New Folder Name Off- Resonance Thermal Noise

Proposal to Measure the Off-Resonance Thermal Noise in a Mechanical System

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Chapter 1 Introduction

Thermal noise could be a fundamental limit to the sensitivity of LIGO in the frequency range of a few hundred Hertz and lower. This prediction is based upon certain models for the loss mechanisms in various substances and how they depend upon frequency. An accurate experimental description of these processes is still necessary to obtain a better idea of the future performance of LIGO. The experiment I propose will attempt to measure the actual mechanical displacement due to thermal noise in fused quartz. This should give a better understanding of loss mechanisms in fused quartz and of the effect of thermal noise in the performance of precise measurement systems such as LIGO.

Chapter 2 Previous Measurements of Thermal Noise

Thermal noise is seen in many different types of systems. It arises because the same mechanisms that cause a system to dissipate energy also cause the system to fluctuate around equilibrium. A well known example is the Johnson noise of a resistor where the voltage noise spectral density across a resistor is:

$$V_{thermal}^2(f) = 4k_B T R$$

Another example is the Brownian motion of small particles. In macroscopic mechanical systems, the effect of thermal noise has never been seen except at some resonant frequency of the system.

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Some of the earliest precision mechanical measurements were made using quartz torsion fibre balances. They include the Roll, Krotkov and Dicke[1] repeat of the Eőtvős experiment and various LaCoste gravimeter experiments to measure normal modes of the Earth[2,3]. In all of these experiments, noise sources other than thermal dominated and limited the sensitivity of these instruments.

Another obvious high precision measurement where thermal noise could be seen is the detection of gravity waves. Acoustic gravity wave detectors can see the on-resonance thermal noise with their current sensitivities. In interferometric detectors, the Garching group has seen the thermal noise at the mirror resonance. No one yet has the sensitivity to observe the off-resonance spectrum. In fact, LIGO might prove to be the first instrument to study off-resonance thermal noise in a mechanical system.

The fact that a measurement of the off-resonance thermal noise in a mechanical apparatus has not been effectively accomplished adds another incentive to proceed with this experiment. While no one doubts the validity of the fluctuation dissipation theorem, especially since it accurately describes the Johnson noise in resistors, an experimental study of thermal noise would be a new scientific observation which also has the bonus of helping to predict the future performance of LIGO.

Chapter 3 Different Models for the Thermal Noise in a Harmonic Oscillator

The Fluctuation-Dissipation theorem gives an expression for the thermal noise in a simple linear system (see Saulson[4] for a thorough review):

$$F^2_{thermal}(\omega) = 4k_B T R(\omega)$$

or

$$x^2_{thermal}(\omega) = \frac{4k_B T \sigma(\omega)}{\omega^2}$$

where $Z = \frac{Force}{velocity}$, $R(\omega) = Re(Z)$ and $\sigma(w) = Re(Z^{-1})$.

1 Roll, P. G., R. Krotkov, and R. H. Dicke, *Ann. Phys.* **26**, 442 (1961).

2 Weiss, R., and B. Block, *J. Geophys. Res.* **70**, 5615 (1965).

3 Block, B., and R. D. Moore, *J. Geophys. Res.* **71**, 4361 (1966).

4 Saulson, P. R., *Phys. Rev. D* **42**, 2437 (1990).

There are two well known models for the damping term in a simple harmonic oscillator- aviscous model and an internal friction model. Viscous damping has a force:

$$F_{damping} = -\beta \dot{x}$$
$$F = m\ddot{x} + \beta \dot{x} + kx$$
$$Z = \beta + i\left(m\omega - \frac{k}{\omega}\right)$$
$$\sigma = \frac{\beta}{\beta^2 + \left(m\omega - \frac{k}{\omega}\right)^2}$$

The damping term for internal friction is:

$$F_{damping} = -k(i\phi(\omega))x$$
$$F = m\ddot{x} + k(1 + i\phi(\omega))x$$
$$Z = \frac{k\phi(\omega)}{\omega} + i\left(m\omega - \frac{k}{\omega}\right)$$
$$\sigma = \frac{k\phi(\omega)\,\omega}{\left(k - m\omega^2\right)^2 + k^2\phi^2(\omega)}$$

The viscous model gives a thermal noise spectrum:

$$F^2(\omega) = 4k_B T\beta$$
$$x^2(\omega) = \left(\frac{4k_B T}{m\omega^2}\right)\frac{\frac{\beta}{m}}{\left(\omega - \frac{\omega_o^2}{\omega}\right)^2 + \left(\frac{\beta}{m}\right)^2}$$

For $\frac{\beta}{m\omega_o} << 1$ *or* $Q >> 1$ ($Q = \frac{\beta}{m\omega_o}$ and is defined as the number of radians the system oscillates through before its energy decays by a factor of 1/*e*) and $\omega >> \omega_o$, the functional dependence of the noise term is:

$$x^2(\omega) \propto \frac{\frac{\beta}{m}}{\omega^4}$$

The internal friction gives:

$$F^2(\omega) = 4k_B T m\omega_o^2 \frac{\phi(\omega)}{\omega}$$
$$x^2(\omega) = \left(\frac{4k_B T}{m}\right)\left(\frac{\phi(\omega)}{\omega}\right)\frac{\omega_o^2}{\left(\omega_o^2 - \omega^2\right)^2 + \phi^2(\omega)\,\omega_o^4}$$

For $\phi << 1$ *or* $Q >> 1$ $(Q = \phi)$ and $\omega >> \omega_o$, the functional dependence of the noise term is:

$$x^2(\omega) \propto \frac{\phi(\omega)}{\omega^5}$$

I have measured the Q of various materials here at M.I.T.. The results tend to indicate that Q remains roughly constant with frequency. This would indicate that either $\beta \propto \frac{1}{\omega}$ or $\phi(\omega) = const.$ The Debye model for single relaxation processes has an explicit frequency dependence:

$$\phi(\omega) = D\frac{\omega\tau}{1 + (\omega\tau)^2}$$

where D is the relaxation strength and τ the relaxation time. One such process is thermal damping as described by Zener[5]. The physical interpretation of this model involves heat transfer across a thin fibre. The heat transfer depends upon the thermal conductivity of the material while the compression and expansion of the fibre (which generates the work) depends upon the frequency of the oscillation. If the period of the oscillation matches the heat transfer time, the dissipation is maximized.

