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77

List of 5/01/89

Summary of present estimates for tube motion and baffling with and without additional tube roughening both for straight tubing and corrugated tubing.

References: "Light Scattering and Proposed Baffle Configuration for the LIGO," K.S. Thorne second draft 1/11/1989.

"Optical Properties of the LIGO Beam Tubes," R. Weiss 1/17/1989 and Error 21989, R. Weiss 2/19/1989

"Analysis of Corrugated Tube as Light Scatterer" R. Weiss 3/1/1989

Assumptions made in numerical estimates

L = length of tube 4km = $4 \times 10^5 cm$
R = radius of tube $24'' \Rightarrow 61 cm$
λ = wavelength of light $0.5 \rightarrow 1.0 \times 10^{-4} cm$
η = quantum efficiency of photodetector = 0.9

Noise

Reference seismic motion (assumed isotropic)

$$x(f) \sim 10^{-7}\ cm/Hz^{1/2} \qquad 1 < f < 10 Hz$$
$$x(f) \sim 10^{-5}/f^2\ \ cm/Hz^{1/2} \qquad f > 10 Hz$$

Reference seismic fluctuation in angle of tube axis

$$\mu(f) \sim \frac{2\pi x(f) f}{c_t} T(f)$$

c_t = transverse sound speed of tube $\sim 3 \times 10^5 cm/sec$

c_{ct} = transverse sound speed of corrugated tube $\sim 3 \times 10^4 cm/sec$

$T(f)$ = transfer function of tube motion/ground motion
~ 10 to account for resonances at low frequencies
($10 - 100 Hz$ straight tubes)
($1 - 10 Hz$ corrugated tubes)

$\mu(f)$ radians/$Hz^{1/2}$

	Straight	Corrugated
$1 < f < 10 Hz$	$2 \times 10^{-11} f$	$2 \times 10^{-10} f$
$f > 10 Hz$	$2 \times 10^{-9}/f$	$2 \times 10^{-8}/f$

Reference sensitivity

Quantum limited antenna optimized for frequency f

$m = 10^6 gm$ T_{store}= storage time of light = $\frac{1}{2\pi f}$ (delay line) = $\frac{1}{4\pi f}$(FP)

$h_{QL}(f) = \left(\frac{8\hbar}{m}\right)^{1/2} \frac{1}{2\pi f L} = \frac{4\times10^{-23}}{f}$ strain$/Hz^{1/2}$

Goal: Kip and I have set a goal that scattering noise should be kept, in these preliminary estimates, to a level

$$h_{\text{scat}}(f) \leq \frac{1}{10} h_{QL}(f)$$

Fundamental scattering and recombination source is mirror scattering function

$$\frac{dP_{sc}(\theta)}{P d\Omega} = \frac{10^{-6}}{\theta^2} = \frac{\alpha}{\theta^2}$$

Caveats

1) Extrapolation is being made to $\theta \sim 10^{-4}$: best present measurements $\theta \geq 10^{-2}$ with poorer mirrors for which $\alpha \sim 3\times10^{-5}$. [Supermirrors are assumed, $\alpha \sim 10^{-6}$]

2) Expectation is that functional dependence will not be stronger than $1/\theta^2$ at smaller angles and may flatten.

3) Clearly an area where project will have to expend effort.

Summary assumes

1) Baffling extends along full 4km tube. I assume that there is no special advantage to place baffles in only 1/2 of the tube length once the decision is taken to employ baffles. (Kip carries both 1/2 and full baffling options in his document.)

2) Retain the option to meet the design goal without output mode filters.

The summary gives baffling and tube properties for interferometers with and without output mode filters. Considering that we have not used output mode filters in the prototypes up to now; it is prudent to consider this option for the LIGO as well, even though it makes more stringent demands on the reduction of scattered light.

3) There will be more baffles than pump stations along the 4km length.

The elegant idea of making the baffles integral with the pump stations and thereby changable is not viable. The results of the preliminary analysis indicate that both tube and baffle motions cause scattering noise; so putting only baffles at firm supports is not adequate. The concept now envisaged is a sheet metal structure placed in the tubes. The design tradeoffs are between

a) Smooth vs absorbing baffles to reduce backscattering

b) The number of baffles vs the attenuation by the tubes themselves

c) The number of baffles vs clear aperature

d) Scattering by baffle edges vs number of baffles

4) Coherence in the scattered field at the output of the interferometer can be neglected (KST)

5) The baseline baffling strategy is to convert glancing rays to larger angles and to then use the absorption by the tubes on multiple encounters to attenuate and isotropize the scattered light. This concept exploits the unique property of the LIGO, the large ratio of L/R.

