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typed version

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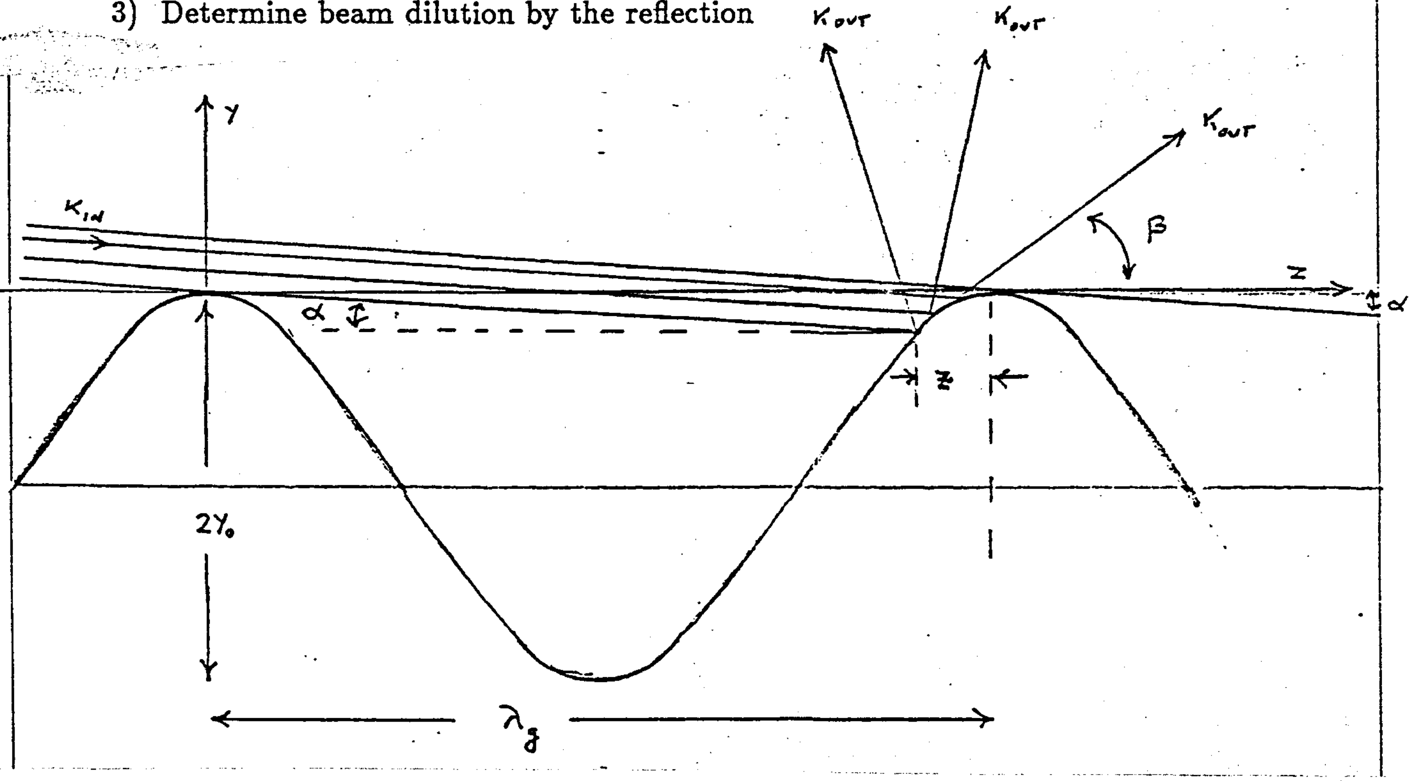
List of 5/01/89

Analysis of corrugated tube as light scatterer

Outline: 1) Determine ray trajectories and intersections with corrugated surface
2) Determine distribution of reflections for single input angle
3) Determine beam dilution by the reflection



Assume plane wave incident, look at individual rays

Equation of corrugated surface

$$y(z) = y_0\left[\cos\left(\frac{2\pi z}{\lambda_g}\right) - 1\right]$$

(Typical values from Larry Jones $\quad y_0 = 1.27cm \quad \lambda_g = 12.7cm$)

Assume $\alpha < .1$ glancing angles

Equation of rays

$y(z) = y(0) - \alpha z$

Limiting rays that hit next convolution after $z = 0$

$y(\lambda - z) = -\alpha(\lambda_g - z)$ $\quad$ grazing at $z = o$

$y(0) = \alpha\lambda_g$ $\quad$ grazing at $z = \lambda$

Intersection of limiting rays with surface

Assume $z/\lambda_g \ll 1$

For surface

$$y(\lambda - z) = y_0\left[\cos\left(2\pi - \frac{2\pi z}{\lambda_g}\right) - 1\right] \cong -\frac{y_0}{2}\left(\frac{2\pi z}{\lambda_g}\right)^2$$

Equation for z

$$-\alpha(\lambda_g - z) = -\frac{y_0}{2}\left(\frac{2\pi z}{\lambda_g}\right)^2 \qquad z^2 + \frac{\alpha}{\frac{y_0}{2}\left(\frac{2\pi}{\lambda_g}\right)^2} z - \frac{\alpha\lambda_g}{\frac{y_0}{2}\left(\frac{2\pi}{\lambda_g}\right)^2} = 0$$

