#67
List of 4/06/89

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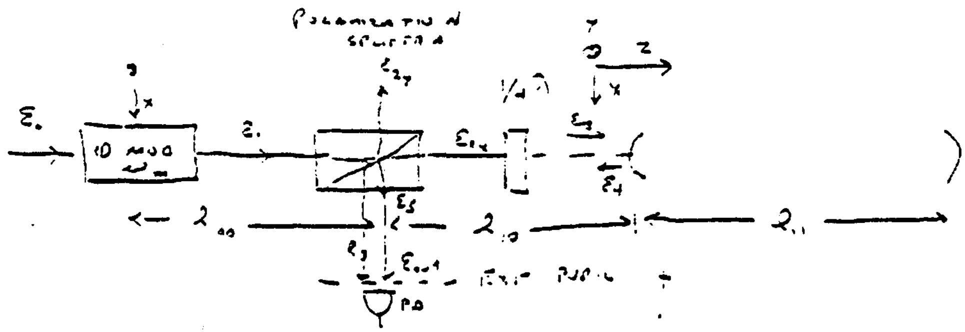
(received 03/14/89, EJF)

ANALYSIS OF STEADY STATE REFLECTION SENSING OF A CAVITY BY POUND/DREVER TECHNIQUE



Jones matrices

$$\begin{pmatrix} E_{x\,out} \\ E_{y\,out} \end{pmatrix} = \begin{pmatrix} O_{xx} & O_{xy} \\ O_{yx} & O_{yy} \end{pmatrix} \begin{pmatrix} E_{x\,in} \\ E_{y\,in} \end{pmatrix}$$

ϕ_{mod} $OP(xy)$

$$OP_\phi(xy) = \begin{pmatrix} e^{i\delta_x(t_o)} & 0 \\ 1 & e^{i\delta_y(t_o)} \end{pmatrix}.$$

$$\delta_x(t_o) = \Gamma_x \cos(\omega_n t + \varphi_m)$$
$$\delta_y(t_o) = \Gamma_y \cos(\omega_m t + \varphi_M)$$

Polarization splitter

$$OP_{ps}(xy) = \begin{pmatrix} t_x\, e^{i\delta_x} & 0 \\ 0 & r_y\, e^{i\delta_y} \end{pmatrix}$$

δ_y may be $\pi + \varphi$ depending on direction of propagation

$1/4\lambda$ plate

in diaganol rep.

$$OP_{1/4\lambda}(x'y') = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

Transformation to xy if fast axis at $\pi/4$ to x

$$OP_{1/4\lambda}(xy) = \frac{1}{2}\begin{pmatrix} 1+i & -1+i \\ -1+i & 1+i \end{pmatrix}$$

$$U_{x'y'\to xy} = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$
$$U_{xy\to x'y'} = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

Cavity reflection at close to normal incidence is a scalar

$$E_R = A(\Delta\omega)\, e^{i\varphi(\Delta\omega)} \qquad \Delta\omega = \omega - \omega_{ocavity}$$

Example: Begin beam at ϕ modulator polarized in x

$$E_{in} = \begin{pmatrix} E_o \\ 0 \end{pmatrix}$$

Output beam of ϕmodulation

$$E_1 = \begin{pmatrix} E_o \ e^{i\delta_x(t_o)} \\ 0 \end{pmatrix}$$

Output beam of polarization splitter

$$E_{2_x} = \begin{pmatrix} E_o t_x \ e^{i(\delta_x(t_o)+\delta_x)} \\ 0 \end{pmatrix} \qquad \text{choose center of splitter as } z = 0$$

Beam incident on cavity

$$E_3 = \begin{pmatrix} E_o t_x \frac{1}{2}(1+i) \ e^{i(\delta_x(t_o)+\delta_x)} \ e^{ik\ell_{10}} \\ E_o t_x \frac{1}{2}(-1+i) \ e^{i(\delta_x(t_o)+\delta_x)} \ e^{ik\ell_{10}} \end{pmatrix}$$

Beam reflected at input mirror of cavity

$$(E_4) = A(\Delta\omega) \ e^{i\varphi(\Delta\omega)}(E_e)$$

Return beam at center of beam splitter

$$E_5 = \frac{1}{4} \begin{pmatrix} 1+i & -1+i \\ -1+i & 1+i \end{pmatrix} \begin{pmatrix} 1+i \\ -1+i \end{pmatrix} A(\Delta\omega) \ e^{i\varphi(\Delta\omega)} E_o t_x \ e^{i(\delta_x(t_o)+\delta_x)} \ e^{ik2\ell_{10}}$$

$$= \begin{pmatrix} 0 \\ -1 \end{pmatrix} A(\Delta\omega) \ e^{i\varphi(\Delta\omega)} E_o t_x \ e^{i(\delta_x(t_A)+\delta_x)} \ e^{ik2\ell_{10}}$$

Beam reflected by polarization splitter at exit pupil

$$E_{\text{out}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} A(\Delta\omega) \ e^{i\varphi(\Delta\omega)} t_x r_y E_o \ e^{i(\delta_x(t_o)+\delta_x+\delta_y)} \ e^{ik(2\ell_{10}+\ell_3)} \ e^{i\omega t}$$

What are the modulation products from the ϕ modulator

$$e^{i\Gamma_x \cos(\omega_m t_A + \varphi_m)} = e^{i(\Gamma_x \cos\omega_m t \cos\varphi_m - \Gamma_x \sin\omega_m t \sin\varphi_m)}$$

Define

$$\Gamma_c = \Gamma_x \cos\varphi_m \qquad \Gamma_s = \Gamma_x \sin\varphi_m$$

$$e^{i\Gamma_x \cos(\omega_m t + \varphi_m)} = e^{i\Gamma_c \cos\omega_m t} \; e^{-i\Gamma_s \sin\omega_m t}$$

$$= \left(\sum_{-\infty}^{+\infty} J_n(-\Gamma_c) \; e^{in(\omega_m t + \frac{\pi}{2})} \right) \left(\sum_{-\infty}^{+\infty} J_n(\Gamma_s) \; e^{-in(\omega_m t)} \right)$$

If Γ_c and $\Gamma_s < 1$

First order terms will be at unshifted frequency and $\pm\omega_m$ side bands, assume for ease of calculation that $\Gamma_c = 0 \Rightarrow \varphi_m = \pi/2 \qquad \Gamma_s = \Gamma$