In every case, assume $\phi(\omega) << 1$. For $\omega >> \omega_o$ and $\omega >> \tau^{-1}$, the functional dependence of the noise term is:

$$x^2(\omega) = \left(\frac{4k_BT}{m}\right)\left(\frac{D\omega_o^2}{\tau}\right)\frac{1}{\omega^6}$$

If $\omega << \tau^{-1}$, then the noise has the form:

$$x^2(\omega) = \left(\frac{4k_BT}{m}\right)(D\omega_o^2\tau)\frac{1}{\omega^4}$$

If $\omega \approx \tau^{-1}$, then one has:

$$x^2(\omega) = \left(\frac{4k_BT}{m}\right)(D\omega_o^2\tau)\frac{1}{\omega^4}\left(\frac{1}{1 + (\omega\tau)^2}\right)$$

On the other side of the resonance, $\omega << \omega_o$, the relation for $\omega >> \tau^{-1}$ is:

$$x^2(\omega) = \left(\frac{4k_BT}{m}\right)\left(\frac{D}{\omega_o^2\tau}\right)\frac{1}{\omega^2}$$

[5] C. Zener, *Phys. Rev.* **52**, 230 (1937); C. Zener, *Phys. Rev.* **53**, 90 (1938).

and $\omega << \tau^{-1}$:

$$x^2(\omega) = \left(\frac{4k_BT}{m}\right)\left(\frac{D\tau}{\omega_o^2}\right)$$

Finally, for $\omega \approx \tau^{-1}$:

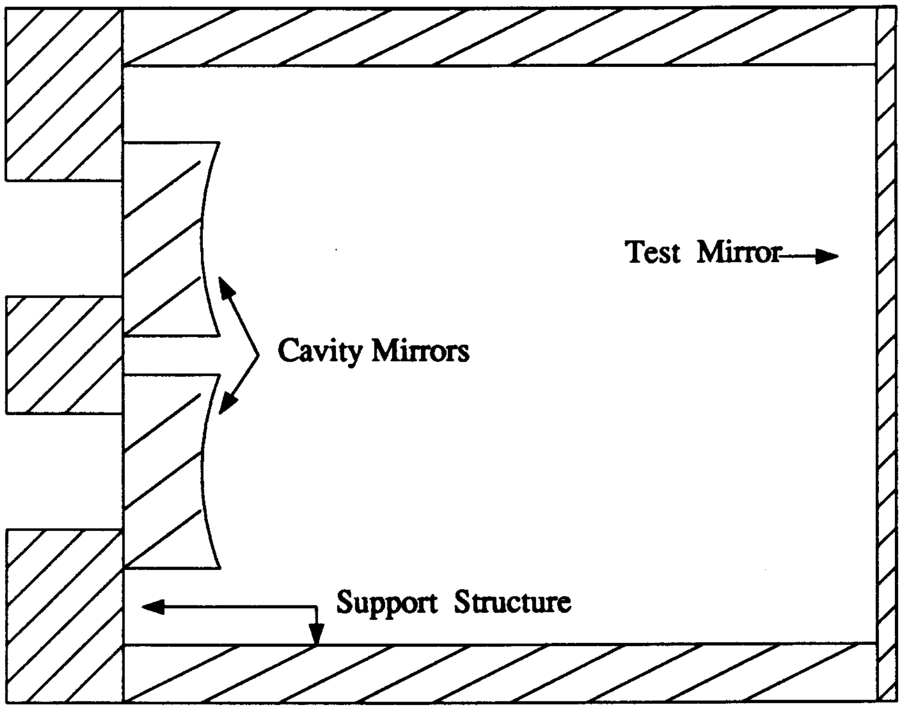
$$x^2(\omega) = \left(\frac{4k_BT}{m}\right)\left(\frac{D\tau}{\omega_o^2}\right)\left(\frac{1}{1+(\omega\tau)^2}\right)$$

This experiment will try to measure the thermal noise spectrum on both sides of the resonance peak. This should give some idea as to the frequency dependence of the loss mechanisms.

Chapter 4 Optical Design

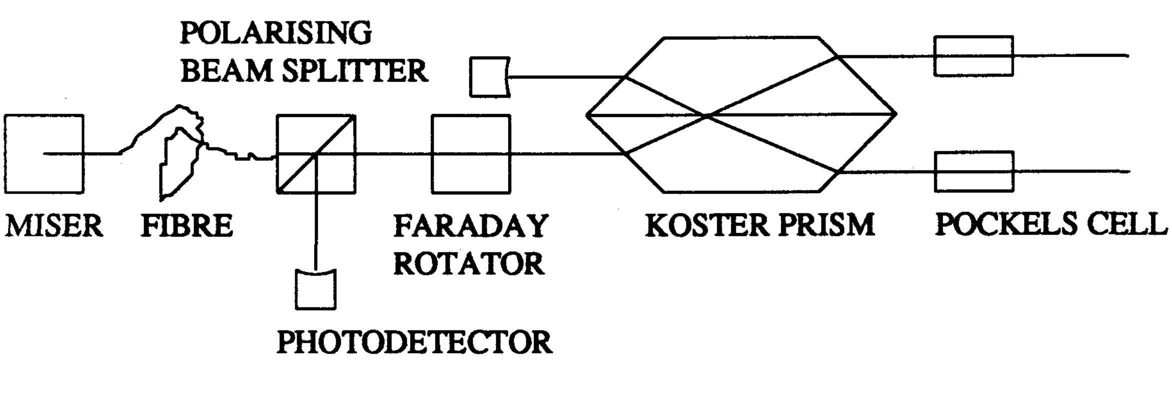
The optical system is basically a recombined Fabry-Perot interferometer where a mechanical mode of a thin, common test mirror produces a differential path length between the two arms (see figure 1). This design should cause the common mode motion to cancel. By making the two spherical mirrors and the fused quartz plate where they are mounted thick enough, their resonant frequencies and mass will be high enough so that their contribution to the thermal noise of the system will be negligible. The normal modes of the cylindrical shell should primarily be common mode in nature. By making the walls sufficiently thick, the frequencies of these modes should be high enough to make their contribution to the thermal noise negligible.

