New Folder Name Servo Topology

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SUBJECT Servo topology and electronic noise of the 40m prototype

Servo topology and its feature of the 40m prototype with the feed-around path is analyzed. The condition to suppress electronic noise below the shot noise in the servo system is also analyzed.

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1. SERVO TOPOLOGY

The servo topology of the current 40m prototype with the feed-around path is shown in Fig.1.

There are three feedback systems: the laser frequency stabilization servo, the primary cavity servo, and the secondary cavity servo. They are shown in the upper, middle, the lower part of the figure, respectively.

The frequency detection system using an rf-reflection technique is represented by a set of blocks consisting of a discriminator (a triangle with + and – inputs), a cavity pole filter (for example C_0), and a frequency-voltage convertor (for example D_0), which can compare the frequency of the incident light with the resonance frequency of the cavity and convert them into the demodulation signal.

For simplicity, all the feedback paths in the laser frequency stabilization servo are represented by G_0. It includes not only the electronics but also the voltage-frequency convertors.

The primary cavity servo has three feedback paths: a feed-around path, a MC PZT path, and a coil path. In the figure, the transfer function of the electronics is represented by g_{FA}, g_{MC}, and g_{C1}, and that of the voltage-frequency convertor is represented by H_{MC} and H_{C1}.

One of the important features in this topology is that the stabilized laser frequency (f_L') is passively filtered by the MC cavity (C_0 between f_L' and f_L'') which is supposed to be equal to the MC cavity pole filter in the laser frequency stabilization (C_0 between Δf_0 and $\Delta f_0'$).

The shot noise and the electronic noise are also incorporated in the figure. The electronic noise is modeled as separate noise injects that drive each of the electronics blocks of Fig.1. Shot noise and electronic noise in photodetectors are also included as drivers of each of the demodulator blocks (i.e. D_0, D_1, and D_2)

Although the secondary cavity servo is also shown in the figure, it won't be analyzed. Instead, the concentration will be on the stabilized frequency of the light after the MC (f_L'') which is measured by the secondary cavity system. Of course, the secondary cavity servo adds shot noise and electronic noise, but they are simple to analyze and can be treated independently.

2. BASIC RESULT

The stabilized frequency of the light after the MC (f_L') is expressed by the following equations:

$$
f_L'' = C_0 \frac{\begin{array}{l} f_L(1+[\mathbf{CL}]) \ + f_0[\mathbf{LS}](1+[\mathbf{CL}]) \\ +f_1\frac{1}{C_0}(1+[\mathbf{LS}])([\mathbf{FA}]+[\mathbf{MC}]) \\ -(s_0+e_0)D_0G_0(1+[\mathbf{CL}]) \ - e_0'G_0(1+[\mathbf{CL}]) \\ -(s_1+e_1)\frac{1}{C_0C_1}(1+[\mathbf{LS}])([\mathbf{FA}]+[\mathbf{MC}]) \\ -\frac{e_{FA}}{D_1}\frac{1}{C_0C_1}(1+[\mathbf{LS}])([\mathbf{FA}]+[\mathbf{MC}]) \\ +\frac{e_{MC}}{D_1 g_{FA}}\frac{1}{C_0C_1}(1+[\mathbf{LS}])([\mathbf{FA}][\mathbf{CL}]-[\mathbf{MC}]) \\ +\frac{e_{C1}}{D_1 g_{FA} g_{MC}}\frac{[\mathbf{CL}]}{C_0C_1}(1+[\mathbf{LS}])([\mathbf{FA}]+[\mathbf{MC}]) \end{array}}{(1+[\mathbf{LS}])(1+[\mathbf{CL}]+[\mathbf{FA}]+[\mathbf{MC}])} \quad \cdots\cdots \quad \text{(Eq.1)},
$$

where

$$
\begin{aligned}
[\mathbf{LS}] &= C_0 D_0 G_0 \\
[\mathbf{FA}] &= \frac{G_0}{1+[\mathbf{LS}]} C_0 C_1 D_1 g_{FA} \\
[\mathbf{MC}] &= \frac{[\mathbf{LS}]}{1+[\mathbf{LS}]} C_0 C_1 D_1 g_{FA} g_{MC} H_{MC} \\
[\mathbf{CL}] &= C_1 D_1 g_{FA} g_{MC} g_{C1} H_{C1}
\end{aligned}
$$

