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LIGO PROJECT

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Considerations of the RMS motion of the LIGO cavity mirrors

We will principally address the question of the LIGO cavities. The physics stays the same for prototypes, but the ground motion is considerably different. The seismic driving spectrum is between 10 (Caltech) and 100 (MIT) times greater than the standard LIGO site seismic noise spectrum, on average, and the noise is non-stationary (like the trucks and other objects which produce it): there are peaks of motion which are much greater than the average. For this reason, some of the qualitative conclusions about LIGO RMS motions do not hold for the prototypes, and some of the solutions for mass control must be different in the prototypes. It would be worthwhile to compare the results given here with the 40 m experience.

I) Excitation of the masses

a) along the optic axis: The mass suspension points will be excited by the seismic noise as filtered by the isolation stack. The standard LIGO seismic noise source spectrum is 10^{-9} m/$\sqrt{\text{Hz}}$ from 0 to 10 Hz, and $10^{-7}/f^2$ m/$\sqrt{\text{Hz}}$ for higher frequencies. This gives an approximate spectrum at the pendulum suspension point of 10^{-9} m/$\sqrt{\text{Hz}}$ from 0 to 10 Hz, $10^{-7}/f^2$ m/$\sqrt{\text{Hz}}$ from 10 to f_{stack} Hz, and $10^{-9}(f_{\text{stack}}/f)^n$ m/$\sqrt{\text{Hz}}$ for higher frequencies, where f_{stack} is 33 and n is 8 for one stack design. The RMS motion due to this excitation will be about 3×10^{-9} m RMS if the excitation is incoherent, or about 0.01 μm peak-to-peak.

The RMS motion of the test mass is effectively determined by the suspension point motion noise spectrum below 10 Hz. This is due to the rapid falloff in the driving spectrum, and because of the test mass pendulum filtering action. Existing passive stack designs and measurements show resonant frequencies in the range of 10 to 35 Hz, with Q factors of 3 to 6. Because their filtering action is above the (approximate) 10 Hz cutoff, the stacks to not influence the test mass motion appreciably.

The tilting motion of the ground is another source of seismic noise. For the low frequencies which are important for the RMS motion, the wavelength of the surface waves (which travel at roughly 3×10^2 m/sec) is large compared with the structure size. Assuming that the surface waves have an amplitude equal to the LIGO standard seismic noise spectrum N_{seismic}, we can calculate the maximum slope of the ground; this gives $\theta_{\text{seismic}} \approx (2\pi f/v_{\text{seismic}})N_{\text{seismic}} = 2\times10^{-11} f$ rad$\cdot$Hz$^{-\frac{1}{2}}$. With a lever arm of 2 m (the approximate height of the suspension point) this gives a contribution of $4\times10^{-11}f$ m/$\sqrt{\text{Hz}}$ at the suspension point. This is smaller than the direct effect for the RMS motion (note that this does not hold true for GW band frequencies).

This excitation is then filtered by the pendulum suspension of the test mass. The resonant frequency is taken to be 1 Hz in the following (corresponding to a 25 cm long pendulum). In the immediate vicinity of the resonance (1 to 10 Hz), the pendulum is critically (viscous) damped electronically (by OSEMs or the equivalent), giving a 1/f transfer function. The spectrum of the mass motion is about $10^{-9}/f$ m/$\sqrt{\text{Hz}}$ from 0 to 10 Hz, and about $10^{-6}/f^4$ m/$\sqrt{\text{Hz}}$ from 10 to 33 Hz; higher frequencies do not contribute significantly to the RMS motion. The net RMS motion of the test mass is about 1.5×10^{-9} m RMS, or 5×10^{-9} m peak-to-peak.

b) along the perpendicular axis: Due to the probable cross-coupling in the stack, the horizontal motion perpendicular to the beam and the vertical motion of the pendulum suspension point will be similar to the above. The pendulum will filter the horizontal-perpendicular motion as it will the horizontal-parallel motion.