Figure 1 Sketch of experimental apparatus. The flat, thin mirror on the right is the one that will be measured. The two small spherical mirrors are optically contacted to a flat piece of fused quartz. Both the thin, flat mirror and the thick plate of fused quartz are optically contacted to a fused quartz cylinder that holds the assembly together.



The fore optics is standard for an internally modulated combined Fabry-Perot system except that the beam splitter is of a special type called a Köster prism.

Figure 2 Fore optics for experiment



Assuming the system sensitivity is shot noise limited, the calculation for the sensitivity goes as follows. The losses in the optics are:

$$loss = (fibre = 0.5)(mode \ \ match = 0.9)(Faraday \ rotator = 0.9)\times$$
$$(beam \ splitter = 0.95)(Koster \ prism = 0.95)^2(Pockels \ cell = 0.8)^2\times$$
$$(photodetector = 0.9) = 0.22$$

The formula for the noise, including the double pass through the Pockels cell, is:

$$noise = \frac{1}{8\pi}\sqrt{\frac{h\lambda c}{IRTn(loss)}}\times$$
$$\left(\frac{\sqrt{1 - KJ_0(2\Gamma\sin(\frac{2\omega_m l}{c}))J_0(2\Gamma(1+\cos(\frac{2\omega_m l}{c})))}}{KJ_1(2\Gamma\sin(\frac{2\omega_m l}{c}))J_1(2\Gamma(1+\cos(\frac{2\omega_m l}{c})))}\right)\frac{1}{A(x)\frac{\partial\varphi(x)}{\partial x}}$$

where I is the intensity of the light, R,T are the reflection and transmission coefficients of the Köster prism, K is the contrast, J_n are the Bessel's functions of the first kind, Γ is the modulation depth, l is the distance between the Pockels cell and the cavity, $A(x)$ is the amplitude function of the cavity, $\varphi(x)$ is the phase function of the cavity and $x = \frac{4\pi\times cavity\ length}{\lambda}$. Putting in values gives:

$$noise = \frac{1}{8\pi}\sqrt{\frac{(6.62618\times10^{-34}J\,s)(1.06\times10^{-6}m)\,(3\times10^{8}m/s)}{(0.04W)(0.5)(0.5)(1)(0.22)}}\times$$
$$\left(\frac{\sqrt{1 - KJ_0(2\Gamma\sin(\frac{2\omega_m l}{c}))J_0(2\Gamma(1+\cos(\frac{2\omega_m l}{c})))}}{KJ_1(2\Gamma\sin(\frac{2\omega_m l}{c}))J_1(2\Gamma(1+\cos(\frac{2\omega_m l}{c})))}\right)\frac{1}{A(x)\frac{\partial\varphi(x)}{\partial x}}$$
$$= 4.1\times10^{-14}cm/\sqrt{Hz}\times$$
$$\left(\frac{\sqrt{1 - KJ_0(2\Gamma\sin(\frac{2\omega_m l}{c}))J_0(2\Gamma(1+\cos(\frac{2\omega_m l}{c})))}}{KJ_1(2\Gamma\sin(\frac{2\omega_m l}{c}))J_1(2\Gamma(1+\cos(\frac{2\omega_m l}{c})))}\right)\frac{1}{A(x)\frac{\partial\varphi(x)}{\partial x}}$$

For a mirror loss of 10^{-4} and a transmission of 2×10^{-4} in the front mirror, $A(x)$ and $\varphi(x)$ are shown in figures 3 and 4.

Figure 3 $\log_{10}(A(x))$ vs. x for a mirror loss of 10^{-4} and a transmission of 2×10^{-4} in the front mirror.

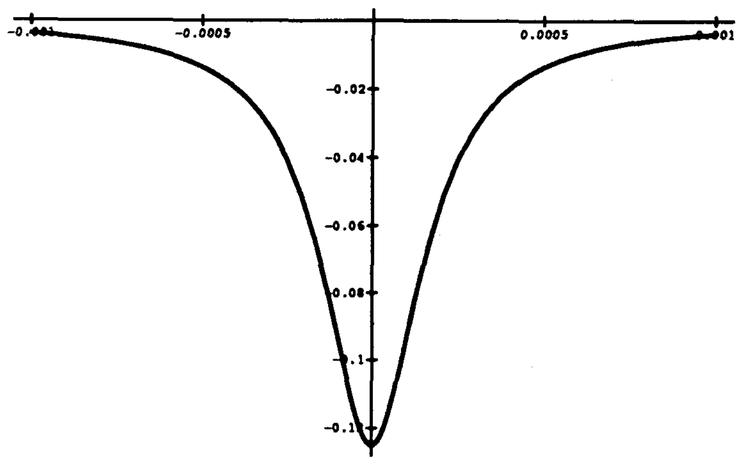
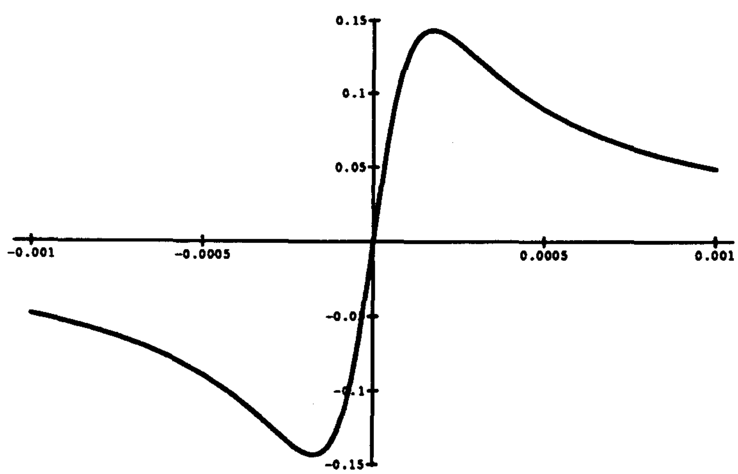


Figure 4 $\varphi(x)$ vs. x for a mirror loss of 10^{-4} and a transmission of 2×10^{-4} in the front mirror.

