: About the vertical axis, the expected RMS motion is 3.5×10^{-7} rad, and about the horizontal axis is 9×10^{-9} rad (for the models above). The horizontal angular motions are already much less than the requirement of $< 0.2\ \mu$rad. The vertical angle will need a servoloop with a gain of several at 0.1 Hz (the main contribution to the motion). The alignment servos must provide the near-critical damping of the masses in angle, as discussed below.

2) signal to noise: The alignment servo must provide the angular damping for running conditions, and so must have a gain of roughly 1 at 1 Hz, and have a noise level compatible with the usual criterion: the influence on the test masses at GW frequencies (>100 Hz) must be negligible. The sensing noise must be less than 0.2μrad for frequencies below 1 Hz, or roughly 2×10^{-8} rad$\cdot$Hz$^{-\frac{1}{2}}$. This corresponds to motions of the beam at the other mirror (4 km distant) of 0.1 mm. The gain of the servo must be much less than unity in the GW band, but this is made easier by the low unity-gain frequency needed in this servo system.



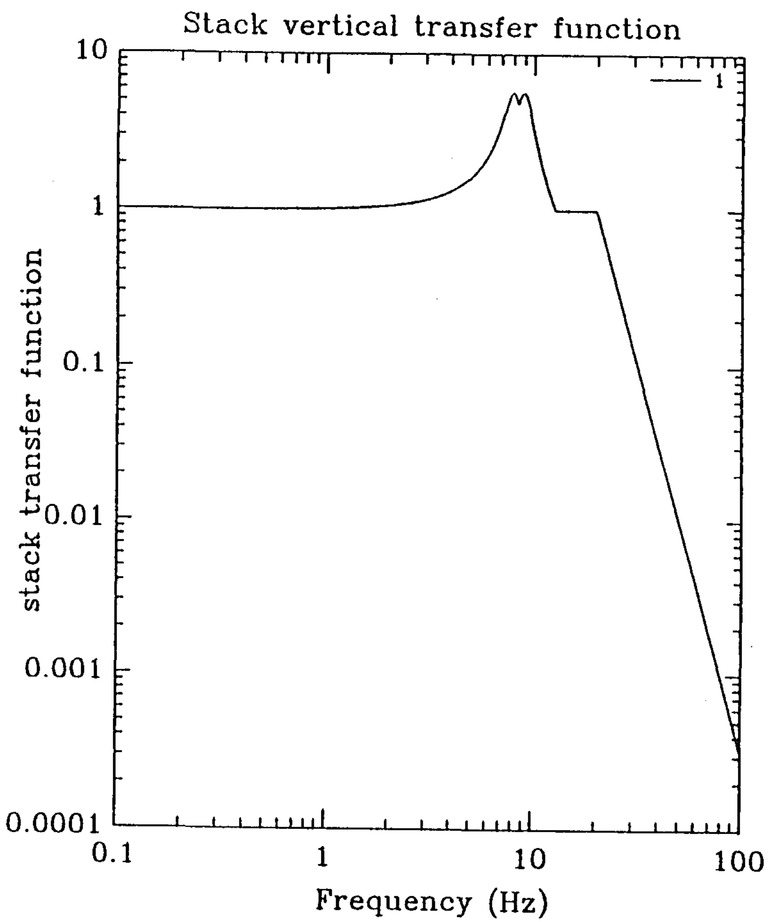
Ground noise model. 3.5×10^{-7} m rms

Figure 1



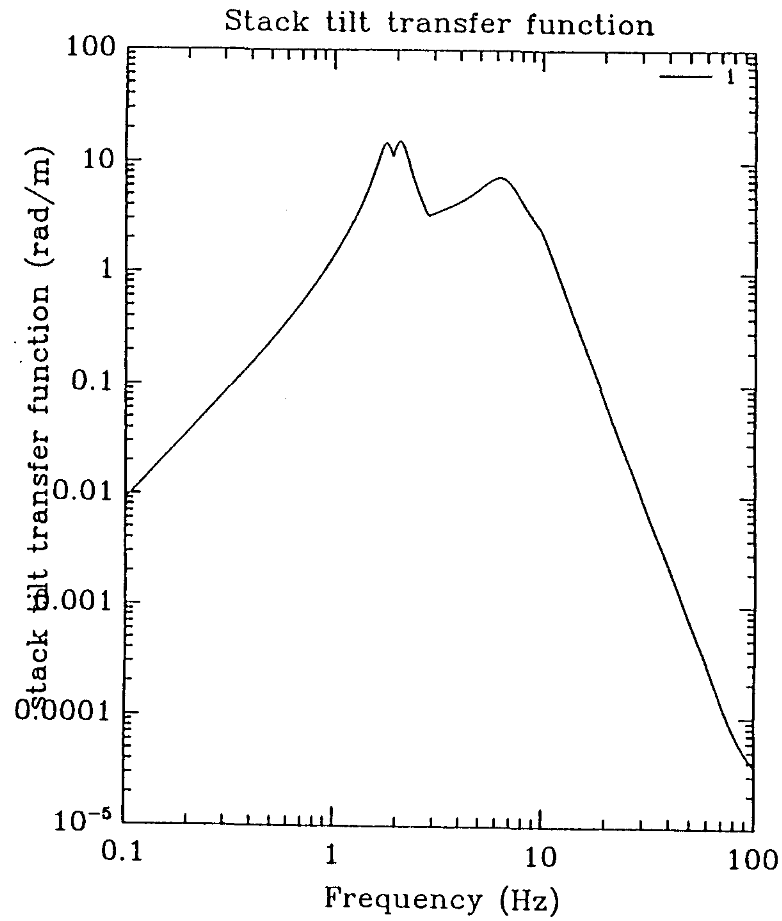
stack model, horizontal–horizontal

Figure 2



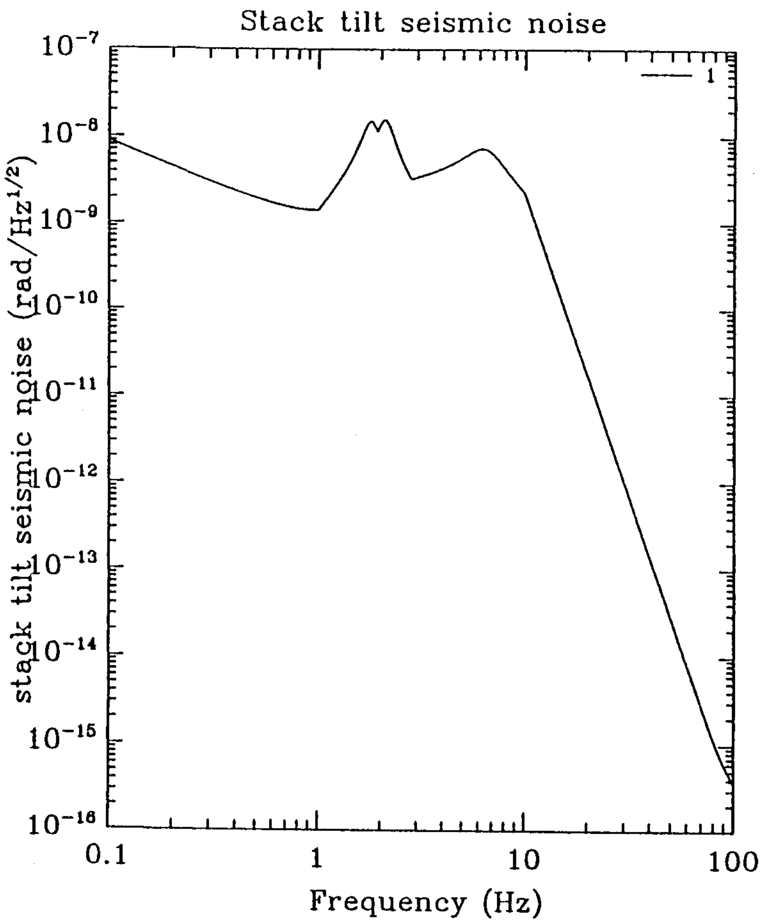
vertical stack transfer function

Figure 3



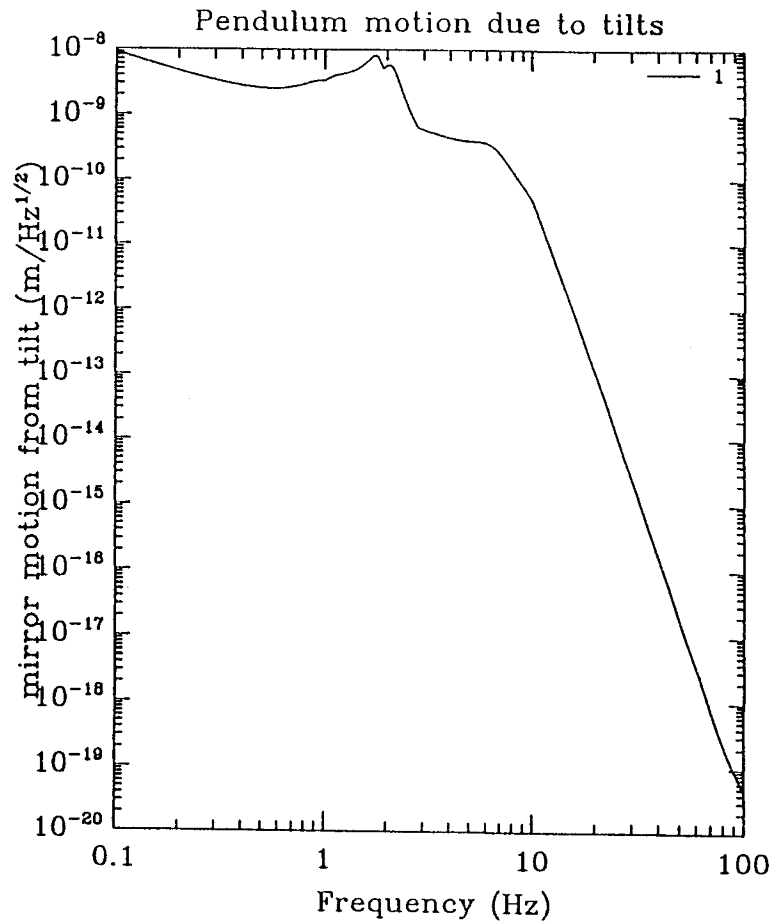
stack tilt transfer function

Figure 4



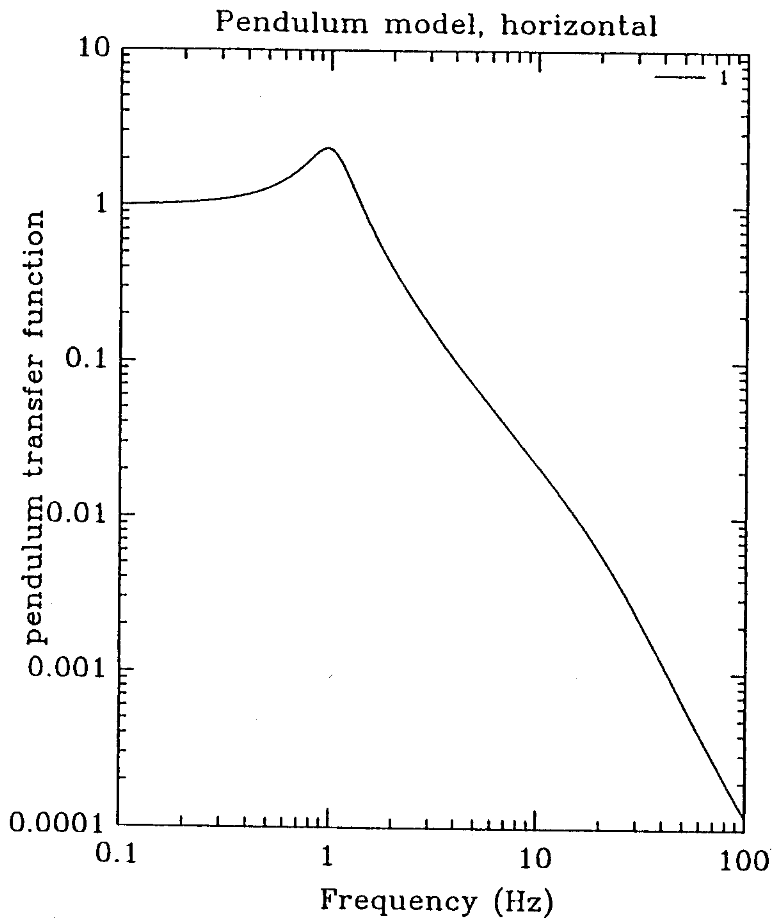
stack tilt seismic noise: 1.9×10^{-8} rad rms