Three important terms are

$$E(\omega) = J_o(\Gamma) \; e^{i\omega t} \qquad E(\omega + \omega_m) = J_{-1}(\Gamma) \; e^{i(\omega+\omega_m)t}$$

$$E(\omega - \omega_m) = J_1(\Gamma) \; e^{i(\omega-\omega_m)t} \qquad J_{-1}(\Gamma) = -J_1(\Gamma)$$

The beam at the output is made of 3 waves in first order

$$E_{\text{out}} = t_x r_y E_o \; e^{ik(2\ell_{10}+\ell_3)} \; e^{i(\delta_x+\delta_y)} \times$$

$$\left\{ J_o(\Gamma) A(\Delta\omega) \; e^{i\varphi(\Delta\omega)} \; e^{i\omega t} + J_1(\Gamma) \Big[A(\Delta\omega - \omega_m) \; e^{i\varphi(\Delta\omega-\omega_m)} \; e^{i(\omega-\omega_m)t} - A(\Delta\omega + \omega_m) \; e^{i\varphi(\Delta\omega+\omega_m)} \; e^{i(\omega+\omega_m)t} \Big] \right\}$$

The intensity at the photodetector (envelope function)

$$I(t) = (E_{\text{out}} E^*_{\text{out}})$$

The output has two interesting terms in first order

The DC term

$$I(\omega=0) \quad = t_x t_x^* r_y r_y^* \; E_o^2\left[J_o^2(\Gamma)A^2(\Delta\omega) + J_1^2(\Gamma)\left(A^2(\Delta\omega-\omega_m)+A^2(\Delta\omega+\omega_m)\right)\right.$$

$$\sim TRI_oJ_o^2(\Gamma)A^2(\Delta\omega) \quad \text{for small modulation} \quad \Gamma < 1$$

$$\sim TRI_oJ_o^2(\Gamma)\left[1-\frac{4(A_1+A_2)}{T_1}\right] \text{at} \;\; \Delta\omega = 0$$

The term at ω_m

$$I(\omega=\omega_m) \quad = TRI_oJ_o(\Gamma)J_1(\Gamma)\Big[A(\Delta\omega)A(\Delta\omega-\omega_m) \;\; e^{i\left[\varphi(\Delta\omega-\omega_m)-\varphi(\Delta\omega)\right]} \;\; e^{-i\omega_m t}$$

$$-A(\Delta\omega)\;A(\Delta\omega+\omega_m)\;\; e^{i\left[\varphi(\Delta\omega+\omega_m)-\varphi(\Delta\omega)\right]} \;\; e^{+i\omega_m t}$$

$$+A(\Delta\omega)\;A(\Delta\omega-\omega_m)\;\; e^{i\left[\varphi(\Delta\omega-\omega_m)-\varphi(\Delta\omega)\right]} \;\; e^{+i\omega_m t}$$

$$-A(\Delta\omega)\;\;A(\Delta\omega+\omega_m)\;\; e^{-i\left[\varphi(\Delta\omega+\omega_m)-\varphi(\Delta\omega)\right]} \;\; e^{-i\omega_m t}\Big]$$

Collecting terms

$$I(\omega=\omega_m) \quad = 2TRI_oJ_o(\Gamma)J_1(\Gamma)\;*$$

$$A(\Delta\omega)\Big[A(\Delta\omega-\omega_m)\;\cos\Big[\omega_m t - \overbrace{(\varphi(\Delta\omega-\omega_m)-\varphi(\Delta\omega))}^{\Delta\varphi(-)}\Big]$$

$$-A(\Delta\omega+\omega_m)\;\cos\Big[\omega_m t + \underbrace{(\varphi(\Delta\omega+\omega_m)-\varphi(\Delta\omega))}_{\Delta\varphi(+)}\Big]\Big]$$

<u>Limiting cases that are important</u>

1) Low loss F.P. so that $\quad A(\Delta\omega) \cong \left(1-\frac{4(\Delta_1+\Delta_2)}{T_1}\right)^{1/2}$

Note: ω_m outside of the resonance $\quad A(\Delta\omega+\omega_m) = A(\Delta\omega-\omega_m) = (1-A)^{1/2}$

Call $B = \left[\left(1-4\left(\frac{A_1+A_2}{T_1}\right)\right)\left(1-A_1\right)\right]^{1/2}$

$$I(\omega=\omega_m) \quad = B2TRI_oJ_o(\Gamma)J_1(\Gamma)\left[\cos\left(\omega_m t-\Delta\varphi(-)\right)-\cos\left(\omega_m t+\Delta\varphi(+)\right]\right.$$

Use $\quad \cos x - \cos y = -2\sin\frac{1}{2}(x+y)\;\sin\frac{1}{2}(x-y)$

$$
\begin{aligned}
I(\omega_m) \quad &= B4TRI_oJ_o(\Gamma)J_1(\Gamma) \quad \Big[\sin\big[\omega_m t + \frac{1}{2}[\Delta\varphi(+) - \Delta\varphi(-)]\big] \\
&\times \sin\big(\frac{1}{2}(\Delta\varphi(-) + \Delta\varphi(+))\big)\Big] \\
I(\omega_m) \quad &= B4TRI_oJ_o(\Gamma)J_1(\Gamma)\sin\frac{1}{2}\Big(\Delta\varphi(-) + \Delta\varphi(+)\Big) \\
&\times \sin\Big(\omega_m t + \frac{1}{2}[\Delta\varphi(+) - \Delta\varphi(-)]\Big)
\end{aligned}
$$

Special cases

Let $\Delta\omega \ll \frac{\omega}{Q_{cavity}}$: The cavity resonance is close to the incident unshifted laser carrier and the modulation sidebands lie $\frac{\omega}{Q} < \omega_m < \frac{c}{4\ell}$ i.e. in the flat part of ϕ curve

Then

$$\Delta\varphi(+) = \varphi(\Delta\omega + \omega_m) - \varphi(\Delta\omega) = 0 + \frac{1}{2}T_1 - \pi + \frac{4}{T_1\left(1 - \left(\frac{A_1+A_2}{T_1}\right)^2\right)}\frac{2\ell}{c}\Delta\omega$$

$$\Delta\varphi(-) = \varphi(\Delta\omega - \omega_m) - \varphi(\Delta\omega) = 2\pi - \frac{1}{2}T_1 - \pi + \frac{4}{T_1\left(1 - \left(\frac{A_1+A_2}{T_1}\right)^2\right)}\frac{2\ell}{c}\Delta\omega$$