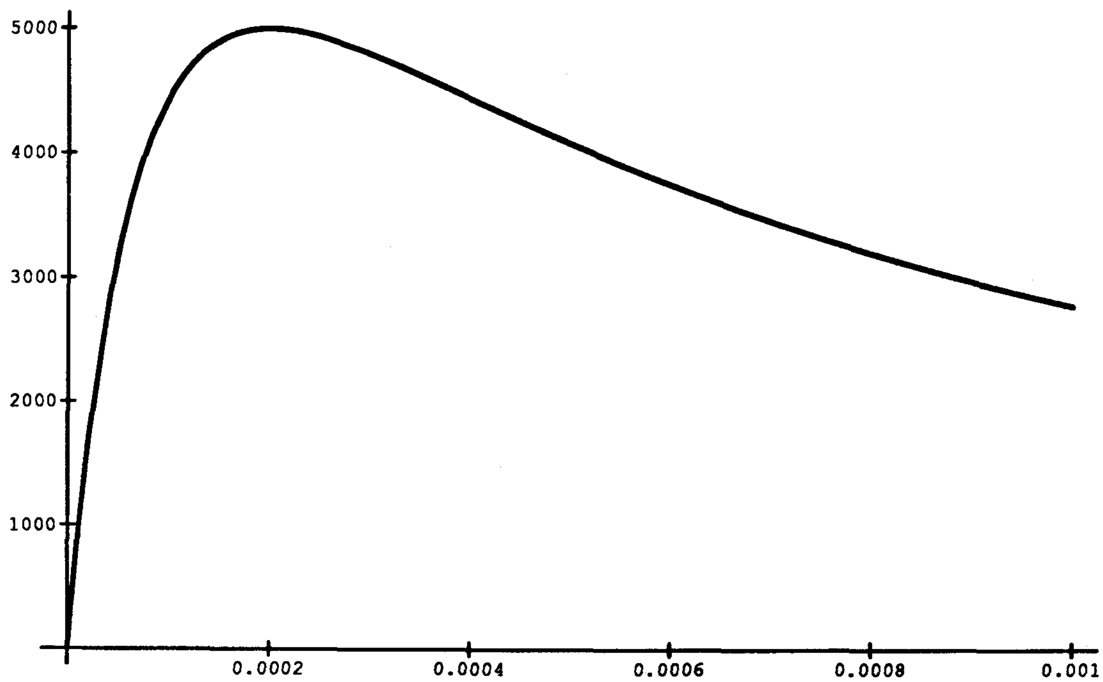


For the cavity on resonance ($x = 0$),

$$A(0)\frac{\partial\varphi(x)}{\partial x}\Big|_{x=0} = \frac{T\sqrt{1-L}}{2-2L+L^2-2\sqrt{1-L}\sqrt{1-L-T}-T+LT}$$

(see figure 5).

Figure 5 $A(0)\frac{\partial\varphi(x)}{\partial x}\big|_{x=0}$ vs. transmission in front mirror for a loss of 10^{-4}.



The maximum of this function occurs for $T = 2L$ and has the value:

$$A(0)\frac{\partial\varphi(x)}{\partial x}\Big|_{x=0} = \frac{2L\sqrt{1-L}}{2-4L+3L^2-2\sqrt{1-L}\sqrt{1-3L}}$$

For perfect contrast ($K = 1$), $\frac{2\omega_m l}{c} << 1$ and modulation index $\Gamma < 1$, the term

$$\left(\frac{\sqrt{1-KJ_0(2\Gamma\sin(\frac{2\omega_m l}{c}))J_0(2\Gamma(1+\cos(\frac{2\omega_m l}{c})))}}{KJ_1(2\Gamma\sin(\frac{2\omega_m l}{c}))J_1(2\Gamma(1+\cos(\frac{2\omega_m l}{c})))}\right)$$

reduces to:

$$1 - x^2\left(\frac{1}{8\Gamma^2} - \frac{1}{4\Gamma} - \Gamma\right),\ x = \frac{2\omega_m l}{c}$$

For a mirror loss of $10^{-4} - 10^{-5}$, the sensitivity is $8 \times 10^{-18} - 8 \times 10^{-20}\ cm/\sqrt{Hz}$. A loss of 10^{-4} is well within the capability of mirror coatings so that the sensitivity of $10^{-17}\ cm/\sqrt{Hz}$ required for the experiment to work seems possible.

The beams from each arm are recombined and locked on a dark fringe. Since the size of the cavity is fixed ($\approx 10cm$) by the pieces optically contacted with each other, the locking technique has to adjust for the overall and differential length of the cavity. The overall resonant frequency can be changed by tuning the MISER.

The differential length between the two arms is planned to be changed by differentially heating the quartz cylinder. A temperature difference of about 5°C between the two parts of the cylinder where the beams are located will provide a change in the differential length equal to $\frac{\lambda}{4}$. This can be achieved by heating the top part of the cylinder with a light source to a temperature about 20°C higher than the ambient temperature. Since the radiaitive cooling from the surface of the cylinder is about 3 times larger than the heat flow across the circumference of the cylinder, a temperature gradient between the top and bottom occurs. The time constant for this to take place is about 40 minutes. The slight tilt in the end pieces that this introduces produces a negligible change in the contrast. The inherent thermal fluctuations in the heating of the cylinder will contribute an effective, but insignificant differential length change noise source.

The alignment of the cavity is tricky, but can be accomplished by moving each beam until it goes through the part of its respective spherical mirror that is normal to the flat, thin mirror. Each beam has to move in the plane parallel to the end of the cylindrical cavity. By rotating the cylinder around its symmetry axis, one can obtain one degree of freedom. The other comes from moving the point where the beam spot hits the Köster prism.