Backscattering by the baffles

K.S.T. page 23 EQ 3.29 no mode filter
EQ 3.30 with mode filter

No mode filter

$$h(f) = 2\alpha^{1/2}(1-\eta)\left(\frac{2\pi fL}{c}\right)\left(\frac{d\sigma}{dA d\Omega}\right)^{1/2}\left(\frac{\lambda}{L}\right)^{1/4}\left(\frac{(\lambda L)^{1/2}}{y_0}\right)\left(\frac{R}{l_1}\right)^{1/2}\ \frac{x(f)}{L}$$

y_0 = closest distance of beam center to a baffle edge

l_1 = distance from mirror to first baffle

$$\frac{d\sigma}{dA d\Omega} = \frac{dP_{\text{sc}}(\theta_{\text{inc}})}{d\Omega P} = \underbrace{G\left(\frac{\pi}{4},\frac{3\pi}{4},0\right)}_{RW\ page\ 7\ EQ\ 1,\ EQ\ 2}$$

With mode filter

$$h(f) = 2\sqrt{2}\alpha\left(\frac{2\pi fL}{c}\right)\left(\frac{d\sigma}{dA d\Omega}\right)^{1/2}\left(\frac{\lambda}{L}\right)^{1/2}\left(\frac{(\lambda L)^{1/2}}{y_0}\right)^{3/2}\left(\frac{H}{\sqrt{\lambda L}}\right)^{1/2} N_b^{1/2}\ \frac{x(f)}{L}$$

N_b = total number of baffles

H = height of baffles

Conditions on baffle surface

Assume standard sheetmetal <u>not</u> optically polished

Assume Gaussian surface: σ=rms surface roughness
T=correlation length of roughness

$$G\left(\pi/4, 3\pi/4, 0\right) = \frac{1}{16\pi}\left(\frac{T}{\sigma}\right)^2 e^{-\left(\frac{T}{2\sigma}\right)^2} \qquad \underset{\textbf{rough surface}}{\left(\frac{2\pi\sigma}{\lambda}\right) > 1}$$

Sample numbers

Typical sheet metal surface aluminium or 304 stainless

$\sigma \sim 100$ microinch roughness $\quad \frac{2\pi\sigma}{\lambda} \sim 32 \quad \lambda = \frac{1}{2}\mu \quad$ if $T/\sigma \sim 1$

$G(\pi/4,\ 3\pi/4,\ 0) \cong 1.5 \times 10^{-2}$

Sample numbers for backscattering noise with this sheet metal surface

No mode filter

Use: $\lambda = 0.5 \qquad y_0 = 20cm$

$h(f) = \frac{3\times10^{-22}}{f l_1^{1/2}}$ strain$/Hz^{1/2}$

The first baffle distance $l \geq 10^4 cm$ so that $h(f) \leq \frac{1}{10} h_{QL}(f)$

With mode filter

$h(f) = \frac{5\times10^{-28}}{f} N_b^{1/2}$ use $H = 6cm$

To stay less than $1/10\ h_{QL}(f)$

Allowed to use $N_b < 6 \times 10^7$ baffles

Conclusions on backscattering by baffles

1) No special care is required in the surface quality of the baffle to reduce backscattering.

2) If the first baffle is placed closer to the mirror than 100 meters, the surface of the baffle should be made less scattering by polish or blackening.

Scatter propagation along the tubes

The fundamental relation for incoherent scattering noise is KST p13 EQ 3.7

$$h_{\rm scat}(f) \;=\; \frac{\lambda f}{c}\left[\int_{\Omega} P_{\rm rec}(\theta)\left(\frac{dP_{sc}\big/dA\,d\Omega\,df}{P_{in}/\lambda L}\right)d\Omega\right]^{1/2}$$

where $P_{\rm rec}(\theta)$ is the recombination probability of scattered light with the main beam.

With output mode filter

$$P_{\rm rec}(\theta) \;=\; \frac{2\alpha}{\theta^2}\,\frac{\lambda}{L} \qquad\qquad \text{KST p12 EQ 3.5}$$

Without output mode filter

$$P_{\rm rec}(\theta) \;=\; \frac{(1-\eta)^2}{2\theta}\left[\frac{\lambda}{L}\right]^{1/2} \qquad\qquad \text{KST p12 EQ 3.6}$$

The dominant noise term with baffling is due to seismically induced slope fluctuations of the tubes. The major process is diffraction at a baffle toward the tube wall followed by multiple reflection transport between the baffles - "diffraction-aided reflection."