This gives photodetector intensity at ω_m as

$$I(\omega_m) \quad = 4\left[\left(1 - 4\frac{(A_1 + A_2)}{T_1}\right)(1 - A_1)^2\right]^{1/2} \quad TRI_oJ_o(\Gamma)J_1(\Gamma)*$$

$$\sin\left[\frac{16\pi\ell}{c}\Delta f\left(\frac{1}{T_1\left(1 - \left(\frac{A_1+A_2}{T_1}\right)^2\right)}\right)\right] \sin(\omega_m t + \frac{T_1}{2})$$

Δf frequency difference of cavity resonance and laser light

or if the frequency of the laser is fixed, a signal for change in the cavity length $\Delta\ell$

$$I(\omega_m) \quad = 4\left[\left(1 - 4\frac{(A_1 + A_2)}{T_1}\right)(1 - A_1)\right]^{1/2} TRI_oJ_o(\Gamma)J_1(\Gamma)*$$

$$\sin\left[\frac{16\pi\Delta\ell}{\lambda}\left(\frac{1}{T_1\left(1-\left(\frac{A_1+A_2}{T_1}\right)^2\right)}\right)\right]\ \sin\left(\omega_m t+\frac{T_1}{2}\right)$$

Modulation products in the reflection locking system

Characterize the FP cavity by the reflection coefficient

$$\frac{E_R}{E_{\text{inc}}}=\left[\frac{A^2-2AB\ \cos x+B^2}{1-2C\ \cos x+C^2}\right]^{1/2}\quad e^{i}\underbrace{\left[\tan^{-1}\left(\frac{B\ \sin x}{A-B\ \cos x}\right)-\tan^{-1}\left(\frac{C\ \sin x}{1-c\ \cos x}\right)\right]}_{\varphi}$$

$$A=\left(1-A_1-T_1\right)^{1/2}\qquad B=\left[(1-A_2)(1-A_1)^2\right]^{1/2}\qquad c=\left[(1-A_1-T_1)(1-A_2)\right]^{1/2}$$

Here assumed mirror 2 $\qquad T_2=0$

$$x=2\omega\ell\big/c$$

Assume for ease of calculation that laser frequency is at ω_0 fixed and cavity length fluctuates around ℓ_0 given by

$$x_0=n2\pi=2\omega_0\ell_0\big/c\qquad\qquad \varphi=\pi$$

Assume further that cavity moves less than a free spectral range $x=n2\pi+z \quad |z|<2\pi$

or $\quad \frac{2\Delta\ell\omega}{c}<2\pi \quad \Delta\ell<\frac{\lambda}{z}$

cavity $\quad E_R\big/E_{\text{inc}}=A(z,\omega)\ e^{i\varphi(z,\omega)}$

The input beam to the cavity has frequency ω_0 at the unmodulated carrier and sidebands at $+\omega_m, -\omega_m$ and $+2\omega_m, -2\omega_m$ leave out higher ϕ modulation products.

Assume the phase modulator produces a ϕ shift

$$\varphi(t)=\Gamma\ \sin\omega_m t$$

Then to order 2

$$\begin{aligned}\frac{E_{\text{refl}}}{E_{\text{in}}}=&J_0(\Gamma)A(z,0)\ e^{i\varphi(z,0)}+J_1(\Gamma)A(z,\omega_m)\ e^{i\varphi(z,\omega_m)}e^{-i\omega_m t}\\&-J_1(\Gamma)A(z,-\omega_m)\ e^{i\varphi(z,-\omega_m)}e^{-i\omega_m t}\\&+J_2(\Gamma)A(z,2\omega_m)\ e^{i\varphi(z,2\omega_m)}e^{-i2\omega_m t}\\&+J_2(\Gamma)A(z,-2\omega_m)\ e^{i\varphi(z,-2\omega_m)}e^{-i2\omega_m t}\end{aligned}$$

[Note: In a long antenna 4km must be careful not to have sidebands fall into another order of the FP. The orders are 30KHz apart]

The operation of detection yields photo current

$i \,\alpha\, (E \times E^*)$

Products at DC

$$\Big[J_0(\Gamma)A(z,0)\Big]^2 + \Big[J_1(\Gamma)A(z,\omega_m)\Big]^2 + \Big[J_1(\Gamma)A(z,-\omega_m)\Big]^2$$
$$\Big[J_2(\Gamma)A(z,2\omega_m)\Big]^2 + \Big[J_2(\Gamma)A(z,-2\omega_m)\Big]^2$$

Products at $\pm\ \omega_m$

$$\Big[J_0(\Gamma)A(z,0)\ e^{i\varphi(z,0)}J_1(\Gamma)A(z,\omega_m)\ e^{-i\varphi(z,\omega_m)}e^{i\omega_m t}\Big]$$
$$-\Big[J_0(\Gamma)A(z,0)\ e^{i\varphi(z,0)}J_1(\Gamma)A(z,-\omega_m)\ e^{-i\varphi(z,-\omega_m)}e^{-i\omega_m t}\Big]$$
$$\Big[J_0(\Gamma)A(z,0)\ e^{-i\varphi(z,0)}J_1(\Gamma)A(z,\omega_m)\ e^{i\varphi(z,\omega_m)}e^{-i\omega_m t}\Big]$$
$$-\Big[J_0(\Gamma)A(z,0)\ e^{-i\varphi(z,0)}J_1(\Gamma)A(z,-\omega_m)\ e^{i\varphi(z,-\omega_m)}e^{i\omega_m t}\Big]$$
$$\Big[J_1(\Gamma)A(z,\omega_m)\ e^{i\varphi(z,\omega_m)}J_2(\Gamma)A(z,2\omega_m)\ e^{-i\varphi(z,2\omega_m)}e^{i\omega_m t}\Big]$$
$$-\Big[J_1(\Gamma)A(z,-\omega_m)\ e^{i\varphi(z,-\omega_m)}J_2(\Gamma)A(z,-2\omega_m)\ e^{-i\varphi(z,-2\omega_m)}e^{-i\omega_m t}\Big]$$
$$\Big[J_1(\Gamma)A(z,\omega_m)\ e^{-i\varphi(z,\omega_m)}J_2(\Gamma)A(z,2\omega_m)\ e^{i\varphi(z,2\omega_m)}e^{-i\omega_m t}\Big]$$
$$-\Big[J_1(\Gamma)A(z,-\omega_m)\ e^{-i\varphi(z,-\omega_m)}J_2(\Gamma)A(z,2\omega_m)\ e^{i\varphi(z,-2\omega_m)}e^{i\omega_m t}\Big]$$

