New Folder Name Step Response

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Step Response of a Fabry-Perot Cavity

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Consider a cavity resonating in a single mode with an input beam of fixed frequency (see Figure 1). Let τ be the round trip time for the cavity. The

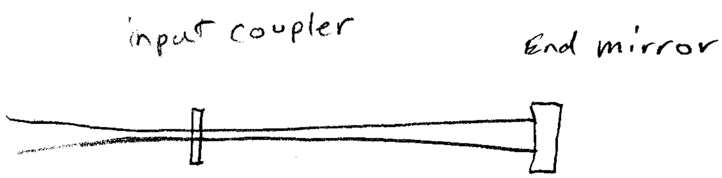

Figure 1:

light leaking out of the input coupler at time $t = 0$ can be thought of as a superposition of the light which entered the cavity at $t = -\tau$ plus that which entered at $t = -2\tau$, and so on, each contribution attenuated to a degree depending on how long it has been in the cavity. If the frequency of the input beam is in the middle of a dark fringe, all of the contributions are in phase, so as to interfere destructively to the maximum extent possible with the incident light reflected from the input mirror.

We can represent the sum of these contributions in a phasor diagram as follows: suppose that during one round trip the amplitude of the light is attenuated by a factor R; then if the electric field due to the light which has made one round trip is E_0, the other contributions will be RE_0, R^2E_0, etc..

Let E be the sum of these contributions, and let α be the phase angle of E (see Figure 2).

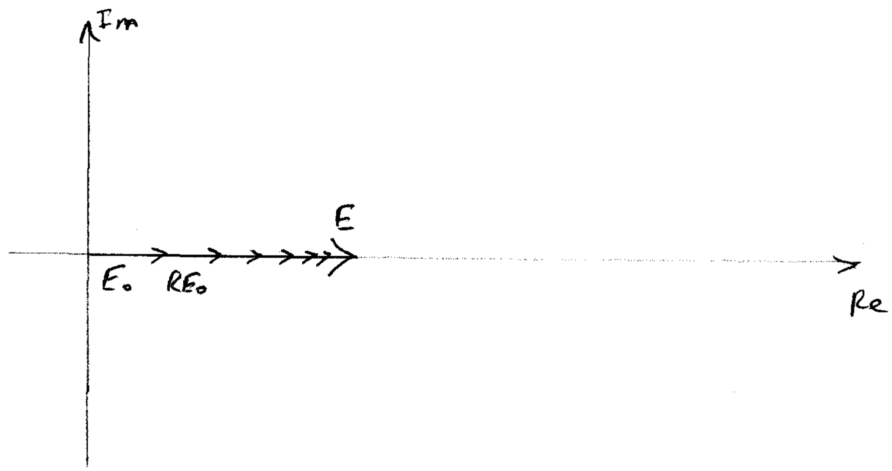

Figure 2:

Now suppose that at $t = 0$ the end mirror is displaced instantaneously to a new position, so that the cavity is shortened by an amount Δl. The light coming out of the cavity for $\frac{\tau}{2} < t < \frac{3\tau}{2}$ will be phase shifted by $\Delta\phi = \frac{4\pi\Delta l}{\lambda}$. This corresponds to the diagram in Figure 3; $\alpha(\tau) = \Delta\phi$. After another round trip time elapses, this light returns, attenuated by R, shifted by another $\Delta\phi$, and augmented by the light that entered the cavity most recently; this last piece has a phase shift of only one $\Delta\phi$ and

$$\alpha(2\tau) = \Delta\phi + R\Delta\phi$$

for $\Delta\phi \ll 1$ (See Figure 4).

Similarly, for $t = 3\tau$, we have (Figure 5)

$$\alpha(3\tau) = \Delta\phi(1 + R + R^2)$$

Finally,

$$\begin{aligned}\alpha(n\tau) &= \Delta\phi + R\Delta\phi + R^2\Delta\phi + \cdots + R^{n-1}\Delta\phi \\ &= \frac{1-R^n}{1-R}\Delta\phi\end{aligned}$$

This can be approximated by

$$\alpha(t) \simeq (1 - e^{-\frac{t}{\tau_c}})\frac{1}{1-R}\Delta\phi$$

where $\tau_c = \frac{\tau}{-\ln R}$.