The noise term for this process is

$$\frac{dP_{\rm sc}}{dA\,df} \;=\; \left.\frac{dP_{\rm sc}}{dA}\right|_{\substack{\text{unmodulated}\\ \text{by motion}}}\left[\frac{\pi(2L-l_n)}{\lambda}\theta_0\;\mu(f)\right]^2 \qquad\qquad \text{KST } p17 \text{ EQ 3.18}$$

Where

$\frac{dP_{sc}}{dA}$ is the unmodulated scattered intensity at the output

l_n is the distance between the scattering point and the output mirror

$\mu^2(f)$ is the power spectrum of seismically driven slope fluctuations of the tube

θ_0 is the minimum grazing angle for which attenuation of multiple reflection propagation exceeds $10^8 \rightarrow 10^9$

The attenuation is given by

$$\left[< R(\theta) > \right]^{L\theta/2R} = \text{attenuation} \qquad \text{RW p3, p10}$$

Where $< R(\theta) >$ is the average power reflectivity which is less than unity due both to absorption and angular redistribution of the incident beam at the wall due to diffuse scattering.

θ_0 establishes baffle number, height, and spacing and is dependent on tube design and roughness.

Assuming uniform baffle spacing along the tube, the separation of baffles is

$$S = \frac{2(H-\delta H)}{\theta_0} \qquad \text{KST p19 EQ 3.24A}$$

Where H is the height of the baffle and δH is the height safety factor to allow for attenuation of diffraction aided reflection. Kip chose $H = 6cm$ and $\delta H = 1cm$. (RW agrees this a good initial choice.)

<u>Note:</u> The non uniform spacing of baffles suggested by Kip was motivated by reducing the baffle number in part because he expected the baffles to be expensive. I see no strong argument now for reducing the number of baffles and see an operational advantage during construction to keep the baffle spacing uniform. The only scientific argument for reducing baffle number comes from the backscattering noise but this has a large margin of safety. (see page 5)

Estimates of θ_0 with attenuation of $80db$ in 4km

$\lambda = 0.5\mu$ $\quad H = 6cm$ $\quad \delta H = 1cm$

Tube Design	σ/λ	σ(cm)	g/θ^2	T/σ	θ_0(rad)	N_b	Reference
Smooth Stainless	0	0	0	∞	0.1	4×10^3	RW p3 fig4
Standard Stainless 100μ inch surface	5.1	2.5×10^{-4}	4.1×10^3	1	5×10^{-2}	2×10^3	RW p7 fig38
Roughened Stainless 1000μinch bumps	51	2.5×10^{-3}	4.1×10^5	1	6.1×10^{-3}	2.4×10^2	RW p7 fig38
Roughened Stainless $10\,000\mu$ inch bumps	510	2.5×10^{-2}	4.1×10^7	1	6.1×10^{-3}	2.4×10^2	RW p7 fig38
Roughened Stainless 1000μ inch bumps	51	2.5×10^{-3}	4.1×10^5	3	2.4×10^{-2}	9.8×10^2	RW p7 fig38
Smooth Corrugated Stainless $y_0 = 1.27cm$ $\lambda = 12.7cm$	0	0	0	∞	2.3×10^{-3}	9.1×10^1	RW CT 3/1/89
Bumpy Corrugated $\sigma_{y_0} = 1\%R = .6cm$ Manufacturing Tolerance	1.2×10^4	0.6	2.3×10^{10}	∞	2.3×10^{-3}	9.1×10^1	RW CT 3/1/89
Roughened Corrugated 1000μ inch bumps	51	2.5×10^{-3}	4.1×10^5	1	2.3×10^{-3}	9.1×10^1	RW CT 3/1/89

Discussion of table

Only diffuse scattering is considered as an attenuation mechanism. It is for this reason that increased roughening does not help. If we are ultimately dominated by coherent scattering increased roughening increases g (RW page 8) which helps enormously.

Taking Kip's estimate that coherent scattering is unimportant, the big advantage in roughening occurs between 100μ inch→ 1000μ inch surfaces for straight tubing.

The correlation length, T is more important than σ once $g \geq 1$.

For corrugated tubes the preliminary estimate for the attenuation is so large that roughening does not add to the attenuation already provided by the angular redistribution of the beam.

The roughening and irregularities in the corrugated tube would increase the attenuation of coherent scattering, if this should become important.