Chapter 5 Normal mode Analysis of a Thin Plate

For a thin circular plate, the general form of the normal modes is:

$$Z(r,\theta) = (J_k(\kappa r) + b\, I_k(\kappa r)) \cos(k\theta)$$

The boundary conditions for the edge rigidly clamped are:

$$Z(r = r_o) = 0$$

$$\frac{\partial Z}{\partial r}\Big|_{r=r_o} = 0$$

Solving these equations give the following:

$$J_k(\kappa r_o)\, I_{k+1}(\kappa r_o) + I_k(\kappa r_o)\, J_{k+1}(\kappa r_o) = 0$$
$$b = \frac{-J_k(\kappa r_o)}{I_k(\kappa r_o)}$$

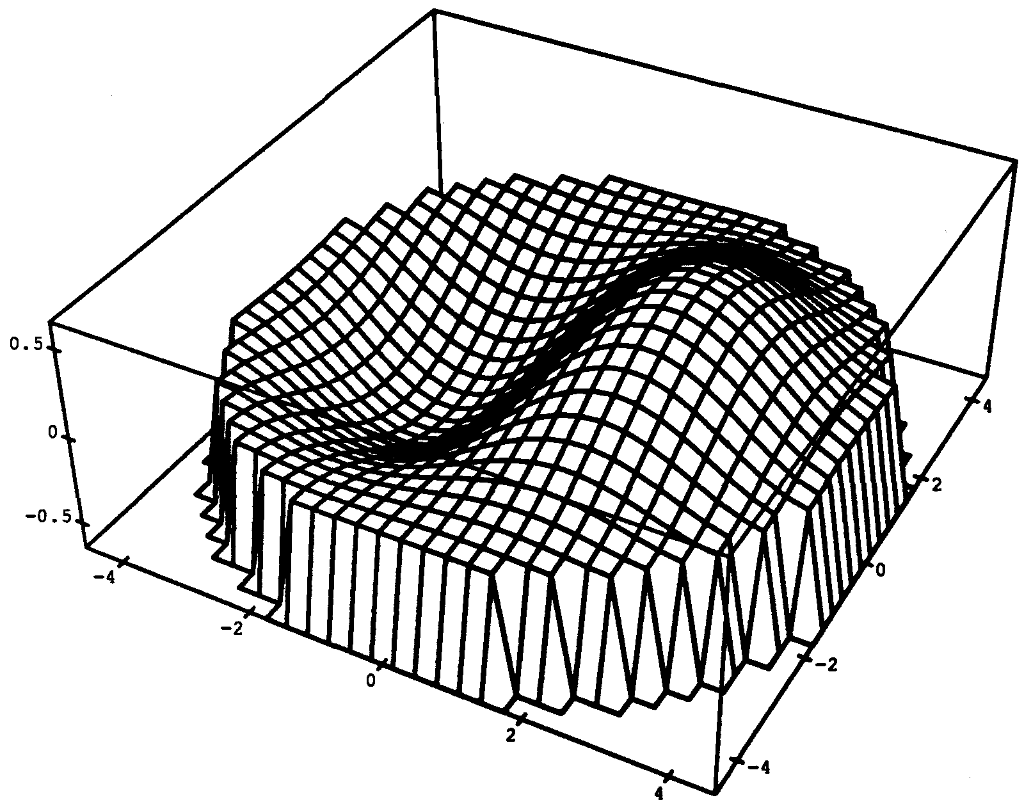
The roots of these equations are given for some values of k (number of radial nodes) and n (number of circular nodes) are given in table 1. The shape of

Table 1 κr_o and b for clamped edge

n	k=0	k=1	k=2	k=3
0	3.19622 0.0557128	4.6109 0.0152162	5.90568 0.00523458	7.14353 0.00201321
1	6.30644 -0.0023015	7.79927 -0.000608146	9.19688 -0.0017531	10.5367 -0.0000565
2	9.4395 0.00110987	10.9581 0.0000254845	12.4022 0.0000675364	κr_o b

the second lowest mode is displayed in figure 6. This is the mode that will be measured.

Figure 6 Second lowest order mode ($n=0$, $k=1$)



The frequencies of these modes are given by:

$$\omega^2 = \frac{Eh^2}{12(1-\sigma^2)\rho}\frac{(\kappa r_o)^4}{r_o^4}$$

$$E = 7.2 \times 10^{11}\ dynes/cm^2$$

$$\sigma = 0.16$$

$$\rho = 2.2\ g/cm^3$$

$$f = 2.66 \times 10^4 \frac{(\kappa r_o)^2 h}{r_o^2}\ Hz\ \ r, h\ in\ cm$$

For the second lowest order mode, the frequency is:

$$f = 5.66 \times 10^5 \frac{h}{r_o^2}\ Hz\ \ r, h\ in\ cm$$

To determine the thermal noise in the normal mode, one must normalize. In generalized coordinates,

$$z(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A_{nk}R_{nk}(r)\Theta_k(\theta)q_{nk}(t)$$

$$\ddot{q}_{nk}(t) + \omega_{nk}^2(1+\phi_{nk}(\omega))q_{nk}(t) = Q_{nk}(t)$$

$$Q_{nk}(t) = \int_0^{r_o}\int_0^{2\pi} f(r,\theta,t)A_{nk}R_{nk}(r)\Theta_k(\theta)r\,d\theta\,dr$$

$$f(r,\theta,t) = pressure$$

$$R_{nk}(r) = J_k(\kappa_{nk}r) + b_{nk}I_k(\kappa_{nk}r)$$

$$\Theta_k(\theta) = \cos(k\theta)$$

$$A_{nk}^2\int_0^{r_o}\int_0^{2\pi} \rho h\left(A_{nk}R_{nk}(r)\Theta_k(\theta)\right)^2 r\,d\theta\,dr = 1$$

An explicit damping term in the form of an internal friction, $\phi(\omega)$, is included. If a force F is applied at r, θ, then a solution of the above system of equations is:

$$Q_{nk} = FA_{nk}R_{nk}(r)\Theta_k(\theta)$$

$$q_{nk} = \frac{FA_{nk}R_{nk}(r)\Theta_k(\theta)}{\omega_{nk}^2 - \omega^2 + i\phi_{nk}^2(\omega)\omega_{nk}^2}$$

$$z(r,\theta) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{F\left(A_{nk}R_{nk}(r)\Theta_k(\theta)\right)^2}{\omega_{nk}^2 - \omega^2 + i\phi_{nk}^2(\omega)\omega_{nk}^2}$$