The vertical motion will be filtered by the vertical resonance of the pendulum, which (for a reasonable safety factor in the pendulum wire breaking strength) will be about 10 Hz. This motion will be more difficult to critically damp due to geometry and stiffness. If critically damped, the vertical motion of the mass will be about 10^{-9} m/$\sqrt{\text{Hz}}$ from 0 to 10 Hz, and about $10^{-5}/f^4$ m/$\sqrt{\text{Hz}}$ from 10 to 100 Hz (where additional stack and pendulum resonances play a role). If damped, the net vertical RMS motion of the test mass is about 4×10^{-9} m RMS, or 1×10^{-8} m peak-to-peak. If not damped, the motion will grow as $\sqrt{Q}$ of the resonance times the ground motion at the resonance frequency. For a Q of 100 and $f_{\text{res}} = 10$ Hz, we have about 3×10^{-8} m RMS.

c) tilts and twists

1) rotations around the vertical axis: Two sources of rotations around the vertical axis can be important. One is the horizontal surface waves, which couple as the tilt above. This results in about 6×10^{-10} rad RMS. The other is cross-coupling of horizontal or vertical translations or tilt motion into rotation. This importance of this effect will be set by the scale size of the stack ring diameter. The perimeter of the stack will execute roughly the horizontal motions described in *1)* above; the rotations associated with these motions will be roughly these motions divided by the stack radius. This will be about one meter, so that the RMS motion due to this excitation will be about 3×10^{-9} rad RMS if the excitation is incoherent, or about 10^{-8} rad peak-to-peak.

This excitation is then filtered by the pendulum suspension of the test mass. The rotational resonant frequency is taken to be 1 Hz in the following, with a transfer function of $1/f$ in this frequency regime (it is assumed that the pendulum is critically damped using OSEMS or the equivalent). The net RMS rotation of the test mass is about 1.5×10^{-9} rad RMS, or 5×10^{-9} rad peak-to-peak.

2) rotations around the horizontal axis: This case depends on the suspension more critically. If the suspension is a simple single wire loop, there is more isolation than for the case *1)* above due to the two resonant systems in series (the pendulum, and the mirror in the sling). If it is a double sling, the isolation depends on the frequency of the rotational resonance, and the construction of the double sling. A simpler model, where the mirror follows the angle of the suspension wire, gives an angular motion $\theta_{\mathrm{horiz}} = x_{\mathrm{sus.pt.}}/l_{\mathrm{pend}}$. For a 1 Hz pendulum, l_{pend}=0.25 m. From *1)* above, and assuming a critically damped motion (OSEMs), the net RMS angular motion of the test mass about the horizontal axis is about 6×10^{-9} rad RMS, or 2×10^{-8} rad peak-to-peak.

II) Sensitivity of the interferometer to RMS motion

In the following it is assumed that the instrument is a recombined-beam broad-band recycled interferometer. Thus the interferometer path length difference (dark fringe) and the two cavity lengths (resonance condition) can be, and must be, separately controlled.

a) along the optic axis

1) deviation of the cavity from resonance: Seismic excitation along the optic axis causes deviations from the resonance condition of the main arm cavities. The full-width at half-maximum of the cavity resonance is $(\lambda/2)$/finesse. Taking a 0.5 msec storage time (rather than the cavity storage time in the Dec 89 proposal of 2 msec, which is for a non-recycled system), we have 2×10^{-9} m. To maintain the power buildup in the cavity, and to keep the cavity in the regime where the phase change in reflection per length change is linear, the cavity should not deviate from resonance by more than about one-tenth of the linewidth. This gives an allowed relative motion of the two test masses of a given cavity of about 2×10^{-10} m RMS.

2) loss of power from the recycling cavity: Deviations of the entire interferometer from the dark fringe allows power to leave the recycling cavity, thus reducing the sensitivity of the interferometer and causing fluctuations in the power level in, and at the output of, the interferometer. With a target recycling gain of 30, the total recycling cavity losses must be less than 3%; losses due to deviations from the Michelson interferometer dark fringe should be maintained such that they do not contribute more than say 0.3% to these losses. More importantly, the shot noise level will vary with changes in the output intensity. To make this effect comparable to the power on the photodetector due to the expected (optically-limited) finite contrast ($C = 1 - 10^{-3}$ is seen in the FMI), this level should be held to to say 10^{-4}. Since the intensity at the output of the interferometer is given by $I_{\mathrm{max}}(1 - C\cos k\delta x)$ where C is the contrast, this corresponds to Michelson path length differences of $\delta x = 1\times10^{-9}$ m. This is a less stringent requirement than *1)* above.