Products at $\pm 2\omega_m$

$$-\Big[J_1(\Gamma)A(z,\omega_m)\ e^{i\varphi(z,-\omega_m)}J_1(\Gamma)A(z,-\omega_m)\ e^{-i\varphi(z,-\omega_m)}e^{-i2\omega_m t}\Big]$$
$$-\Big[J_1(\Gamma)A(z,\omega_m)\ e^{-i\varphi(z,\omega_m)}J_1(\Gamma)A(z,-\omega_m)\ e^{i\varphi(z,-\omega_m)}e^{i2\omega_m t}\Big]$$
$$+\Big[J_0(\Gamma)A(z,0)\ e^{-i\varphi(z,0)}J_2(\Gamma)A(z,2\omega_m)\ e^{-i\varphi(z,2\omega_m)}e^{i2\omega_m t}\Big]$$
$$+\Big[J_0(\Gamma)A(z,0)\ e^{-i\varphi(z,0)}J_2(\Gamma)A(z,2\omega_m)\ e^{i\varphi(z,2\omega_m)}e^{-i2\omega_m t}\Big]$$
$$+\Big[J_0(\Gamma)A(z,0)\ e^{-i\varphi(z,0)}J_2(\Gamma)A(z,-2\omega_m)\ e^{i\varphi(z,-2\omega_m)}e^{i2\omega_m t}\Big]$$
$$+\Big[J_0(\Gamma)A(z,0)\ e^{i\varphi(z,0)}J_2(\Gamma)A(z,-2\omega_m)\ e^{-i\varphi(z,-2\omega_m)}e^{-i2\omega_m t}\Big]$$

Try combining

$\pm\omega$ terms

$$J_0(\Gamma)J_1(\Gamma)A(z,0)A(z,\omega_m)\left[e^{i\varphi(z,0)-i\varphi(z,\omega_m)+i\omega_m t}+cc\right]$$
$$-J_0(\Gamma)J_1(\Gamma)A(z,0)A(z,-\omega_m)\left[e^{i\varphi(z,-\omega_m)-i\varphi(z,0)+i\omega_m t}+cc\right]$$
$$J_1(\Gamma)J_2(\Gamma)A(z,\omega_m)A(z,2\omega_m)\left[e^{i\varphi(z,\omega_m)-i\varphi(z,2\omega_m)+i\omega_m t}+cc\right]$$
$$-J_1(\Gamma)J_2(\Gamma)A(z,-\omega_m)A(z,-2\omega_m)\left[e^{i\varphi(z,-2\omega_m)-i\varphi(z,-\omega_m)+i\omega_m t}+cc\right]$$

$\pm 2\omega_m$ terms

$$-J_1^2(\Gamma)A^2(z,\omega_m)A^2(z,-\omega_m)\left[e^{i\varphi(z,-\omega_m)-i\varphi(z,-\omega_m)+i2\omega_m t}+cc\right]$$
$$+J_0(\Gamma)J_2(\Gamma)A(z,0)A(z,2\omega_m)\left[e^{i\varphi(z,0)-i\varphi(z,2\omega_m)+i2\omega_m t}+cc\right]$$
$$+J_0(\Gamma)J_2(\Gamma)A(z,0)A(z,-2\omega_m)\left[e^{i\varphi(z,-2\omega_m)-i\varphi(z,0)+i\omega_m t}+cc\right]$$

The terms converted to real functions

$\underline{\omega_m}$

$$2J_0(\Gamma)J_1(\Gamma)A(z,0)A(z,\omega_m)\ \cos\left[\varphi(z,0)-\varphi(z,\omega_m)+\omega_m t\right]$$
$$-2J_0(\Gamma)J_1(\Gamma)A(z,0)A(z,-\omega_m)\ \cos\left[\varphi(z,-\omega_m)-\varphi(z,0)+\omega_m t\right]$$
$$2J_1(\Gamma)J_2(\Gamma)A(z,\omega_m)A(z,2\omega_m)\ \cos\left[\varphi(z,\omega_m)-\varphi(z,2\omega_m)+\omega_m t\right]$$
$$-2J_1(\Gamma)J_2(\Gamma)A(z,-\omega_m)A(z,-2\omega_m)\ \cos\left[\varphi(z,-2\omega_m)-\varphi(z,-\omega_m)+\omega_m t\right]$$

$\underline{2\omega_m}$

$$-2J_1^2(\Gamma)A^2(z,\omega_m)A^2(z,-\omega_m)\ \cos\left[\varphi(z,-\omega_m)-\varphi(z,\omega_m)+2\omega_m t\right]$$
$$2J_0(\Gamma)J_2(\Gamma)A(z,0)A(z,2\omega_m)\ \cos\left[\varphi(z,0)-\varphi(z,2\omega_m)+2\omega_m t\right]$$
$$2J_0(\Gamma)J_2(\Gamma)A(z,0)A(z,-2\omega_m)\ \cos\left[\varphi(z,-2\omega_m)-\varphi(z,0)+2\omega_m t\right]$$

Using $\cos(A+B) = \cos\ A\ \cos\ B\ -\ \sin\ A \sin\ B$ and that $< \sin^2 x >= 1/2$

ω_m terms in mixer output

$$-J_0(\Gamma)J_1(\Gamma)A(z,0)\Big[A(z,\omega_m)\sin[\varphi(z,0)-\varphi(z,\omega_m)] \qquad \underline{\sin\ \omega t}$$

$$-A(z,-\omega_m)\ \sin[\varphi(z,-\omega_m)-\varphi(z,0)]\Big]$$

$$J_0(\Gamma)J_1(\Gamma)A(z,0)\Big[A(z,\omega_m)\cos[\varphi(z,0)-\varphi(z,\omega_m)] \qquad \underline{\cos\ \omega t}$$

$$-A(z,-\omega_m)\ \cos[\varphi(z,-\omega_m)-\varphi(z,0)]\Big]$$

$$-J_1(\Gamma)J_2(\Gamma)\Big[A(z,\omega_m)A(z,2\omega_m)\ \sin[\varphi(z,\omega_m)-\varphi(z,2\omega_m)] \qquad \underline{\sin\ \omega t}$$

$$-A(z,-\omega_m)A(z,-2\omega_m)\ \sin[\varphi(z,-2\omega_m)-\varphi(z,-\omega_m)]\Big]$$

$$J_1(\Gamma)J_2(\Gamma)\Big[A(z,\omega_m)A(z,2\omega_m)\ \cos[\varphi(z,\omega_m)-\varphi(z,2\omega_m)] \qquad \underline{\cos\ \omega t}$$

$$-A(z,-\omega_m)A(z,-2\omega_m)\ \cos[\varphi(z,-2\omega_m)-\varphi(z,-\omega_m)]\Big]$$