Recalling that the thermal noise in a mechanical system is:

$$z_{thermal}^2(\omega) = \frac{4k_BT\sigma(\omega)}{\omega^2}$$

and combining the above gives

$$z^2(\omega) = \frac{4k_BT}{\pi\rho h\omega}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{\cos^2(k\theta)\left(J_k(\kappa_{nk}r) + b_{nk}I_k(\kappa_{nk}r)\right)^2}{M_{nk}}\times$$

$$\frac{\phi_{nk}(\omega)\,\omega_{nk}^2}{\left(\omega_{nk}^2 - \omega^2\right)^2 + \phi_{nk}^2(\omega)\,\omega_{nk}^4}$$

$$M_{nk} = \frac{1}{\kappa_{nk}^2}\int_0^{\kappa_{nk}r_o}\left(J_k(u) + b_{nk}I_k(u)\right)^2 u\,du$$

Table 2 gives some values for M_{nk}/r_o^2. By putting in the values for fused quartz

Table 2 $\frac{M_{nk}}{r_o^2} = \frac{1}{\kappa_{nk} r_o^2} \int_0^{\kappa_{nk} r_o^2} (J_k(u) + b_{nk} I_k(u))^2 u \, du$

n	k=0	k=1	k=2	k=3
0	0.10887	0.067164	0.0498746	0.0394756
1	0.0506907	0.0404892	0.0336565	0.0287584
2	0.211237	0.0289387	0.124823	

and doing some simplification, one has a workable form for the thermal noise contribution of a specific mode:

$$z_{nk}^2(f) = \frac{3.81 \times 10^{-15}}{h\,f} \frac{\cos^2(k\theta)(J_k(\kappa_{nk} r) + b_{nk} I_k(\kappa_{nk} r))^2}{r_o^2\left(\frac{M_{nk}}{r_o^2}\right)} \times$$

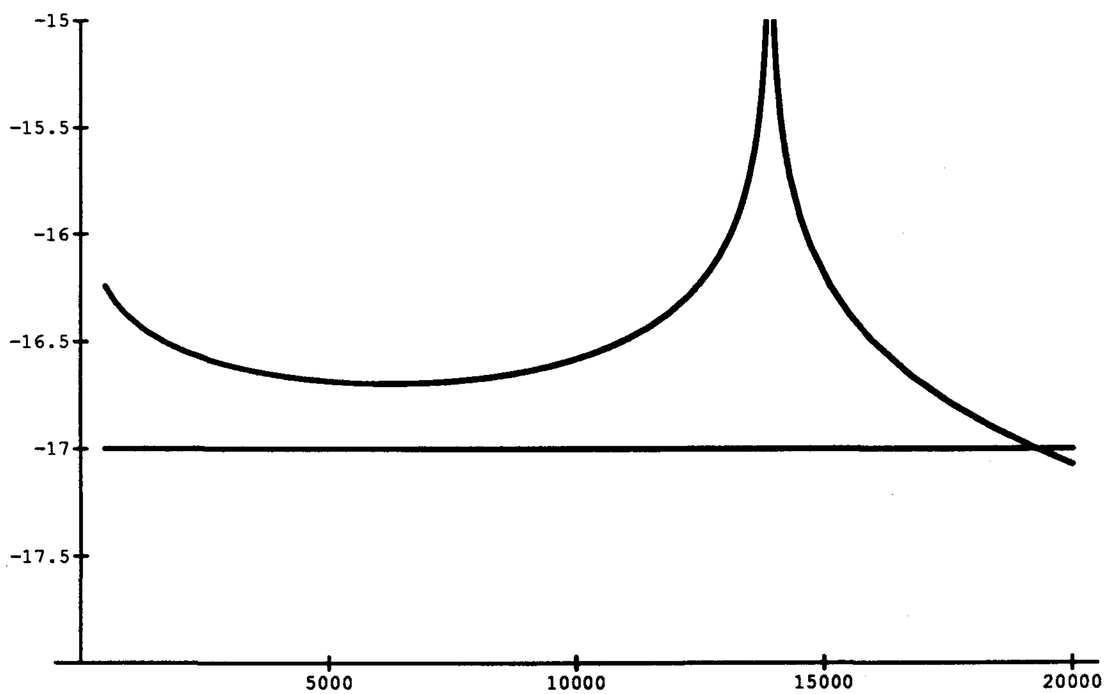
$$\frac{\phi(\omega)}{\left(1.67 \times 10^5 \frac{(\kappa_{nk} r_o)^2 h}{r_o^2} - 2.36 \times 10^{-4} \frac{r_o^2 f^2}{(\kappa_{nk} r_o)^2 h}\right)^2 + 2.79 \times 10^{10} \phi^2(\omega) \frac{(\kappa_{nk} r_o)^4 h^2}{r_o 4}}$$

$$in\ cm^2/Hz,\ for\ r_o, r, h\ in\ cm,\ f\ in\ Hz$$

The parameters that determine the size of the mirror are chosen to give a resonant frequency between 1 and 20 *kHz*. There is a trade off here because a low resonance frequency (below 1 *kHz*) is more easily excited by acoustic and seismic noise. A high resonance frequency implies a bigger mass for the mirror which decreases the effect of the thermal noise. There are two ways to achieve a resonant frequency in this range and each has a problem associated with it. The mirror can be made thin which causes problems in the manufacturing process since a certain aspect ratio is necessary in order to obtain a good surface figure. The surface figure is constrained by two different factors. The mirror will be optically contacted which requires a minimum surface figure error. Also, the high finesse of the optical cavities demands a very small loss in the mirror which constrains the surface figure. The other dimension that can be changed is the diameter. Practical considerations limit this size.

Standard industry practice uses a 9:1 aspect ratio for a mirror to have a surface figure of $\lambda/10$. For a frequency of 10 *kHz*, the diameter of the mirror will be 10″. This is too large. A reasonable size for the diameter is about 4″. Since the surface figure is only important over the area that is optically contacted and over the beam spot (which is small), the aspect ratio and surface figure can be relaxed somewhat. The numbers that I will use are a diameter of 4″ (actually 4.5″ including the surface that is optically contacted) and a thickness of 1/4″. This has a resonance frequency of $14\,KHz$ for the second lowest mode. Using $\phi(\omega) = 10^{-5}$ or $Q = 10^5$ gives the curve in figure 7. The spot on the mirror is chosen to be at the maximum displacement point or at $r = 0.38r_o$. If the predicted sensitivity of this experiment does reach the conservative calculated value of $10^{-17}\,cm/\sqrt{Hz}$, then the off resonance thermal noise should be seen.