3) sensitivity to laser power fluctuations: Deviations of the entire interferometer from the dark fringe allow power fluctuations in the laser power to appear as a signal at the interferometer output. This should be less than the shot-noise limited sensitivity, thus putting an upper limit on the product of the root mean square (rms) deviations from the dark fringe $\delta\phi_{\mathrm{rms}}$ and the power fluctuations $\widetilde{\delta I}(f)/I$ at the measurement frequency f:

$$\delta\phi_{\mathrm{rms}}\widetilde{\delta I}(f)/I < \sqrt{\frac{h\nu}{\eta P}}$$

A preliminary estimate can be made for the upper limit of the power fluctuations allowed. The main interferometer (ICD600) will be specified to have rms deviations from the dark fringe of less than $\delta x = 1\times10^{-9}$ m (see *2)* above), or 10^{-2} rad. The shot noise limited sensitivity for the main interferometer is $6\times10^{-10}\delta I(f)/I$ Hz$^{-\frac{1}{2}}$ ($h \approx 10^{-21}$ in a 1 kHz band). Thus the allowed power fluctuations of the circulating power in the interferometer are $\delta I(f)/I$ Hz$^{-\frac{1}{2}}$ of $< 6\times10^{-8}$ in the GW band. The power fluctuations of the pre-stabilized laser are filtered by the mode cleaner and recycling cavity ICD300-ICD500 transfer function. This gives an maximum allowed power fluctuation spectrum from ICD100 of a $\delta I(f)/I$ Hz$^{-\frac{1}{2}}$ of roughly $6\times10^{-8}(f^2/160)$. For example, the laser must exhibit a $\delta I(f)/I$ Hz$^{-\frac{1}{2}}$ of $< 3\times10^{-5}$ at 300 Hz, a reasonable goal but requiring some active power stabilization.

b) cross-coupling to longitudinal motion:

All of these 'indirect' effects (those not directly affecting the length of the cavities), and any servo corrections applied to correct for them, will be seen with some reduced effect in the length of the cavities. Some examples, probably not exaustive:

1) angular changes: If the beam is not centered on the axis of rotation of the mirror, then there will be a linear coupling from mass rotations to cavity length. Given the expected ratio of angular to translational motion, and probably offsets from the axis of rotation, this should be small: $x = \theta y_{\mathrm{offset}} \approx 10^{-9}$ rad 0.01 m $= 10^{-11}$ m, a factor of 100 less than the expected direct translations.

2) horizontal or vertical translations: The cavity length is a quadratic function of the displacement perpendicular to the beam of a curved mirror. If this motion is locally damped (OSEMS) it should be a negligible effect.

3) upconversion: If there are surface irregularities on the mirror, motions of the mirror perpendicular to the beam may cause modulation of the length of the cavity with characteristic frequencies much higher than the spectrum of the mirror motion. No attempt to model this analytically is made here; measurements should be made, with numerical simulation to back up the numbers.

c) tilts and twists: Changes in the pointing of the test mass due to excitation by seismic noise have (at least) the following effects:

1) TEM_{00} *mode coupling:* The amount of power coupled into the fundamental mode of the cavity varies as a function of mirror angle. As one example, assume a flat-concave cavity and take motions of the concave mirror of radius R. Rotations θ of that mirror make translations in the optic axis of the cavity of $y = \theta R$ (for small angles). The overlap integral of the optic axis of the cavity and the optic axis of the light is quadratic with a scale of the $1/e$ diameter of the beam: $M \approx \frac{1}{2}(1-(y/w)^2)$ for $y << w$. The mode matching M as a function of rotations of the far concave mirror will be $M \approx 1-(\theta R/w)^2$. For R=4 km, w=5 cm, and M >0.99, θ must remain less than 1μrad. This should also be worked for rotations of the near flat mirror.

2) fluctuations of the contrast: The contrast of the recombined beam interferometer varies with angle variations. Taking as an example the motion of a near, flat, mirror, rotations θ of that mirror make rotations in the optic axis of the cavity of θ. The contrast C of a simple non-recycled interferometer varies as $C = 1 - (k\theta w/2)^2$ (for small angles); we take this model for simplicity. Choosing to maintain the contrast as limited by this effect to be greater than $C < (1 - 1\times10^{-3})$ (this makes the power lost to a recycling gain of 30 negligible, and keeps the power on the photodetectors at a reasonable level) requires that the angular motion in both axis be less than 0.2 μrad.

III) Servo systems

¿From the above estimates for the motion of the test mass and the sensitivity of the interferometer to those motions, the transfer functions of the servo systems needed to bring the motions to an acceptable level can be described.

a) along the optic axis: The primary requirement of this servo system is to bring the deviations of the main cavities from resonance to an acceptable level. A further question is whether the servo loop should have a unity-gain frequency considerably greater than, or less than, the GW band of frequencies. Without entering into this discussion, we calculate the servo system needed for the low unity-gain case, because this is the minimum condition to address the RMS motion problem.

