New Folder Name Coincedence Sensitivity

Interoffice Memorandum

CALIFORNIA INSTITUTE OF TECHNOLOGY

To:	R. Vogt, Director – LIGO project	**Date:**	September 5, 1990
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Cc:	Alex, Bill, Bob, David, Fred, Kip, Mike B., Mike, Rai, Ron, Seiji		
Subject:	**Coincidence Sensitivity for a Pair of Interferometers**		

This is a short memorandum explaining the adopted definition for the sensitivity of a pair of laser interferometric gravitational wave detectors operating in coincidence. This definition is fully developed in the memorandum titled "SENSITIVITY OF A PAIR OF LASER INTERFEROMETRIC GRAVITATIONAL WAVE DETECTORS OPERATING IN COINCIDENCE AS A FUNCTION OF THEIR ORIENTATIONS" by Yekta Gürsel and Massimo Tinto dated August 31, 1990. It is chosen by the group of people consisting of Yekta Gürsel, Robert Spero, Kip Thorne and Rainer Weiss. Both Spero and Weiss introduced their own definitions for the sensitivity for a pair of interferometers operating in coincidence. Spero has shown that the values of the sensitivity predicted by his definition agrees with the values predicted by the adopted definition within 5 percent or so. Weiss' predictions of the locations of the minima and the maxima of the coincidence sensitivity also agree very closely with the ones predicted by the adopted definition. For further details, please consult the manuscripts written by Spero and Weiss which are referenced in the memorandum mentioned above. Thorne has written a short memorandum explaining why the Gürsel-Tinto definition should be adopted. In that memorandum, he also gives a simple way to look at the Gürsel-Tinto definition.

The likely sources to be detected by the initial LIGO interferometers are the burst sources. In this memorandum, we will consider burst signals detected in coincidence by a network of two detectors which are separated by a large distance. The responses of two interferometers widely separated on the Earth to a given burst signal are different due to the dependence of the detector antenna patterns on the relative orientations of the detectors. This prevents one from directly cross-correlating the two detectors outputs with each other for arbitrarily polarized bursts. The reason for this is that the source location is a free parameter constrained to a circular band in the sky by the time delay between the two detectors. If the burst is linearly or perfectly circularly polarized, such a cross correlation becomes feasible as an indicator of the presence of the signal in the data. A similar calculation may be performed in the search for the stochastic gravitational wave background, but the wide separation of the detectors reduces the sensitivity in this case. The complete determination of waves' parameters requires at least three widely separated detectors.

The antenna patterns of laser interferometric gravitational wave detectors are more sensitive to the locations and the orientations of the detectors for linearly polarized bursts. For this reason, we will develop a measure of coincidence sensitivity for a pair of widely separated detectors in the case of linearly polarized bursts.

Consider the signals detected in coincidence by a network of two detectors which may be separated from each other by a large distance. The detectors are oriented in different ways. Assume that the detectors are equal in construction and in performance. For detection of bursts, certain thresholds have to be set in each detector to distinguish the signal from the (ideally Gaussian) noise. Since the gravitational wave bursts are relatively rare events, setting of sharp thresholds is justified. The reason for this is that the confidence level of having such events solely due to the Gaussian noise is a very sensitive function of their amplitude. Few tens of percent of change in the amplitude implies a change of a factor of ten in the confidence level. In order for the signals to be detected in coincidence, both of the pulses in the detectors have to be above their respective thresholds. Since the detectors are identical in construction and in performance by our assumption, their thresholds are also assumed to be equal.

Let a linearly polarized gravitational wave impinge on the detectors. Let the amplitude of the gravitational wave be h. The ratio F of the response R of the detector to the amplitude h of the impinging gravitational wave is called the antenna pattern of the detector:

$$F = \frac{R}{h}$$

The antenna pattern F for a given detector is a function of the orientation and the location of the detector on the Earth as well as the angular location of the source in the sky. In order to distinguish between the two detectors under consideration, we will call their antenna patterns F_{1+} and F_{2+} where 1 and 2 are the detector indices, and $+$ signifies the fact that the wave is linearly polarized. We assume that the other polarization component of the gravitational wave which is customarily labeled with $\times$ is zero.

The amplitude of the gravitational wave impinging on the Earth is of the form:

$$h = \frac{f}{r}$$

where r is the radial distance to the source. f is a function of the parameters of the source, like the mass, the rotation rate, etc. The function f is a constant since we assume that the sources are identical. Let the common positive threshold of the detectors be T. The radius of L of the "observable" universe is defined as the distance to the sources which generate a response of precisely T at the detectors on the Earth:

$$T = |F_{max}| \frac{|f|}{L}$$

where F_{max} is the maximum of the antenna pattern with respect to the source direction in the sky and the polarization angle. Note that $|F_{max}| = 1$. Any source which lies at this radius will be barely detected by the corresponding detectors if it also lies at the correct angular position and it has the correct polarization angle. All of the sources within this radius would have been detected if the antenna patterns of the detectors were independent of the angular location of the source in the sky and the polarization angle of the wave.