Figure 7 Plot of thermal noise in second lowest order mode $\log_{10}\left(cm/\sqrt{Hz}\right)$ $vs.\ frequency\ (Hz)$ for $\phi(\omega) = 10^{-5}$. The horizontal line shows the predicted sensitivity of $10^{-17}\,cm/\sqrt{Hz}$.

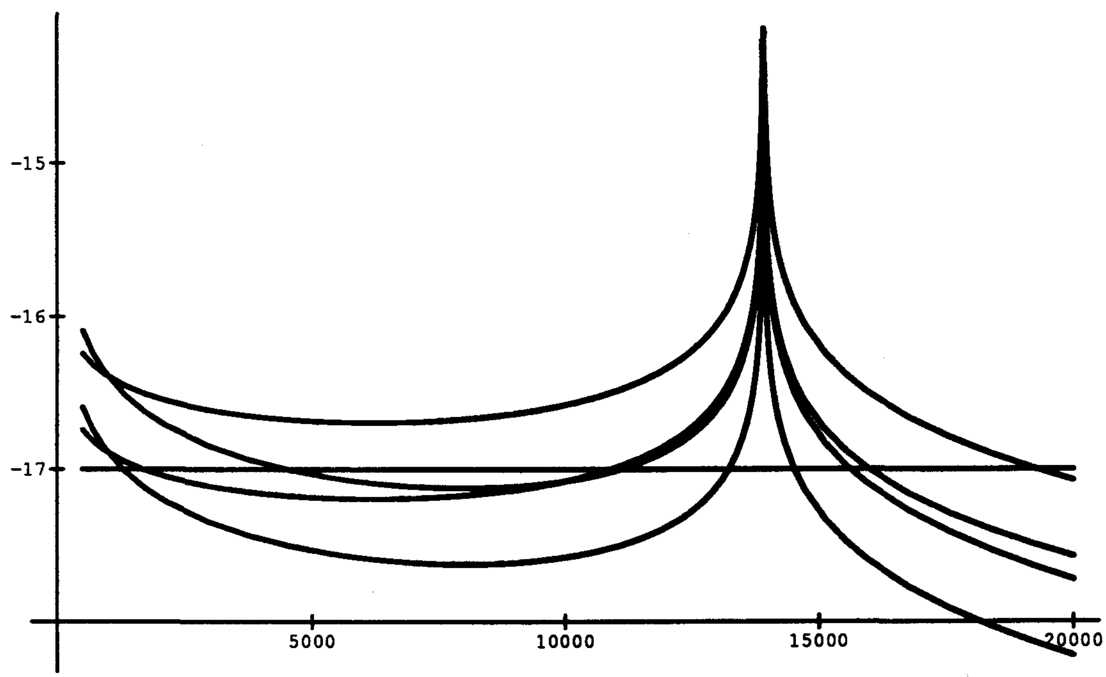


One of the important characteristics of this experiment involves determining the shape of the off-resonance thermal noise curve. Figure 9 shows four different plots that describe:

$$\phi(\omega) = 10^{-5},\ 10^{-6},\ \left(10^2\frac{\omega}{2\pi}\right)^{-1},\ \left(10^3\frac{\omega}{2\pi}\right)^{-1}$$

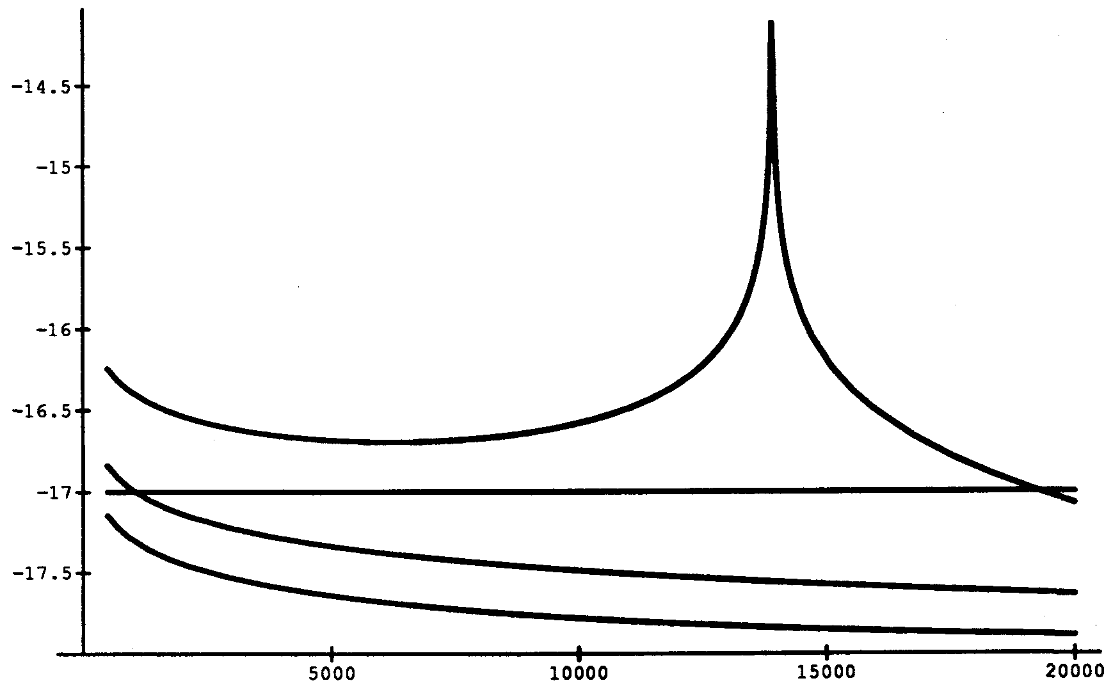
The last two expressions are similar to viscous damping force with a $Q = 10^5$, 10^6 at 1 *kHz*. It is clear that there are substantial differences between the two models at frequencies above and below the resonance.

