New Folder Name Sensitivity to a Stochastic Gravitational Radiation Background

Nelson Christensen
3-28-90

Sensitivity to a Stochastic Gravitational Radiation Background Using Dual Recycling Fabry-Perot Interferometers

The purpose of this memo is to point out an error in Figure A-4c of the LIGO proposal. This figure shows the expected sensitivity to a stochastic gravity wave background using the early LIGO detector, the advanced LIGO broadband recycling detector, and the advanced LIGO resonant recycling detector. The plot for the dual recycling envelope is wrong, and this memo will explain why.

For two full length interferometers at the same site, aligned with one another, with identical dual recycling systems, and with optimum filter techniques applied to the output data, the signal to noise ratio is given by

$$\frac{S}{N} = \frac{8G\rho_c}{5\pi c^2}\sqrt{\frac{T}{2}\int_0^\infty \frac{\Omega_{gw}^2(f)\,df}{f^6 h_n^4(f)}} \quad ,$$

where G is Newton's constant, c is the speed of light, ρ_c is the critical energy density of the universe, $\Omega_{gw}(f)$ is the ratio of the energy density of the stochastic gravity wave background per unit logarithmic interval to the critical energy density, and $h_n(f)$ is the interferometer's spectral density of strain noise.

The above equation can be written in terms of the rms value of the gravity wave. If one has

$$h_{rms}^2 = \left\langle \sum_{i,j} h_{ij}h_{ij} \right\rangle = \int_0^\infty S_h(f)\,df \quad ,$$

where $S_h(f)$ is the gravity wave background's spectral density, then

$$S_h(f) = \frac{8G\rho_c\Omega_{gw}(f)}{\pi c^2 f^3} \quad .$$

The limits on the energy density background of the gravity waves, or the rms value of the strain can be derived from these equations. Normally one claims that for some small frequency band spanning Δf around a frequency f, and with S/N = 1, the limit on the energy density would be

$$\Omega_{gw}(f) = \frac{5\pi c^2 f^3}{8\rho_c G}\sqrt{\frac{2}{\Delta f\, T}}\,h_n^2(f) \quad .$$

The limit on the rms value of the strain would be

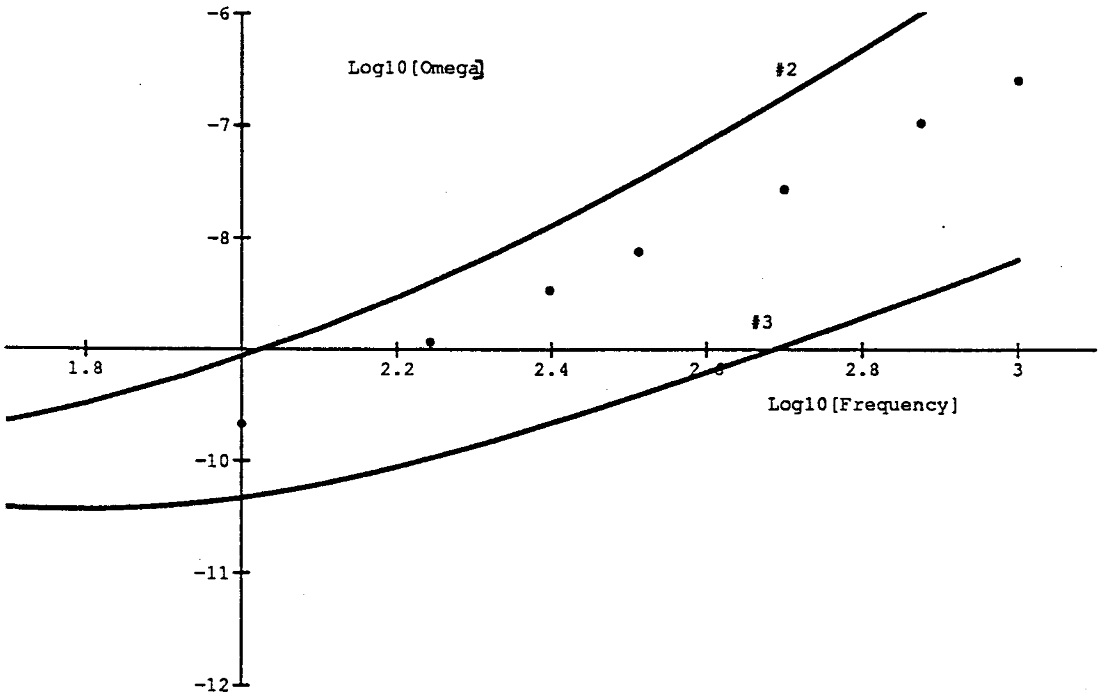
$$h_{rms} = \sqrt{5}\left(\frac{2\Delta f}{T}\right)^{1/4} h_n(f) \quad .$$

The integration has been approximated over a band Δf. For Figure A-4c of the proposal one assumes that the bandwidth of the measurement Δf is about the same as the frequency at which the measurement is concentrating. This is an approximation that works well for broadband detection, but it isn't necessarily correct for a narrowband measurement, like dual recycling. With dual recycling, a measurement centered somewhere in the hundreds of Hertz will have a full width at half maximum for the transfer function of tens of Hertz. Remember, when optimum filter techniques are applied to the outputs of each antenna, then

$$\frac{S}{N} = \frac{8G\rho_c}{5\pi c^2}\sqrt{\frac{T}{2}\int_0^\infty \frac{\Omega_{gw}^2(f)\,df}{f^6 h_n^4(f)}} \quad .$$

The bottom line is that for dual recycling the effective bandwidth is much smaller than the frequency where the measurement is taking place. Figure 1.1 shows the result of this. The sensitivity of the advanced recycling Fabry-Perot (#2) and the advanced dual recycling Fabry-Perot (#3) systems according to the approximation Δf=f are plotted against Ω_{gw}. The dots show the result when the optimum filter output is properly integrated over frequency. Therefore, the actual dual recycling limit is not quite as good as what one would normally think.