Figure 5



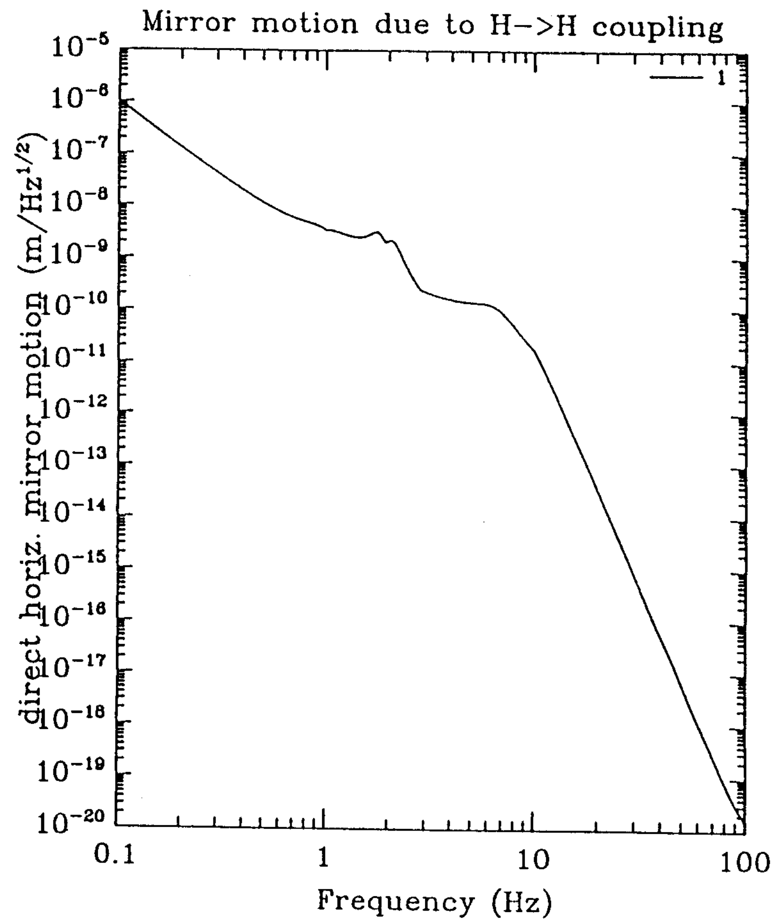
motion due to tilts and down-tube: 7.9×10^{-9} m rms