If the frequency of the input beam, instead of being fixed, is controlled so as to keep it interfering destructively with the light leaking out of the cavity, the same arguments hold up to the diagram for $\alpha(2\tau)$. In this case, the value of $\alpha(2\tau)$ is $2\Delta\phi$ since the incoming light will have its phase adjusted so as to continue interfering constructively with the light inside the cavity. Then

$$\alpha(n\tau) = n\Delta\phi$$

which can be approximated $\alpha(t) = t\frac{\Delta\phi}{\tau}$, or as a frequency shift:

$$\begin{aligned}\nu' &= \nu + \frac{1}{2\pi}\frac{d\alpha(t)}{dt} \\ \frac{\Delta\nu}{\nu} &= \frac{\Delta l}{l}.\end{aligned}$$

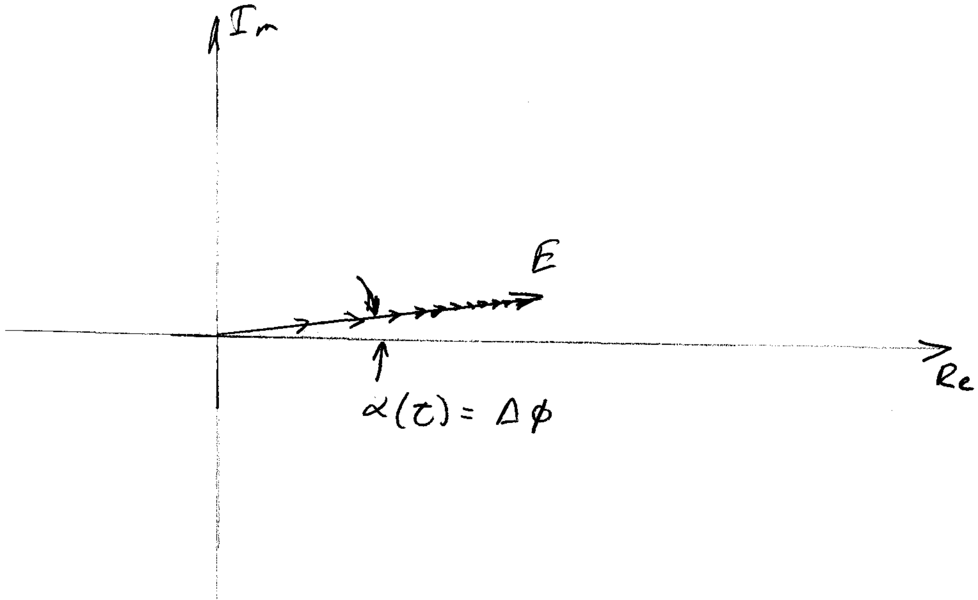

Figure 3:

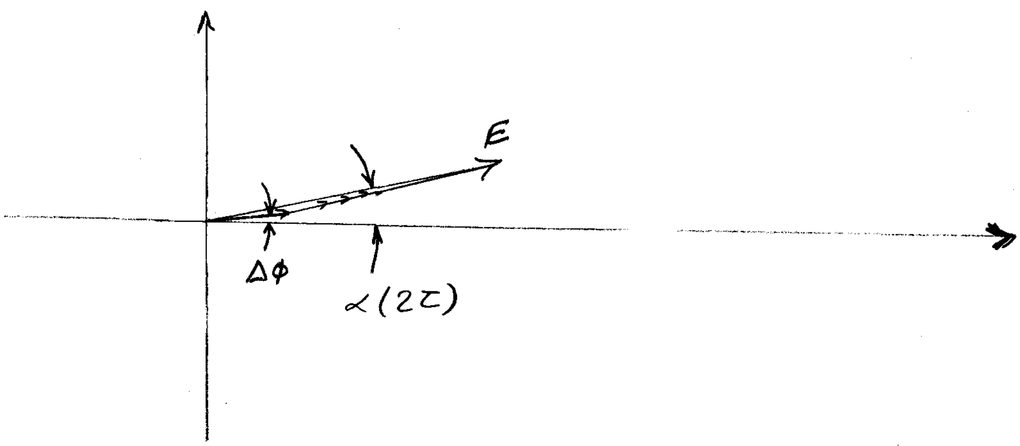

Figure 4:

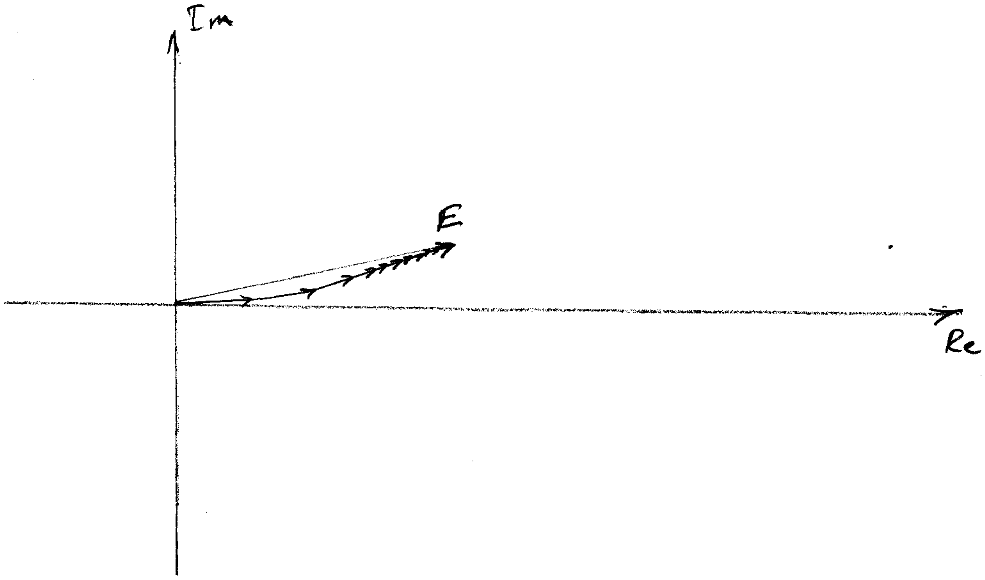

Figure 5:

One nonlinear effect ~~which~~ produces in the response is the change to the round trip time ~~produced~~ caused by the motion of the end mirrors. Let $\Delta\alpha(t)$ be the additional phase, in the light leaking out of the cavity, generated through this mechanism. Let $\gamma(t)$ be the phase of the light entering the cavity (assumed of constant amplitude). Then the contribution to the $\vec{E}$ of light which has made one round trip, instead of being $E_0 e^{-i\gamma(t)}$ is

$E_0 e^{-i\gamma(\tau-\Delta\tau)}$ where $\Delta\tau = \frac{2\Delta l}{c}$.

Approximate $\gamma(\tau-\Delta\tau)$ by $\gamma(\tau) - \left.\frac{d\gamma}{dt}\right|_{t=-\tau} \Delta\tau$.

Summing the contributions to $\Delta\alpha$ gives

$$\Delta\alpha = \frac{E_0}{E} \sum_{n=1}^{\infty} \left(\left.\frac{d\gamma}{dt}\right|_{t=-n\tau} \right) R^{n-1}\, n\Delta\tau$$

let $\gamma' = \max_n \left[\left| \left.\frac{d\gamma}{dt}\right|_{t=n\tau} \right| \right]$

then $|\Delta\alpha| \le \frac{E_0}{E} \Delta\tau \gamma' \sum_{n=1}^{\infty} n R^{n-1}$

$$\frac{R}{1-R} = \frac{R-1+1}{1-R} = -1 + \frac{1}{1-R}$$

$$= \frac{E_0}{E} \Delta\tau \gamma' \frac{d}{dR} \sum_{n=1}^{\infty} R^n$$

$$= \frac{E_0}{E} \Delta\tau \frac{d}{dR}\left(\frac{R}{1-R}\right)\gamma' = \frac{+E_0 \Delta\tau}{(1-R)^2}\gamma'$$

$$\Delta\tau = \frac{2\Delta l}{c}$$

$$\Delta l = \frac{\lambda\Delta\phi}{4\pi}$$

$$\Delta\tau = \frac{\Delta\phi\,\lambda}{2\pi c}$$

$$\alpha = \frac{1}{1-R}\Delta\phi$$

$$\Delta\tau = \frac{(1-R)}{2\pi\nu}\alpha$$

$$= \frac{E_0 \alpha}{E\, 2\pi\nu(1-R)}\gamma'$$

$$= \frac{\cancel{E}\alpha}{2\pi\nu}\gamma'$$