Figure 1.1 The solid lines show the sensitivity to Ω_{gw} for the advanced broadband recycling Fabry-Perot (#2) and the advanced dual recycling Fabry-Perot (#3) according to the approximate solution where $\Delta f = f$. The dots show the result when the optimum filter output is properly integrated over frequency.



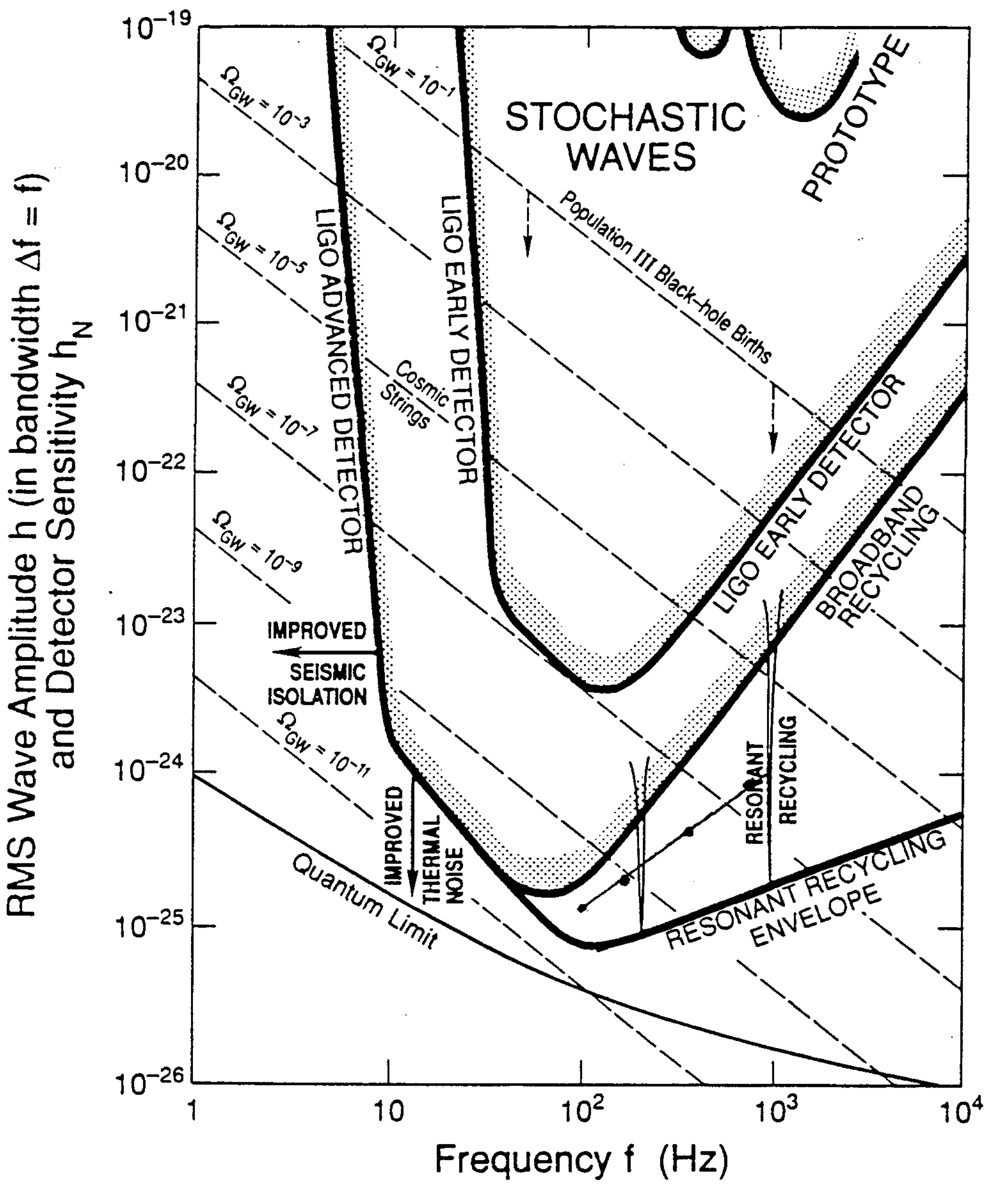


Figure A–4c The estimated rms amplitude h in a bandwidth $\Delta f = f$ for stochastic backgrounds of gravitational waves from various sources (dashed lines); and benchmark sensitivities $h_N \simeq \sqrt{5}\tilde{h}(f)[2f/\hat{\tau}]^{1/4}[1 + fD/c]^{1/2}$ (see Equation (A.20)) (solid curves and stippled strips atop them) for interferometric detectors today and in the proposed LIGO. For each detector the solid curve corresponds to the rms amplitude in a bandwidth equal to the frequency for a stochastic background which would be detected at unity signal–to–noise ratio in a cross-correlation experiment between two interferometers at different LIGO sites, after integration for $\hat{\tau} = 10^7$ seconds. The top of the stippled strip corresponds to the rms amplitude in a bandwidth equal to the frequency of a stochastic background which would give significant detection in 10^7 seconds. Stated more precisely, it corresponds to a confidence level of 90% that the signal is not a false alarm due to a statistical fluctuation in Gaussian noise. The lower right-hand part of the advanced detector curve indicates the envelope of peak responses of a resonant recycling detector system when tuned for optimal operation at each frequency in the range shown. Each of the resonance curves above this indicates response of a particular resonant recycling system. The solid line near the bottom of the figure indicates the limit to the advanced detector sensitivity curves set by the quantum limit for the test masses.