Figure 6



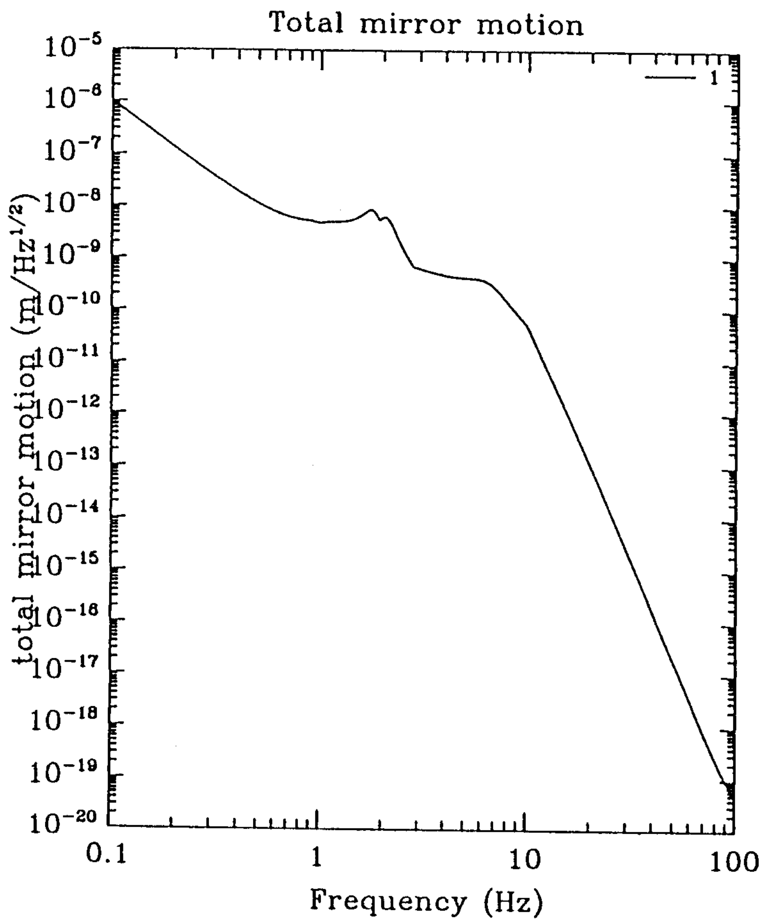
pendulum model, horizontal

Figure 7



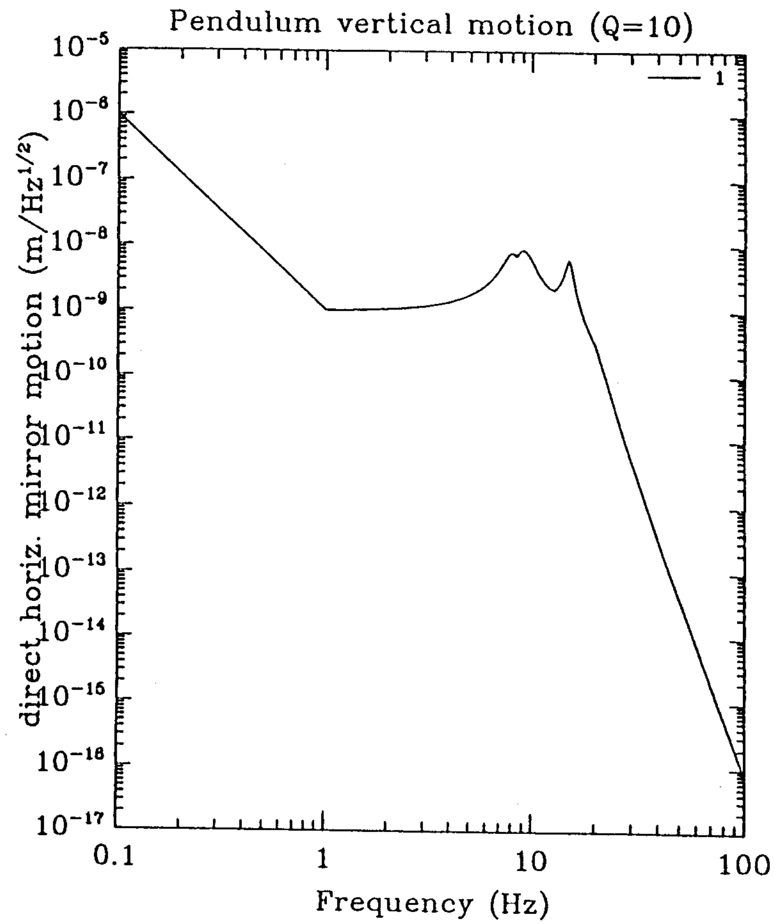
horizontal mirror motion from direct H->H coupling; 3.5×10^{-7} m rms