$2\omega_m$ terms

$$-J_1^2(\Gamma)A^2(z,\omega_m)A^2(z,-\omega_m)\Big[\cos[\varphi(z,-\omega_m)-\varphi(z,\omega_m)]\Big] \qquad \underline{\cos\ 2\omega t}$$

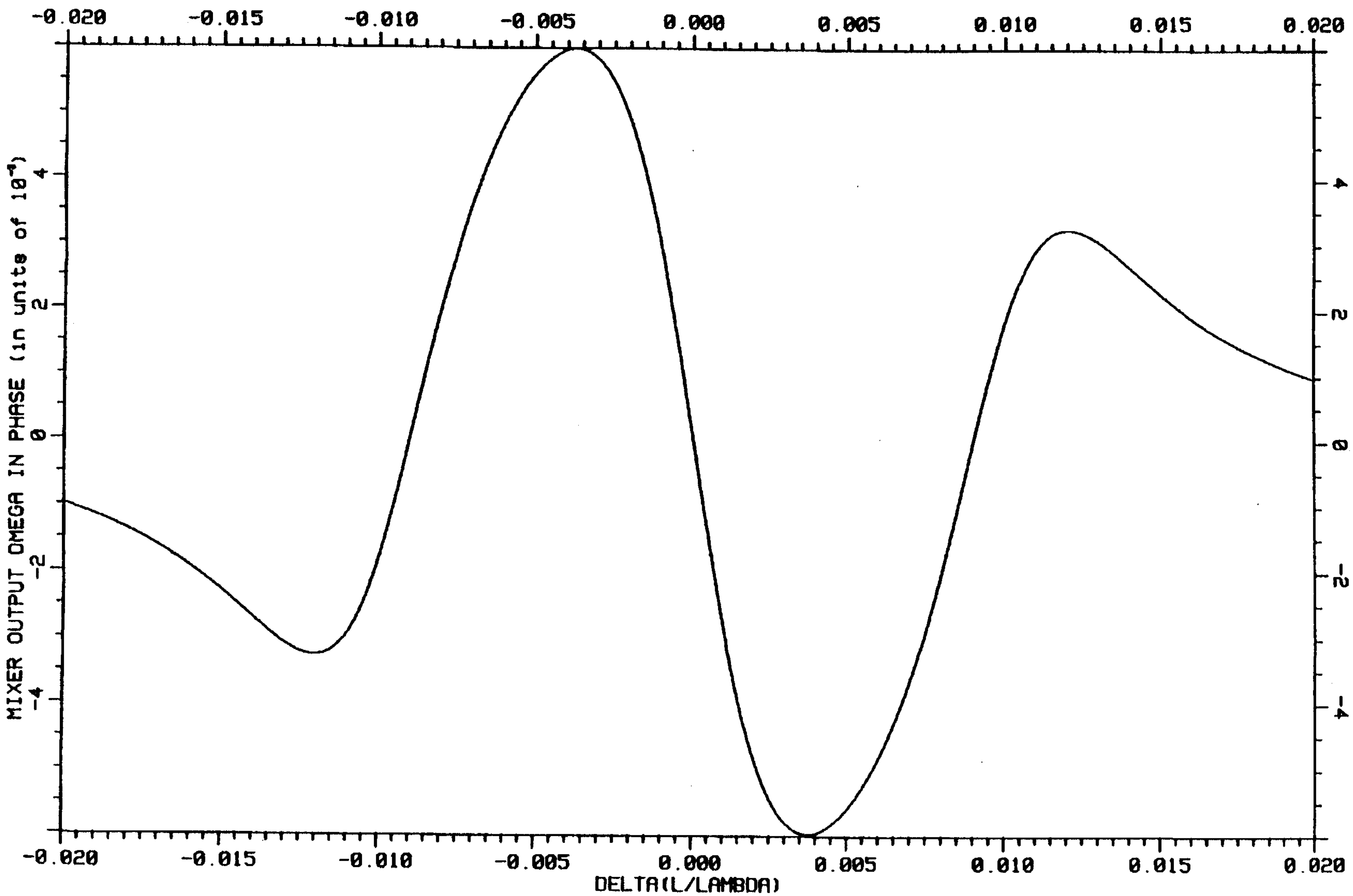
$$+J_1^2(\Gamma)A^2(z,\omega_m)A^2(z,-\omega_m)\Big[\sin[\varphi(z,-\omega_m)-\varphi(z,\omega_m)]\Big] \qquad \underline{\sin\ 2\omega t}$$

$$+J_0(\Gamma)J_2(\Gamma)A(z,0)\Big[A(z,2\omega_m)\ \cos[\varphi(z,0)-\varphi(z,2\omega_m)] \qquad \underline{\cos\ 2\omega t}$$