Solution of quadratic equation $\alpha < 1$, $y_0/\lambda_g < 1$

$$z = \left(\frac{2\alpha\lambda_g}{y_0}\right)^{1/2} \frac{\lambda_g}{2\pi}$$

Good to 10% up to $\alpha \sim 10^{-1}$

The intersection for rays between limits

$$z = \left(\frac{2(y(0) - \alpha\lambda_g)}{y_0}\right)^{1/2} \frac{\lambda_g}{2\pi}$$

The tangent of the corrugated surface at the point of intersection

$$\left.\frac{dy}{dz}\right|_{\text{surface}} = \frac{2\pi y_0}{\lambda_g} \sin\frac{2\pi z}{\lambda_g} \xrightarrow[z/\lambda_g < 1]{} \left(\frac{2\pi}{\lambda_g}\right)^2 z y_0$$

Eliminate z and get useful expression in terms of α and $y(0)$

$$\left.\frac{dy}{dz}\right|_{\text{surface}} = \frac{2\pi}{\lambda_g}\left(2y_0(y(0) - \alpha\lambda_g)\right)^{1/2}$$

The distribution of reflected angles

Unit normal vector to the surface

$$\hat{n} = \frac{\hat{y} - \frac{dy}{dz}\hat{z}}{\left(1+\left(\frac{dy}{dz}\right)^2\right)^{1/2}}$$

Law of reflection written in vector form

$$\hat{k}_{\text{out}} = \hat{k}_{\text{in}} - 2(\hat{k}_{\text{in}} \cdot \hat{n})\hat{n} \qquad (1)$$

Define

$$\hat{k}_{\text{out}} = \sin\beta\hat{y} + \cos\beta\hat{z}$$

$$\hat{k}_{\text{in}} = -\sin\alpha\hat{y} + \cos\alpha\hat{z}$$

Look at change in y component only (transverse) from equation 1

$$\sin\beta = -\sin\alpha + \frac{2(\sin\alpha + \frac{dy}{dz}\cos\alpha)}{\left(1+\left(\frac{dy}{dz}\right)^2\right)}$$

In the limit

$\frac{dy}{dz} \ll 1 \qquad \alpha \ll 1 \qquad \beta \ll 1$

$$\beta = \alpha + 2\frac{dy}{dz} \qquad \text{(as expected)}$$

$$\beta = \alpha + \frac{4\pi}{\lambda_g}\left(2y_0\left|\left(y(0) - \alpha\lambda_g\right)\right|\right)^{1/2}$$

Redistribution: beam at input angle α

gets redistributed $\qquad \alpha \leq \beta \leq \alpha + 4\pi\left(\frac{2y_0}{\lambda_g}\right)^{1/2}\alpha^{1/2}$

For corrugated pipe with

$y_0 = 1.27 cm$

$$\beta_{\max} = \alpha + 5.62\ \alpha^{1/2}$$

$\lambda_g = 12.7 cm$

α	$\beta_{\max}$
10^{-2}	0.57
3×10^{-3}	0.31
10^{-3}	0.18
3×10^{-4}	0.097
1×10^{-4}	0.056

The corrugations certainly do a job on the grazing indicence rays

Redistribution of an input beam by the corrugations

The problem is one dimensional since only the transverse angles are redistributed.

The redistribution is always toward larger angles because of the shadowing by the corrugations, exactly the right way to increase the overall attenuation by the tube for grazing rays.

The exact calculation of the propagation along a corrugated tube seems difficult for me so I will approximate by assuming that the redistribution takes place uniformly over the band of angles $\Delta\beta$

$$\Delta\beta = 4\pi \left(\frac{2y_0}{\lambda_g}\right)^{1/2} \alpha^{1/2} \qquad \text{in one dimension}$$

But with no redistribution in the orthogonal direction

The total two dimensional beam attenuation for rays at all incidence angles $\alpha \leq \alpha_0$ will be approximately given by $\left(\frac{\alpha_0}{\Delta\beta}\right)^{1/2} = \left[\frac{\alpha_0^{1/2}}{4\pi\left(2y_0/\lambda_g\right)^{1/2}}\right]^{1/2}$ per encounter with the corrugated tube

The attenuation from multiple encounters with walls is given as before (RW p3, p10)

$$\text{attenuation}(\alpha > \alpha_0) = \left[\left[\frac{\alpha_0^{1/2}}{4\pi(\frac{2y_0}{\lambda_g})}\right]^{1/2}\right]^{\frac{L\alpha_0}{2R}}$$

with the parameters

$y_0 = 1.27cm$ $\quad$ $L = 4km = 4 \times 10^5 cm$

$\lambda_g = 12.7cm$ $\quad$ $R = 61cm$

α_0 (rad)	Attenuation db in 4km
1×10^{-2}	292
8×10^{-3}	240
6×10^{-3}	186
4×10^{-3}	130
3×10^{-3}	101
2×10^{-3}	70
1×10^{-3}	37.5
8×10^{-4}	30.6
6×10^{-4}	23.6
4×10^{-4}	16.3
2×10^{-4}	8.6
1×10^{-4}	4.6

Corrugated tubes look very good in reducing the critical angle for 80db attenuation along the 4km tube.

They are not by themselves good enough to eliminate the need for baffles. As can be seen from the table above, rays at angles $\alpha < 2 \times 10^{-3}$ radians must still be blocked by baffles.