Noise due to diffraction aided reflection

<u>No output mode filter</u> KST p21 EQ 3.25b

$$h_{\rm scat}(f) = \frac{1}{\sqrt{3}}\,\frac{1}{16\pi^2}\,\frac{\lambda}{\delta H}\,\alpha^{1/2}\,(1-\eta)\,\frac{2\pi f L}{c}\left[\frac{(\lambda L)^{1/2}}{R}\right]^{3/2}\left[ln\left(\frac{L\theta_0}{4R}\right)\right]^{1/2}\mu(f)\;.$$

Using $\theta_0 \sim 6 \times 10^{-3}$ radians [very insensitive to θ_0]

$\underset{\text{straight tube}}{h(f)} = 1.4 \times 10^{-25}$ strain/$Hz^{1/2}$ $f > 10Hz$

crosses $\frac{1}{10} h_{\rm QL}(f)$ at about 30Hz

$\underset{\text{corrugated tube}}{h(f)} = 1.4 \times 10^{-24}$ strain/$Hz^{1/2}$ $f > 10Hz$

crosses $\frac{1}{10} h_{\rm QL}(f)$ at about 3Hz

<u>With output mode filter</u> KST p22 EQ 3.27b

$$h_{\rm scat}(f) = \frac{1}{\sqrt{3}}\,\frac{1}{64\pi^2}\,\frac{\lambda}{\delta H}\,\alpha\,\frac{2\pi f L}{c}\left[\frac{(\lambda L)^{1/2}}{R}\right]^{2}\mu(f)$$

$\underset{\text{straight tubing}}{h(f)} \cong 4 \times 10^{-29}$ strain/$Hz^{1/2}$

$\underset{\text{corrugated tubing}}{h(f)} \cong 4 \times 10^{-28}$ strain/$Hz^{1/2}$

Other considerations

1) Tube alignment and out of roundness

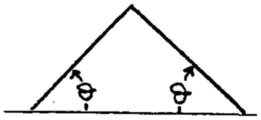
Primary concern is the effect of coherent scattering for beams on the tube axis and second order terms in the diffuse scattering due to slope fluctuations of the tube

In Sec III, KST the recommendation is made to put linear offsets of the tubes of $\sim 1cm$ in the variation around the average line of sight down the tubes. The recommendation should be adopted, it raises the baffle height from 6 to 7cm keeping the δH of 1cm, in further tradeoff studies one could recover the aperature by putting approximately 15% more baffles in.

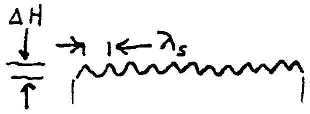
An out of roundness of $\frac{r_l - r_p}{<r>} \sim 1/100$ is also recommended to reduce the possibility of coherent scattering

2) Concept of the baffles

My present concept of the baffles is a sheet metal triangle placed as a helix in the beam tubes. The surface finish back toward the mirrors can be as rough as 100μ inches. The angle of the baffle sides is not critical. I believe it can be $\theta = 45 \pm 5^o$. (this needs further evaluation)



The baffle edge could be roughened to reduce coherent scattering. If it is roughened the variation in height



$$\Delta H \geq \frac{\lambda L}{2R} \sim 3.5mm \text{ at } 1.06\mu \text{ and } \lambda_s < \Delta H$$

The roughening can be periodic or random, it will get scrambled by the multiple baffles.

3) Corrugated tubes have a great appeal since they have substantial margin for the scattered noise. The only penalty paid is the lower flexural frequencies of the tubes themselves, relative to straight tubing. The helical baffle design can be easily adapted to the "culvert" type of corrugated tube design.

Uncertainties and future work

1) I believe there is enough information to make a first cut costing and conceptual design of the baffling.

2) It is extremely unlikely that we will improve the baffling design or get mirrors with small enough α so that a cover for the tubes becomes unnecessary. If we do not intend to compromise the LIGO at the quantum limit and wish to retain the option of not using output mode filters a cover is needed.

   Neither Kip nor I are sufficiently confident of the estimates that we would bank on a factor of 10 in the noise estimate of $h(f)_{\text{scat}}$

3) Continuing uncertainties in the estimates are

   1) have all mechanisms been uncovered;
   2) is coherence really unimportant;
   3) the effect of scattering by interferometer components - mirror edges, wires, magnetic controllers etc.

   Once the heat of the present proposal and conceptual design activities has attenuated, Kip and I recommend that a computer model of the baffling strategy with an installed interferometer be carried out. The modeling should include both coherent and diffuse scattering.

4) Direct measurements that would help future estimates

   1) photodetector uniformity
   2) measurment $\frac{dP_{\text{scat}}(\theta)}{Pd\Omega}$ for LIGO type mirrors at $\theta < 1 \times 10^{-2}$ radians
   3) direct diffuse scattering measurements at grazing incidence of candidate baffle and tube material

5) It would be useful for the project to find out the availibility of the scattering measurment facilities at China Lake. Dr. H Bennett at China Lake is a useful contact.