$$+A(z,-2\omega_m)\ \cos[\varphi(z,-2\omega_m)-\varphi(z,0)]\Big]$$

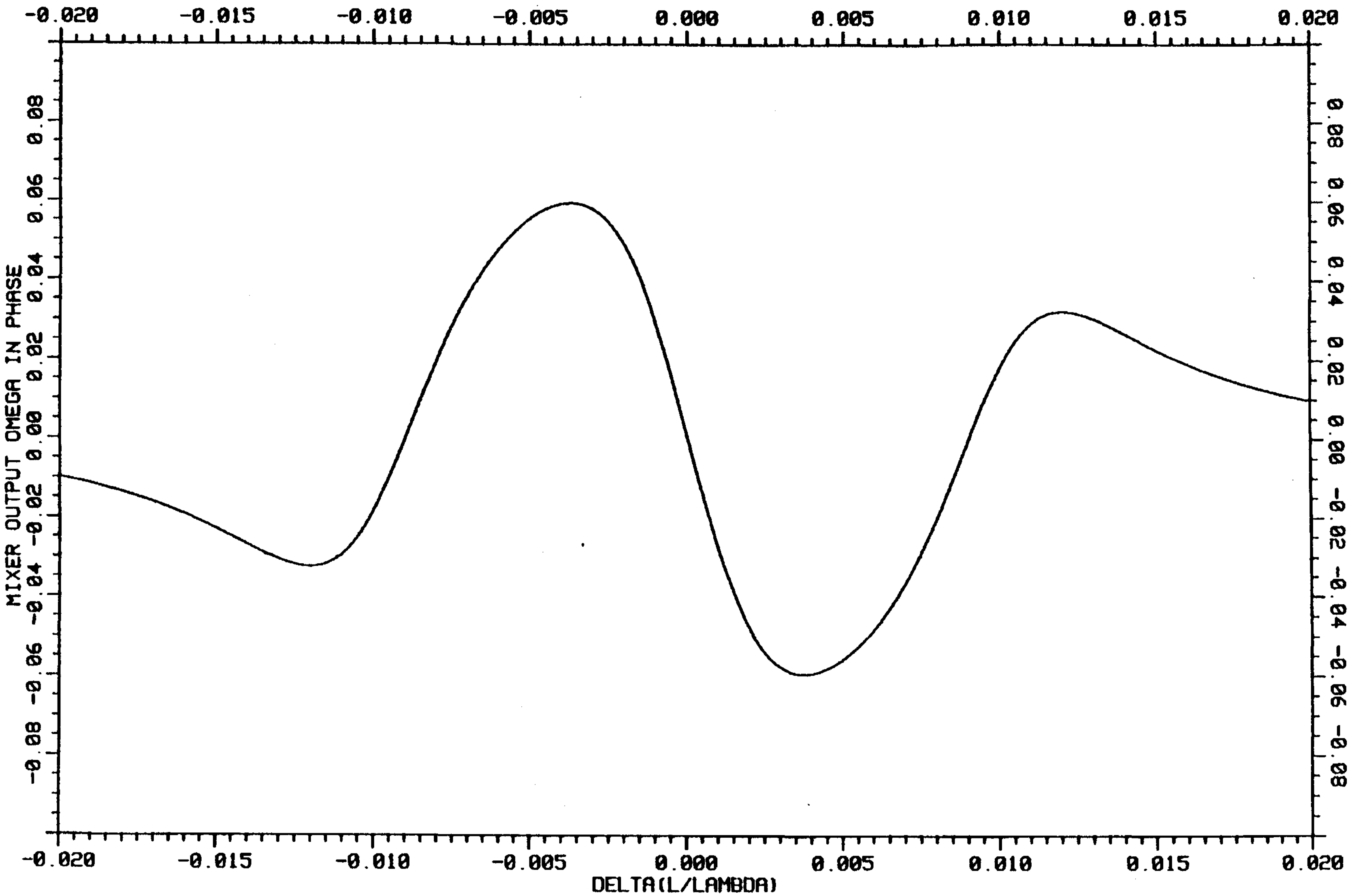
$$-J_0(\Gamma)J_2(\Gamma)A(z,0)\Big[A(z,2\omega_m)\ \sin[\varphi(z,0)-\varphi(z,2\omega_m)] \qquad \underline{\sin\ 2\omega t}$$

$$+A(z,-2\omega_m)\ \sin[\varphi(z,-2\omega_m)-\varphi(z,0)]\Big]$$



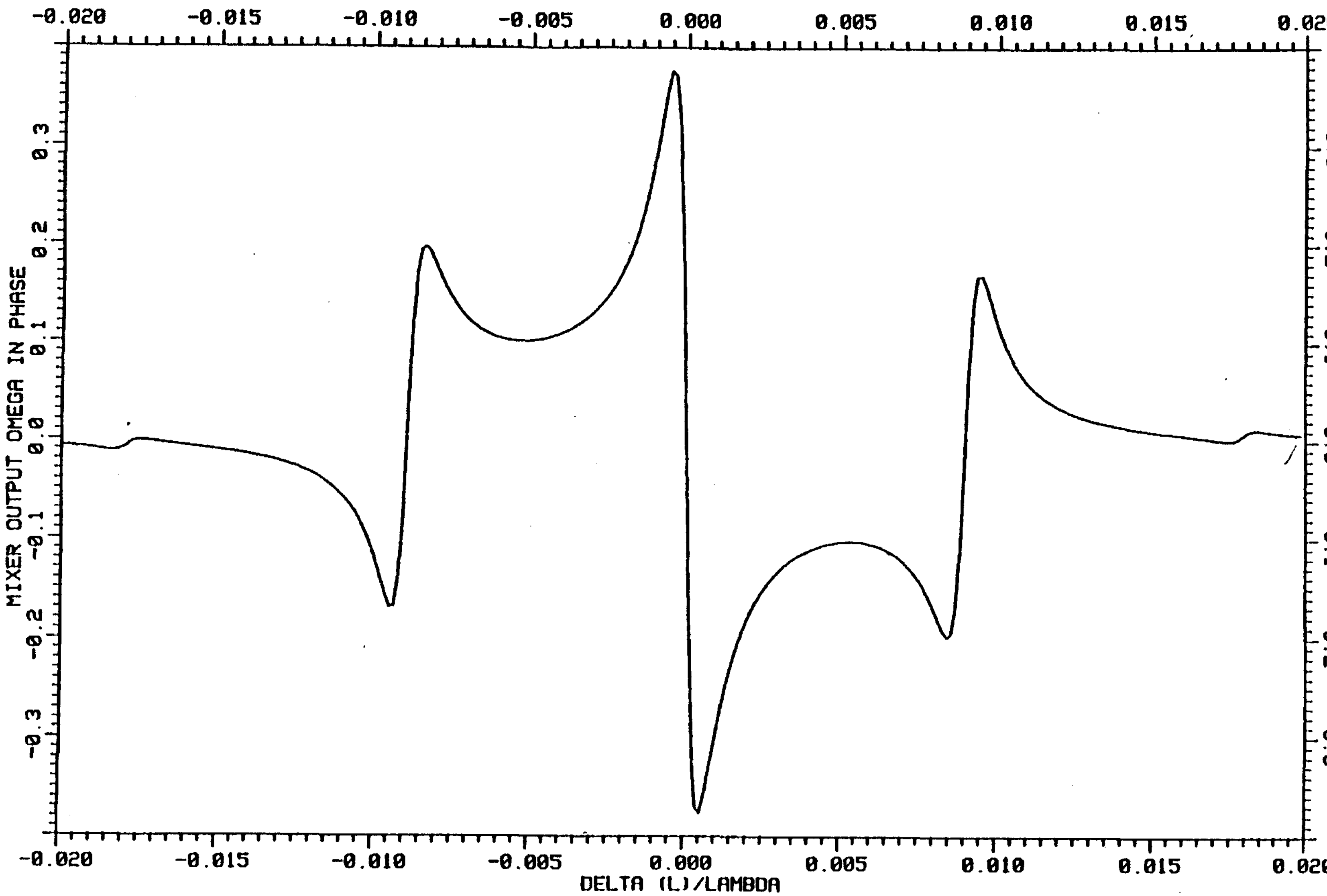
DISCRIMINANT FOR GAMMA =.01 IN PHASE OMEGA