Figure 8 Plot of thermal noise in second lowest order mode $\log_{10}\left(cm/\sqrt{Hz}\right)$ *vs. frequency* (Hz). The top curve is $\phi(\omega) = 10^{-5}$; the next is $\phi(\omega) = \left(10^2 \frac{\omega}{2\pi}\right)^{-1}$, then $\phi(\omega) = 10^{-6}$; the bottom curve is $\phi(\omega) = \left(10^3 \frac{\omega}{2\pi}\right)^{-1}$. The horizontal line represents the predicted sensitivity of $10^{-17}\ cm/\sqrt{Hz}$.



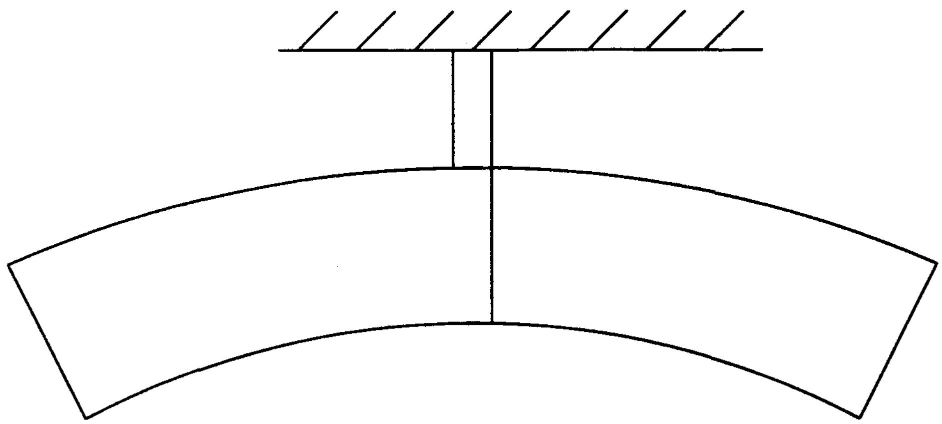
The two spherical mirrors and the plate upon which they are attached do contribute to the thermal noise being measured, but not significantly. A spherical mirror 1″ in diameter and 1/2″ thick has its lowest order mode at 214 *kHz*. Each spherical mirror contributes uncorrelated noise from this mode. Since the plate is much thicker than the thin mirror, it has a much higher resonant frequency (55.6 *kHz* for a thickness of 1″ and diameter of 4″) for the mode that will contribute to the thermal noise. Figure 9 shows a plot of the thermal noise contribution from different parts of the interferometer.

Figure 9 Plot of $\log_{10}\left(cm/\sqrt{Hz}\right)$ *vs. frequency* (Hz). The bottom curve is the contribution from a spherical mirror. The next one up is the contribution from the thick plate. The horizontal line represents the predicted sensitivity of $10^{-17}\ cm/\sqrt{Hz}$. Finally, the top curve is the signal that will be measured. In all cases, $\phi(\omega) = 10^{-5}$.



Another source that can introduce an error in the signal is the first flexural mode in the cylinder. It is excited by ground motion through the wire loop that suspends the cylinder (see figure 10).

Figure 10 First flexural mode of cylinder excited by wire suspension.



For a shell with an inner diameter of 4″, wall thickness of $1cm$ and length of 10 *cm*, the lowest frequency of this mode is 10 *kHz*. The coupling between the vertical motion at the wire to the tilting motion at the ends of the cylinder is about 1. Taking into account the position of the beam spots and the differential nature of the measurement gives a factor of 1.6 for the coupling of the vertical motion at the wire to the actual signal being measured. An optimistic spectrum for the ground noise in the lab is:

$$x(f) = \frac{10^{-4}}{f^2}\, cm/\sqrt{Hz}$$

At 10 *kHz*, this gives a value of $10^{-12}\, cm/\sqrt{Hz}$ which is too large until one considers the filtering effect of the wire suspension.

The numbers given above for the dimensions of the cavity give it a mass of 700 g. Making the length of the wires 10 *cm* and their diameter (0.074 *mm*) which is five times the yield strength for tungsten ($4 \times 10^{10}\, dynes/cm^2$) gives a resonant frequency of 23.3 *Hz* to the suspension. At 10 *kHz*, this offers an isolation of 5×10^{-6} which makes the noise $7 \times 10^{-18}\, cm/\sqrt{Hz}$. This is just below the predicted sensitivity of the apparatus. An isolation stage can further reduce this until it becomes negligible.

Another source of excitation for this mode is the thermal noise in the wire suspension. A quick calculation shows that it is not important. For the wire dimensions given above, the noise contribution at 10 *kHz* (assuming $\phi(\omega) = 2 \times 10^{-5}$) is $8 \times 10^{-20}\, cm/\sqrt{Hz}$ which is not significant.

Cost Estimate

- ☐ 1 Flat 4.5″ diameter mirror........................$2500* ←
- ☐ 2 Curved 1″ diameter mirors......................$4500*
- ☐ 1 Plate for mounting small mirrors............$1000*
- ☐ 1 Fused Quartz cylinder.............................$4000* ←
- ☐ Optical Contacting......................................$5000* ←
- ☐ 1 Köster prism..$4500* ←
- ☐ Sub-Total...$21500*
- ☐ 1 MISER

  40 *mW*..$12000
  300 *mW*..$22000

- ☐ 2 Pockels Cells
- ☐ 1 Polarising beam splitter
- ☐ 1 Faraday Rotator
- ☐ 2 Photodetectors and electronics
- ☐ Miscellaneous lenses

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* Estimated

"Proposal to Measure the Off-Resonance Thermal Noise in a Mechanical System," by Joe Kovalik

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Circulated by R. Vogt 3/5/91

PLEASE PASS ALONG PROMPTLY

A. Abramovici Passed 9-March-91
Y. Gürsel — Passed 12-March-91
S. Kawamura – Passed – 20-March-91
L. Sievers L.S. 4/12/91
R. Spero RS 6 MAR 91
M. Zucker MEZ 4/9/91
R. Drever ✓

C. Akutagawa (Science Library) -last

cc: F. Raab
S. Whitcomb

To: R. Vogt

From: R. Weiss (c/o DHS) 22 Feb 91

Subject: J. Kovalik's PhD experiment

Robbie,

This is the proposal to the MIT physics department for Joe Kovalik's experiment on thermal noise. Comments from the science team are welcome.