1) signal to noise: We assume that the cavities will be locked using the RF reflection locking technique, using a signal from a near-Brewster pickoff plate between the cavity near mirror and the beamsplitter. To allow recycling, the the power that is taken out of the system by the pickoff plate must be limited. On the other hand, the signal to noise ratio in the detection of the cavity length will (hopefully) be shot noise limited, and thus requires a certain amount of power and modulation level. The position noise associated with this detection process will be necessarily greater than the position noise in the main interferometer (since is necessarily uses less power than the main interferometer). Alternatively, a dithering system could be used, with the light transmitted through the cavity; this is the same situation from the standpoint of signal-to-noise.

The recycling gain condition (for a recycling gain of 30) limits the fractional power taken by the pickoff plate to something of the order of 3×10^{-3} (about 1/10 the total losses in the recycling cavity). If the modulation in the recycling cavity is optimized, the position sensitivity will be then about $1/\sqrt{3\times10^{-3}} \approx 20$ times that of the main interferometer. Where the gain of the servo-loop that uses this error signal is greater than 1, the mirror motion will be constrained to follow the noise of the detection, which is in excess of the shot-noise limited sensitivity of the main LIGO interferometer. Thus the gain of the servo-loop must be much less than 1 (in fact, less than 1/20 in our signal-to-noise example; we take 0.01) in the GW band.

2) required gain as a function of frequency: In **I a** and **II a** above, we found that the motion of the test masses must be reduced from 1.5×10^{-9} m to roughly 2×10^{-10} m RMS. Most of the motion is in the frequency range from 0 to 2 Hz. This gives a requirement of a gain of 7.5 in the servo-loop at 1 Hz, and a gain of 0.01 at 300 Hz. This is not difficult to achieve.

3) 'locking': The expected damped motion of one test mass is 1.5×10^{-9} m RMS. The relative motion of two test masses, or the variation in the length of the cavity, is about $\sqrt{2}$ times this or 3×10^{-9} m peak-to-peak. This is about 1.5 cavity linewidths (the FWHM of the cavity is 2×10^{-9} m), and 1/120 of an FSR. The principal frequency of these motions is 0 to 2 Hz, which means that the cavity changes length slowly compared to its storage time of 0.5 msec. This means that the during 'locking' period, the servo loop receives a continuous error signal with the correct sign to drive the system into resonance ('lock'). The cavity should 'leap into lock'.

4) required forces: There are two regimes: initial damping, and then damped or locked. The first regime forces will be determined by the time that we wish to wait before the mass is damped. If the undamped pendulum Q_{f_0} at its resonant frequency is, say, $Q_{f_0} = 10^5$, then the undamped motions will be $\sqrt{Q_{f_0}}x_{\mathbf{pend}} = 5\times10^{-7}$ m RMS. In the second regime, forces must be exerted to hold the pendulum in position against forces exerted by changes in the pendulum suspension point. This force is $F = mgx_{\mathbf{sus.pt.}}/l = 10 \times 10 \times 1.5\times10^{-9}/0.25 = 6\times10^{-7}$ N RMS, or 2×10^{-6} N peak. This does not take into account static forces which may be required to offset drifts in the pendulum suspension point position.

b) transverse motion: To be determined. Upconversion mechanisms must be studied experimentally.

c) rotations: The primary requirement of these servo systems is to bring the deviations of the main cavities from alignment to an acceptable level.

1) required gain as a function of frequency: About the vertical axis, the expected RMS motion is 1.5×10^{-9} rad, and about the horizontal axis is 6×10^{-9} rad (for the models above). Both the vertical axis and horizontal motions are already much less than the requirement of $< 1\mu$rad. This suggests that any active pointing system must only maintain the correct alignment over times comparable to drift rates in the system (order of seconds).

2) signal to noise: The sensing noise must be less than 1μrad for frequencies below 1 Hz, or roughly 1×10^{-7} rad·Hz$^{-\frac{1}{2}}$. This corresponds to motions of the beam at the other mirror of 0.4 mm, which should be easy to achieve. The gain of the servo will be much less than unity in the GW band, so that the pointing servo will not influence the main interferometer noise budget. Attention must be given to eliminate driver amplifier noise in the GW band, however. This is made easier by the low unity-gain frequency needed in this servo system.