03/14/89 14:18 617 253 7014 MIT GRAVITY GRP. → LIGO 043



DISCRIMINANT FOR GAMMA=.1 IN PHASE OMEGA

03/14/89 14:19 ☎617 253 7014 MIT GRAVITY GRP. →→→ LIGO ☑044



DISCRIMINANT FOR GAMMA =.5 IN PHASE OMEGA

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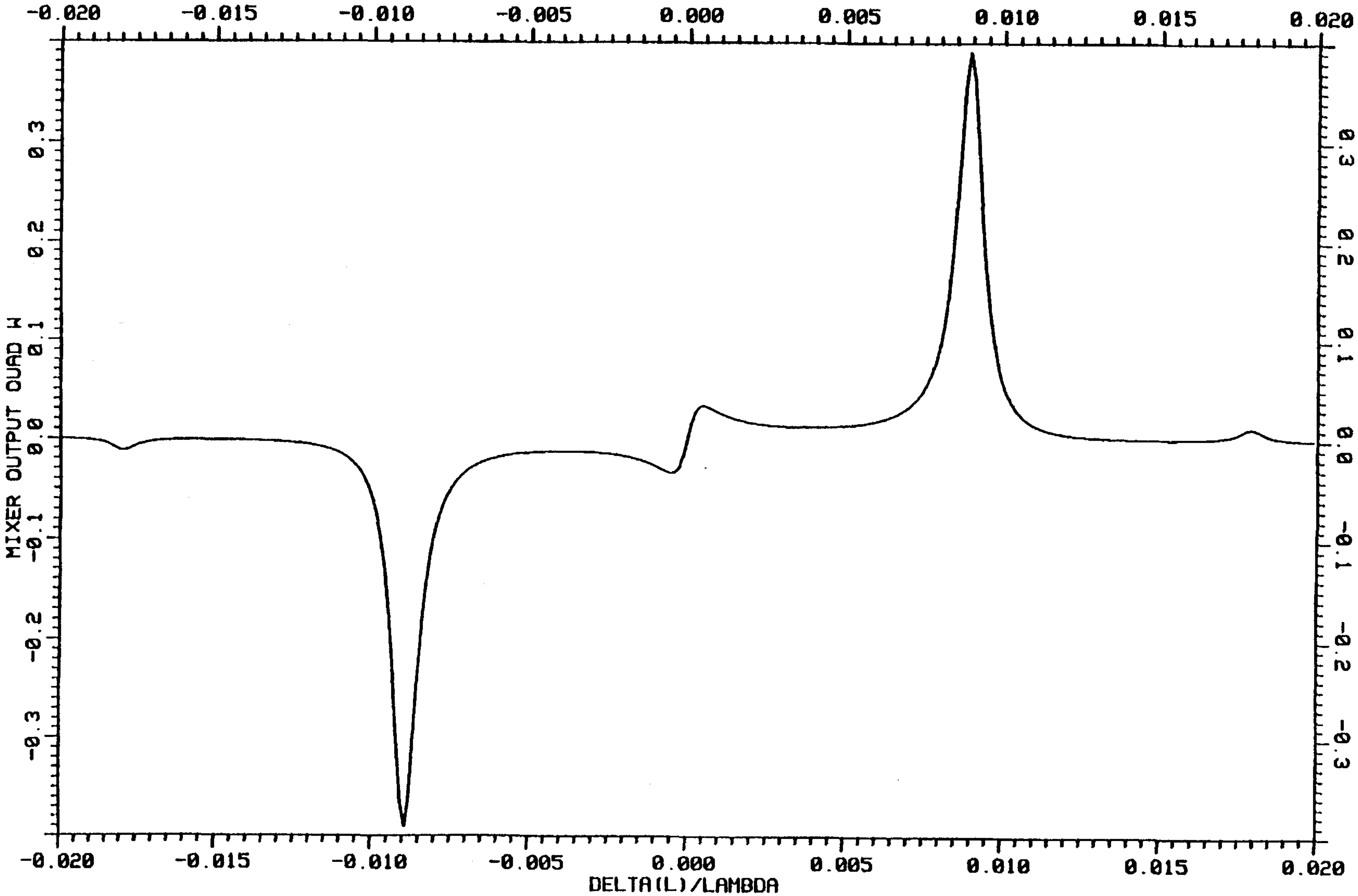
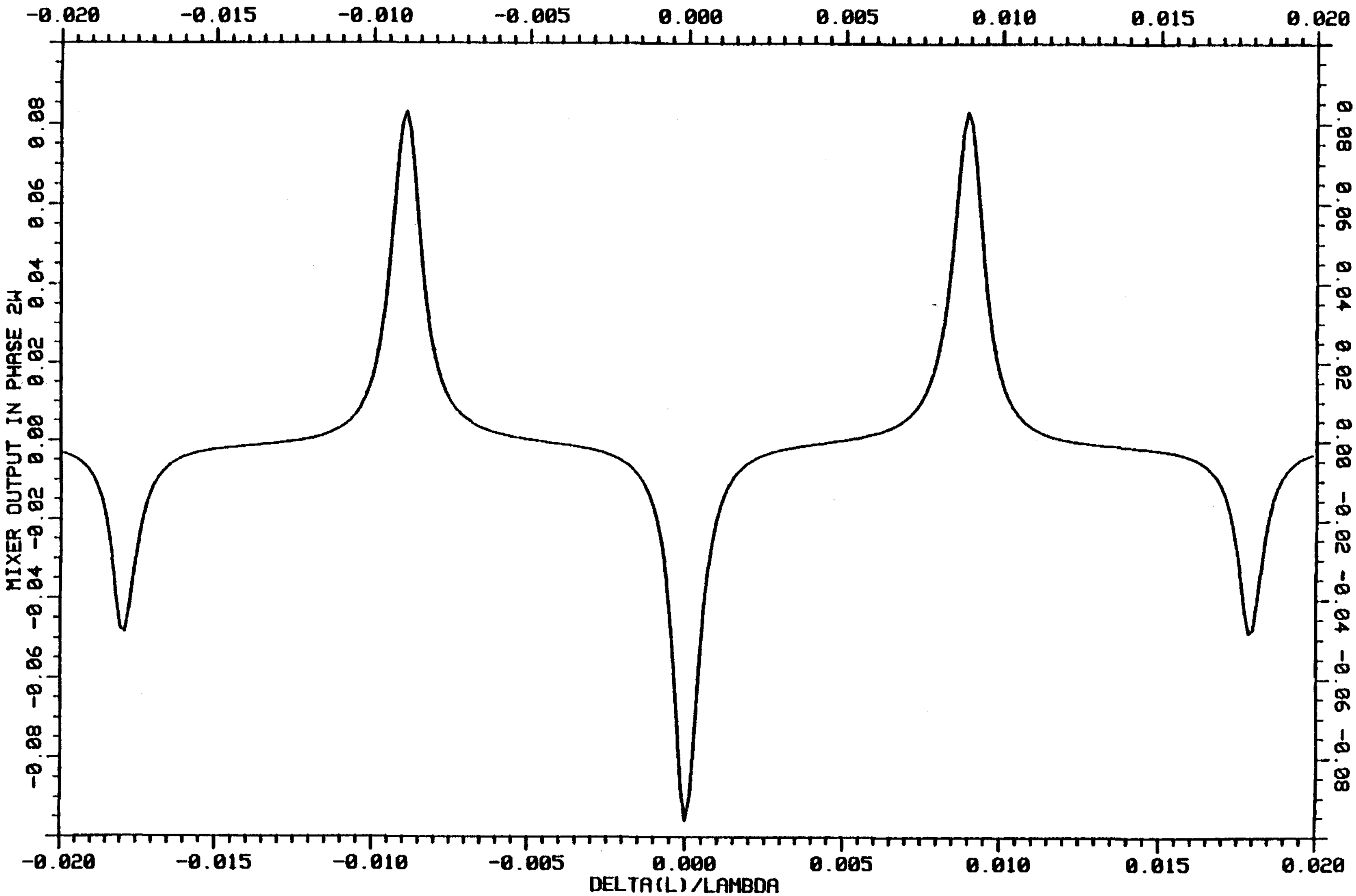