Let the identical sources of gravitational waves be uniformly distributed in the observable universe. Then, the radial density ρ of the sources at a distance r from the Earth is given by:

$$\rho = 3 \frac{r^2}{L^3} M$$

where M is the total number of sources in the observable universe. Note that

$$M = \int_0^L \rho dr.$$

The number of the sources detected in coincidence by the two detectors is then given by:

$$K = \langle \int_0^{\min(r_{1max}, r_{2max})} \rho dr \rangle$$

where $\langle\rangle$ denotes the average over the angular location of the source in the sky and the polarization angle of the linearly polarized gravitational wave. r_{1max} and r_{2max} are radii below which the sources produce above threshold amplitudes at the corresponding detectors. Note that these radii are functions of the angular location of the source in the sky and the polarization angle of the gravitational wave. The minimum of these radii is taken since both responses of the detectors have to be above the threshold level T.

The detection efficiency N is defined to be the ratio of the number K of the sources detected in coincidence to the total number M of the sources which are detectable. We then obtain:

$$N = \frac{K}{M} = \langle \int_0^{\min(r_{1max}, r_{2max})} 3\frac{r^2}{L^3} dr \rangle$$

The detection efficiency N is precisely proportional to the coincidence event rate. For this reason, Thorne suggested calling it the "coincidence rate factor".

Performing the radial integral, we get:

$$N = \langle \text{minimum}(\frac{{r_{1max}}^3}{L^3}, \frac{{r_{2max}}^3}{L^3}) \rangle$$

Using the equations:

$$T = \frac{|f|}{L},$$

$$T = |F_{1+}|\frac{|f|}{r_{1max}}$$

and

$$T = |F_{2+}|\frac{|f|}{r_{2max}}$$

leads to:

$$\frac{r_{1max}}{L} = |F_{1+}|, \qquad \frac{r_{2max}}{L} = |F_{2+}|.$$

The final expression for the detection efficiency becomes:

$$N = \langle \text{minimum}(|F_{1+}|^3, |F_{2+}|^3) \rangle.$$

Thorne suggested that one should define the "coincidence sensitivity" H as the inverse cube root of the detection efficiency N:

$$H = N^{-1/3}$$

The sensitivity H is directly proportional to the average amplitude noise in the detectors. Hence, the LOWER sensitivity corresponds to a better network of two detectors. This is in the same spirit as the noise curves in our construction proposal.

We have written a program which computes and optionally plots the coincidence sensitivity as defined above. The program is very user-friendly and robust. It resides in the directory "~yekta/fast-orient" on the "ligo" computer and it is called "fast-orient". It runs on SPARCstations and Sun 4 computers. A graphics terminal is NOT necessary to run

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Here is an example of the dialogue with the program.  The user
inputs are indicated by "<---" on the right.

***************START OF DIALOGUE*****************************

feline[1]% fast-orient
           Welcome to the fast-orient program.
    Yekta Gursel and  Massimo Tinto  August 31, 1990

  This program computes and plots the coincidence sensitivity
of two laser interferometer gravitational wave detectors
running in coincidence.

  The orientation of one of the detectors is fixed.  This
detector is called the reference detector.  The orientation
of the other detector is stepped through a range of values
and the coincidence sensitivity is computed as a function of
the orientation of the rotating detector.

  The computation of the coincidence sensitivity for a given
pair of orientations takes about 6.5 seconds on a Sun
SPARCstation 1+.  The total time it takes to step through
a 90 degree range of orientations for the moving detector
with a 1 degree step size is about 10 minutes.  The accuracy
of the computation is within 5 percent with the default  grid.

Ready?
yes                                             <---
Do you want to change the integration grid size?
no                                              <---
Do you want the orientation to be defined as the angle
between the bisector of the arms of the detector and
the local North direction? (Clockwise is positive)
no                                              <---
The orientation is now defined to be the angle between
the bisector of the arms and the local East direction.
(Counter-clockwise is positive)

Enter the latitude of the reference detector in degrees:
36.0                                            <---
Enter the longitude of the reference detector in degrees:
-115.0                                          <---
Enter the orientation of the reference detector in degrees:
12.0                                            <---
Enter the latitude of the rotating detector in degrees:
45.0                                            <---
Enter the longitude of the rotating detector in degrees:
-67.5                                           <---
Enter the starting angle for the orientation of this detector:
(in degrees)
0                                               <---
Enter the final angle for the orientation of this detector:
(in degrees)
90                                              <---
Enter the number of steps for the rotating detector:
90                                              <---
Do you want to compute inverse of the sensitivity?
(H=N^(1/3) as opposed to H=N^(-1/3), H is the sensitivity,
N is the detection efficiency as defined in the memo.)
no                                                             <---
Do you want the results normalized to the case of two
overlapping detectors?
no                                                             <---
Enter the file name for the output:
Maine-California                                               <---
Do you want the output plotted?
yes                                                            <---
Enter the graph title:
Maine-California                                               <---
Do you want to send the plot only to the printer?
yes                                                            <---
Enter the plot layout:
(Type either portrait or landscape)
landscape                                                      <---
Start of computation: Wed Sep  5 12:53:04 1990
Percent done: 100    [Wed Sep  5 13:04:39 1990]
End of computation:   Wed Sep  5 13:04:39 1990
feline[2]%
```

the program. In this case the graphics output can be sent to the printer directly. The full listing and some details about the operation of the program is given in our memorandum mentioned above. A sample annotated dialogue with the program is given in the following pages.

Attachment
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