Here we can reasonably interpret the above quantity as follows:

[LS] : The open loop gain of the laser frequency stabilization

[FA] : The open loop gain of the feed-around path in the primary cavity servo

[MC] : The open loop gain of the MC PZT path in the primary cavity servo

[CL] : The open loop gain of the coil path in the primary cavity servo

3. ANALYSIS OF SERVO

In this section, using the equation of f_L'', some intuitive understanding about the servo will be obtained. All shot noise and electronic noise are neglected here.

(1) When f_0=0, f_1=0, and g_{C1}=0,

$$f_L'' = C_0 \frac{f_L}{(1+[\mathbf{LS}])(1+[\mathbf{FA}]+[\mathbf{MC}])}$$

When the MC and the primary cavity are perfectly quiet, and there's no coil path, the laser frequency is stabilized by **[LS]**, and then by **[FA]** + **[MC]** as well as the MC cavity pole passive filter, C_0.

(2) When **[LS]**>>1,

$$[\mathbf{FA}] = \frac{C_1 D_1}{D_0} g_{FA}$$

$$[\mathbf{MC}] = C_0 C_1 D_1 g_{FA} g_{MC} H_{MC}$$

When **[LS]** is much bigger than 1, **[FA]** contains C_1, not C_0. The open loop (for the feed-around path) transfer function from the injection point (v_{FA}) to the demodulation signal (Δv_1) is $\frac{C_1 D_1}{D_0}$.

(3) When f_0=0 and f_1=0,

$$f_L'' = C_0 \frac{f_L}{(1+[\mathbf{LS}])\left(1+\frac{[\mathbf{FA}]+[\mathbf{MC}]}{1+[\mathbf{CL}]}\right)}$$

The coil path acts as if it reduces the gain of the primary cavity servo (for the frequency) in such a way that the gain is divided by 1 + **[CL]**. Where **[CL]** is comparable to **[FA]** +**[MC]**, the primary cavity servo is inactive to the frequency.

(4) When $\frac{f_L}{[\mathbf{LS}]} \ll f_0$,

$$f_L'' = C_0 \frac{f_0}{\left(1 + \frac{[\mathbf{FA}] + [\mathbf{MC}]}{1 + [\mathbf{CL}]}\right)}$$

When the MC is not so quiet as compared with [LS], the laser frequency is stabilized up to f_0 by the laser frequency stabilization.

4. ANALYSIS OF ELECTRONICS NOISE

In this section, the condition for suppressing the effect of the electronic noise in each stage below the shot noise of the primary cavity servo will be shown. The strategy is to use Eq. 1 and to compare the terms multiplying each of the electronic noise sources with the term multiplying the shot noise.

(1) For e_1,

$$|e_1| < |s_1|$$

Obvious !

(2) For e_{FA},

$$|e_{FA}| < |s_1 D_1|$$

Obvious !

(3) For e_{MC},

$$|e_{MC}| < \left| s_1 D_1 g_{FA} \frac{1 + \frac{[\mathrm{MC}]}{[\mathrm{FA}]}}{[\mathrm{CL}] - \frac{[\mathrm{MC}]}{[\mathrm{FA}]}} \right|$$

The fraction on the right side of inequality can be greater or smaller than 1. For example when $[\mathrm{CL}] > \frac{[\mathrm{MC}]}{[\mathrm{FA}]} > 1$, the fraction is smaller than 1. For this condition it is more difficult to minimize the effect of the electronic noise relative to the shot noise (i.e. the condition is more severe).

(4) For e_{C1},

$$|e_{C1}| < \left| s_1 D_1 g_{FA} g_{MC} \frac{1}{[\mathbf{CL}]} \right|$$

The condition is more severe when **[CL]** >1.