Figure 8



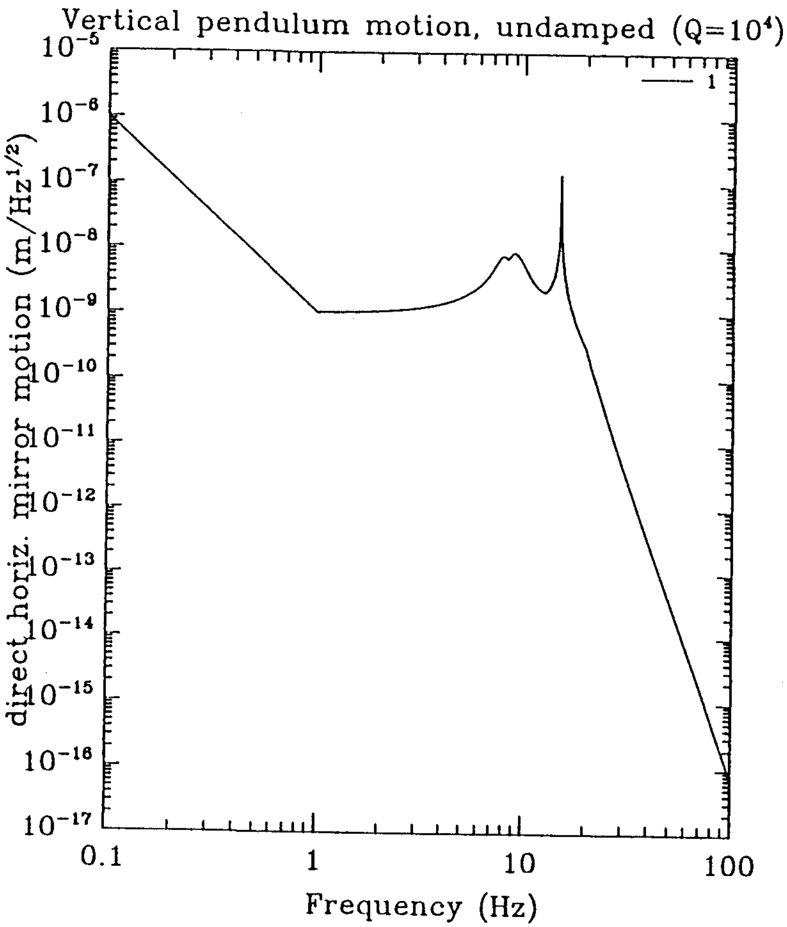
total mirror motion: 3.5×10^{-7} m rms

Figure 9



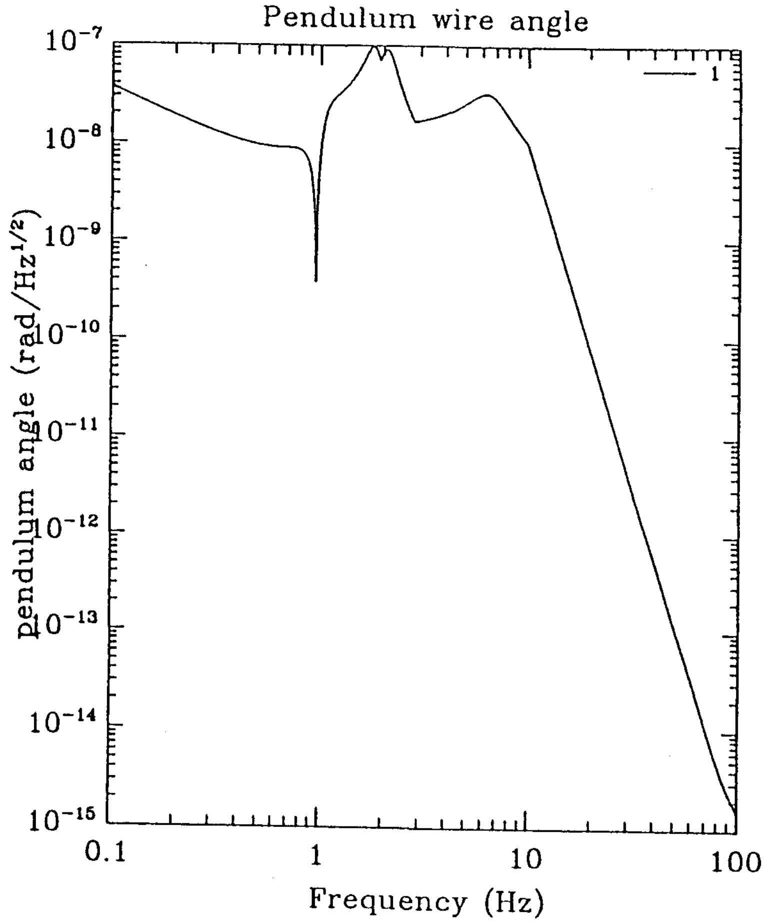
pendulum vertical motion: 3.5×10^{-7} m rms

