New Folder Name Overcoupled Cavities and Sensitivity Formula

Overcoupled Cavities and Sensitivity Formula

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New Mirror Order

The mirrors currently installed are close to optimally coupled: the transmission T_1 of the input mirror is about the same as the total of the other losses. It looks like mirrors are getting better, and scattering and absorption losses might be reduced by a factor of 5 or more, perhaps to 20 ppm per mirror. Alex has suggested[1] that the order for new test masses specify maximum reflectivity for the end masses, and 200 ppm for the input couplers. As shown below, although the visibility would be degraded in such an overcoupled configuration, the shot-noise limited sensitivity would be almost unchanged relative to an optimal cavity.

Visibility

Define $\mathcal{L}$ as the total loss in a two-mirror cavity (excluding the transmission of the input mirror): $\mathcal{L} = T_2 + L_1 + L_2$ Then the visibility (for perfect mode matching) is given by

$$V = \frac{4\frac{\mathcal{L}}{T_1}}{\left(1+\frac{\mathcal{L}}{T_1}\right)^2}.$$

Note that $V\left(\frac{\mathcal{L}}{T_1}\right) = V\left(\frac{T_1}{\mathcal{L}}\right)$. With β the ratio of storage times defined in an earlier document[2], $\frac{\mathcal{L}}{T_1} \equiv \beta - 1$. Figure 1 shows how visibility degrades as the cavity becomes over-coupled.

[1] *Specifying Transmission for Monolithic Test Masses*, Alex Abramovici, 3 October 1990.

[2] *State of the Mirrors: Input Coupling vs. Losses*, 24 August, 1990.

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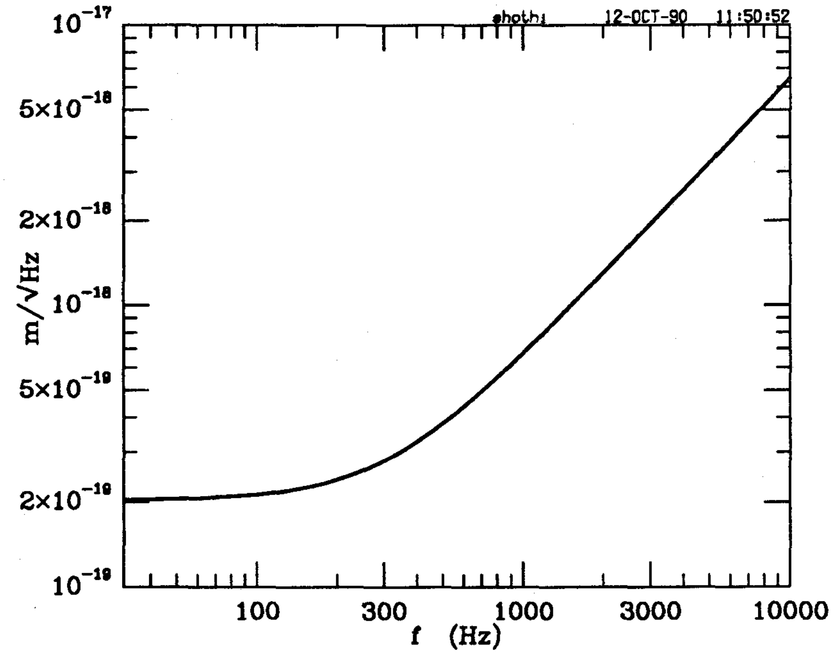

Figure 2: Shot-noise limited sensitivity in the present configuration The parameters for storage time, visibility, and input coupler transmission are 254μsec, 85%, and 650 ppm. The quantum-efficiency corrected bright fringe power is 20 mW on each arm, and the modulation index is 0.9.

Equation 1 may be rewritten

$$\tilde{x}(f) = \frac{L}{\pi \tau_e} \left[\frac{3\lambda \mathrm{h}}{8cP} \mathcal{M} \left(1 + [f/f_k]^2 \right) \right]^{1/2} \tag{2}$$

where h is Planck's constant and the modulation function

$$\mathcal{M} = \frac{1}{3} \left[\frac{M^{-1} + A^2 J_0^2 - 2AJ_0^2 + 2AJ_0 J_2}{M A^2 J_0^2 J_1^2} \right]$$

is equal to 1 when $M = 1$, $A = 1$, and $\Phi = 0$.

The limits for short and long storage times, respectively, are

$$\tilde{x}(f) = \sqrt{\frac{\mathrm{Watt}}{\mathrm{P}}} \sqrt{\mathcal{M}} \times \begin{cases} (L/\mathrm{meter})\,(f/\mathrm{Hz}) & \cdot\, 2.61 \times 10^{-24}\ \mathrm{m}/\sqrt{\mathrm{Hz}}, \quad f \gg f_k \\ (T_1 + \mathcal{L}) & \cdot\, 3.12 \times 10^{-17}\ \mathrm{m}/\sqrt{\mathrm{Hz}}, \quad f \ll f_k \end{cases}$$

Optimum Modulation

The optimum depth of modulation depends weakly on the mode matching M and on the cavity coupling. The tables below show the spectral density of the displacement noise, $\tilde{x}(f)$, in units of $\mathrm{m}/\sqrt{\mathrm{Hz}}$, evaluated at signal frequency $f = 1$ kHz. Four values of M are used: 1.0, 0.92 (estimate of mode matching in present configuration), 0.8, and 0.6. Φ_0 is the modulation index that gives the best sensitivity. The rightmost column is the noise in dB, relative to 10^{-18} $\mathrm{m}/\sqrt{\mathrm{Hz}}$. In all cases, $\mathcal{L}$ is taken to be 100 ppm, and the bright fringe power is 20 mW

$M = 0.6$

T_1 (ppm)	f_k	Visibility	Φ_0	$\tilde{x}(f = 1\text{kHz})$	dB
20	35.8	0.33	1.002	4.11E-18	12.27
50	44.8	0.53	0.931	1.94E-18	5.77
100	59.7	0.60	0.896	1.29E-18	2.18
200	89.5	0.53	0.913	1.02E-18	0.13
300	119.4	0.45	0.937	9.44E-19	-0.50
500	179.0	0.33	0.966	9.00E-19	-0.92
650	223.8	0.28	0.979	8.91E-19	-1.00
1000	328.3	0.20	0.995	8.96E-19	-0.95
2000	626.7	0.11	1.011	9.86E-19	-0.12