(5) For e_0,

$$|e_0 D_0| < \left| s_1 D_1 g_{FA} \frac{1 + \frac{[\mathbf{MC}]}{[\mathbf{FA}]}}{1 + [\mathbf{CL}]} \right|$$

The fraction can be smaller than 1, when $[\mathrm{CL}] > \frac{[\mathbf{MC}]}{[\mathbf{FA}]} > 1$.

(6) For e_0',

$$|e_0'| < \left| s_1 D_1 g_{FA} \frac{1 + \frac{[\mathbf{MC}]}{[\mathbf{FA}]}}{1 + [\mathbf{CL}]} \right|$$

The same as the condition for e_0.

5. CONCLUSION

In a single feedback loop, it is self-evident to specify the condition for suppressing the effect of noise in one stage below the effect of noise in another stage. But in the complicated servo topology like the current 40m prototype, the condition is also complicated.

In this particular case, the existence of the coil path generally adds the difficulty to the condition. In addition, the coil path reduces the gain of the primary cavity servo for the frequency.

It is worth considering carefully the possibility and necessity of removing the coil path from the primary cavity servo.

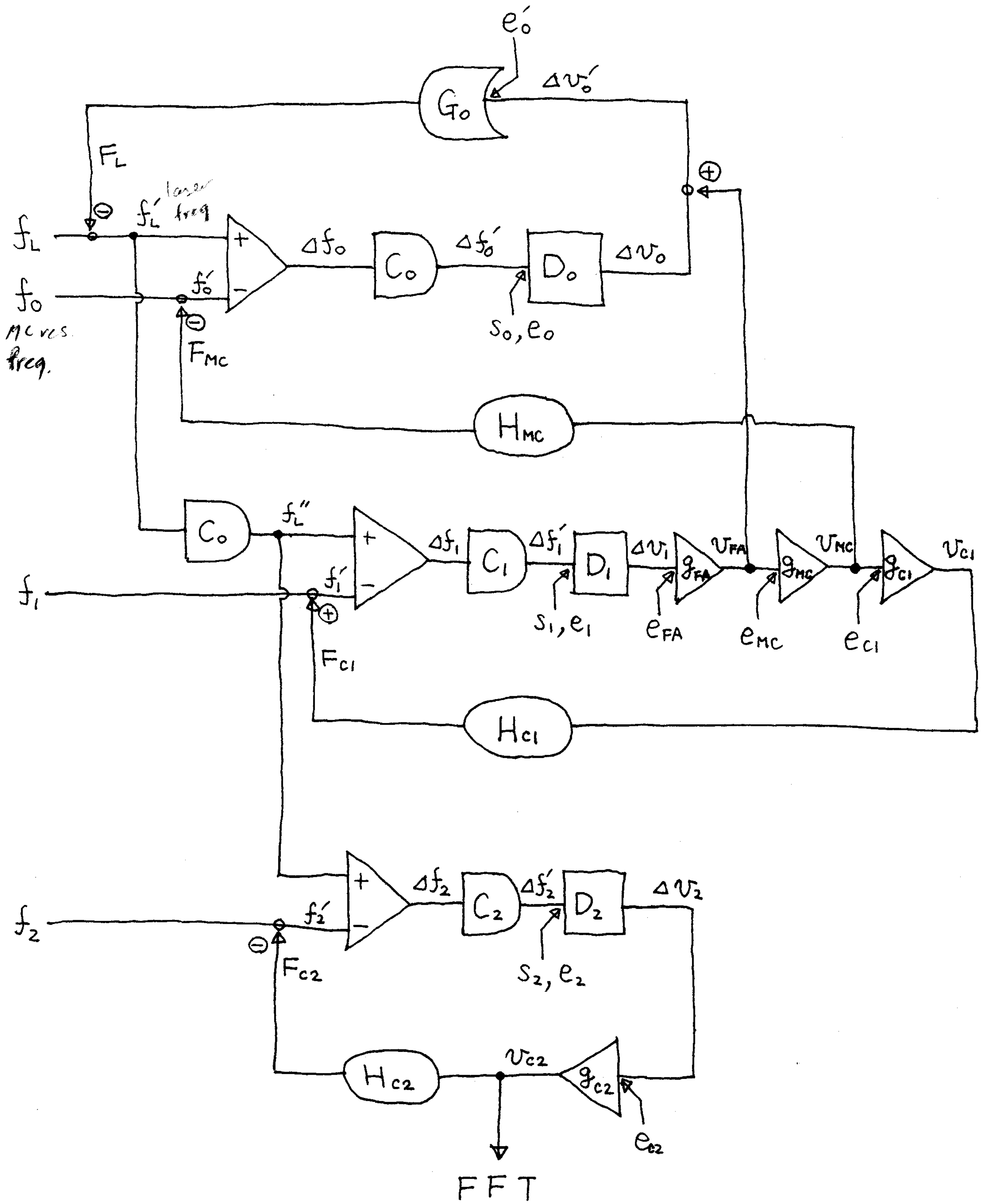

Fig. 1 Servo topology.

f_L: Original frequency of the laser light

f_L': Stabilized frequency of the laser light after the PC

f_0: Resonance frequency corresponding to the original length of the MC

f_0': Resonance frequency corresponding to the stabilized length of the MC

Δf_0: Difference of frequencies between f_L' and f_0'

$\Delta f_0'$: Measurable difference of frequencies between f_L' and f_0'

f_L'': Stabilized frequency of the light after the MC

f_1: Resonance frequency corresponding to the original length of the primary cavity

f_1': Resonance frequency corresponding to the stabilized length of the primary cavity

Δf_1: Difference of frequencies between f_L'' and f_1'

$\Delta f_1'$: Measurable difference of frequencies between f_L'' and f_1'

f_2: Resonance frequency corresponding to the original length of the secondary cavity

f_2': Resonance frequency corresponding to the stabilized length of the secondary cavity

Δf_2: Difference of frequencies between f_L'' and f_2'

$\Delta f_2'$: Measurable difference of frequencies between f_L'' and f_2'