Figure 10



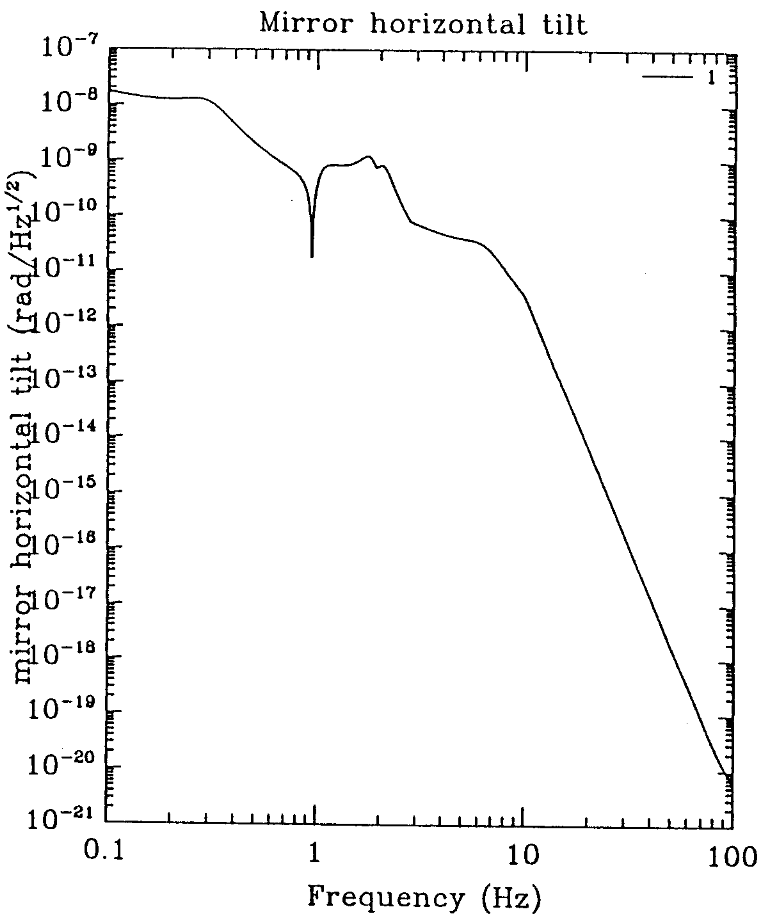
undamped vertical motion, peak unresolved; real total 5.3×10^{-7} m rms

Figure 11



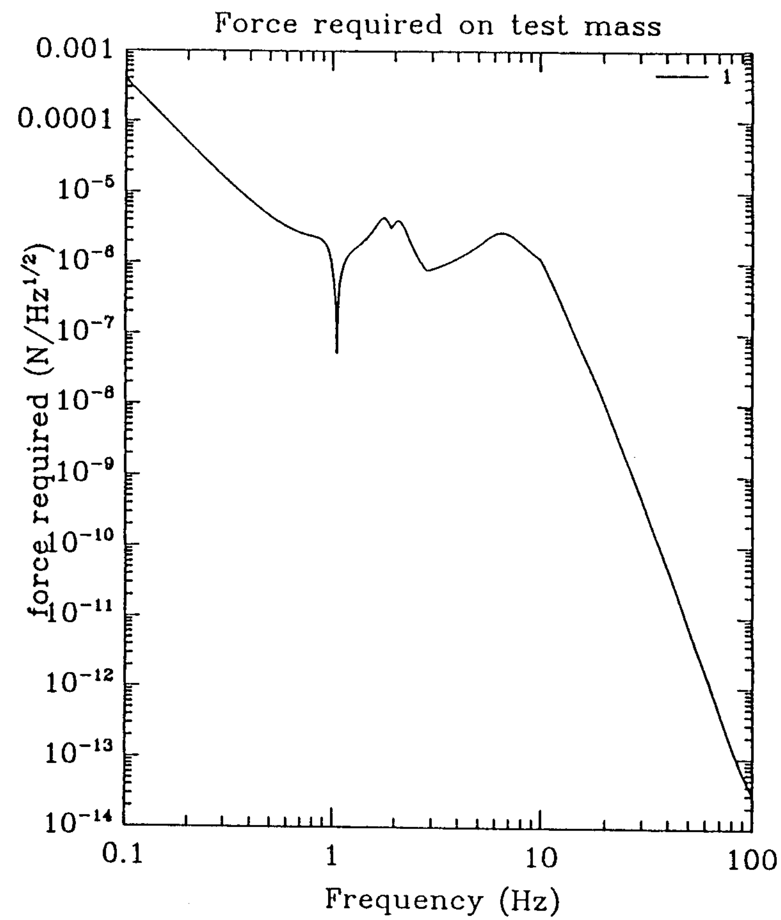
RMS angle 1.0×10^{-7} rad rms

Figure 12



mirror horizontal tilt: 8.6×10^{-9} rad rms

Figure 13



RMS force required 1.4×10^{-4} N rms

Figure 14