Δv_0: Demodulation signal of the MC servo

$\Delta v_0'$: Demodulation signal of the MC servo added by the feed-around signal

Δv_1: Demodulation signal of the primary cavity servo

v_{FA}: Feedback signal to the feed-around injection point

v_{MC}: Feedback signal to the MC PZT

v_{C1}: Feedback signal to the arm 1 coil

Δv_2: Demodulation signal of the secondary cavity servo

v_{C2}: Feedback signal to the arm 2 coil

F_L: Equivalent correction frequency to the laser light

F_{MC}: Equivalent correction frequency to the MC

F_{C1}: Equivalent correction frequency to the primary cavity

F_{C2}: Equivalent correction frequency to the secondary cavity

C_0: Cavity pole filter of the MC

D_0: Convertor from frequency to demodulation signal in frequency detection system of the MC servo

G_0: Transfer function of the total MC servo including electronics and optics

C_1: Cavity pole filter of the primary cavity

D_1: Convertor from frequency to demodulation signal in frequency detection system of the primary cavity servo

g_{FA}: Electronics for the feed-around path

g_{MC}: Electronics for the MC PZT path

g_{C1}: Electronics for the arm 1 coil path

H_{MC}: Conversion factor of the MC PZT system

H_{C1}: Conversion factor of the arm 1 coil system

C_2: Cavity pole filter of the secondary cavity

D_2: Convertor from frequency to demodulation signal in frequency detection system of the secondary cavity servo

g_{C2}: Electronics for the arm 2 coil path

H_{C2}: Conversion factor of the arm 2 coil system

s_0: Shot noise of the MC servo

s_1: Shot noise of the primary cavity servo

s_2: Shot noise of the secondary cavity servo

e_0: Electronic noise produced before demodulation in the MC servo

e_0': Electronic noise produced in G_0

e_{FA}: Electronic noise produced in g_{FA}

e_{MC}: Electronic noise produced in g_{MC}

e_{C1}: Electronic noise produced in g_{C1}

e_1: Electronic noise produced before demodulation in the primary cavity servo

e_2: Electronic noise produced before demodulation in the secondary cavity servo

e_{C2}: Electronic noise produced in g_{C2}