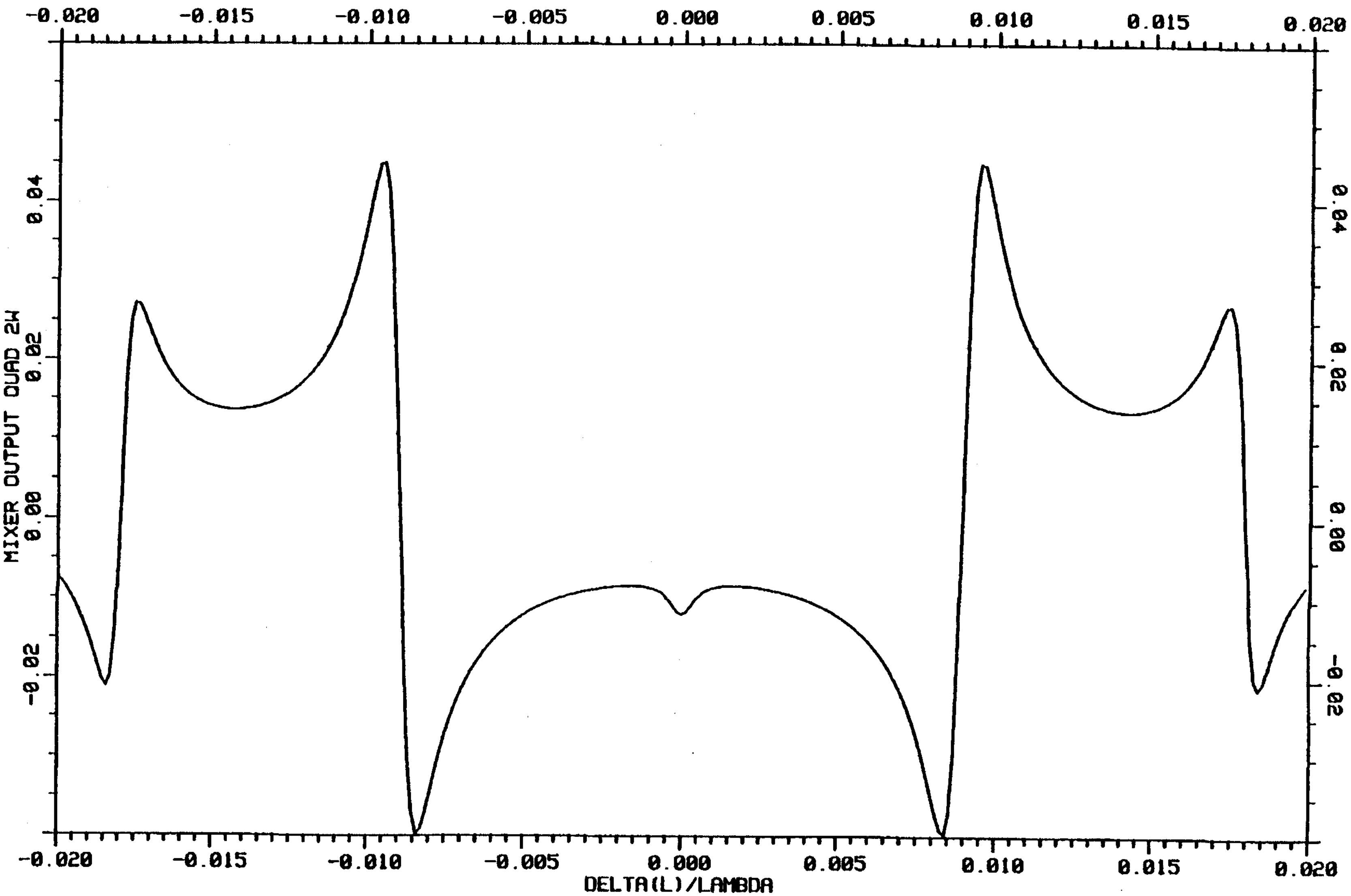
DISCRIMINANT AT GAMMA =.5 QUAD PHASEW

03/14/89 14:21 ☎617 253 7014 MIT GRAVITY GRP. →→→ LIGO ☑046



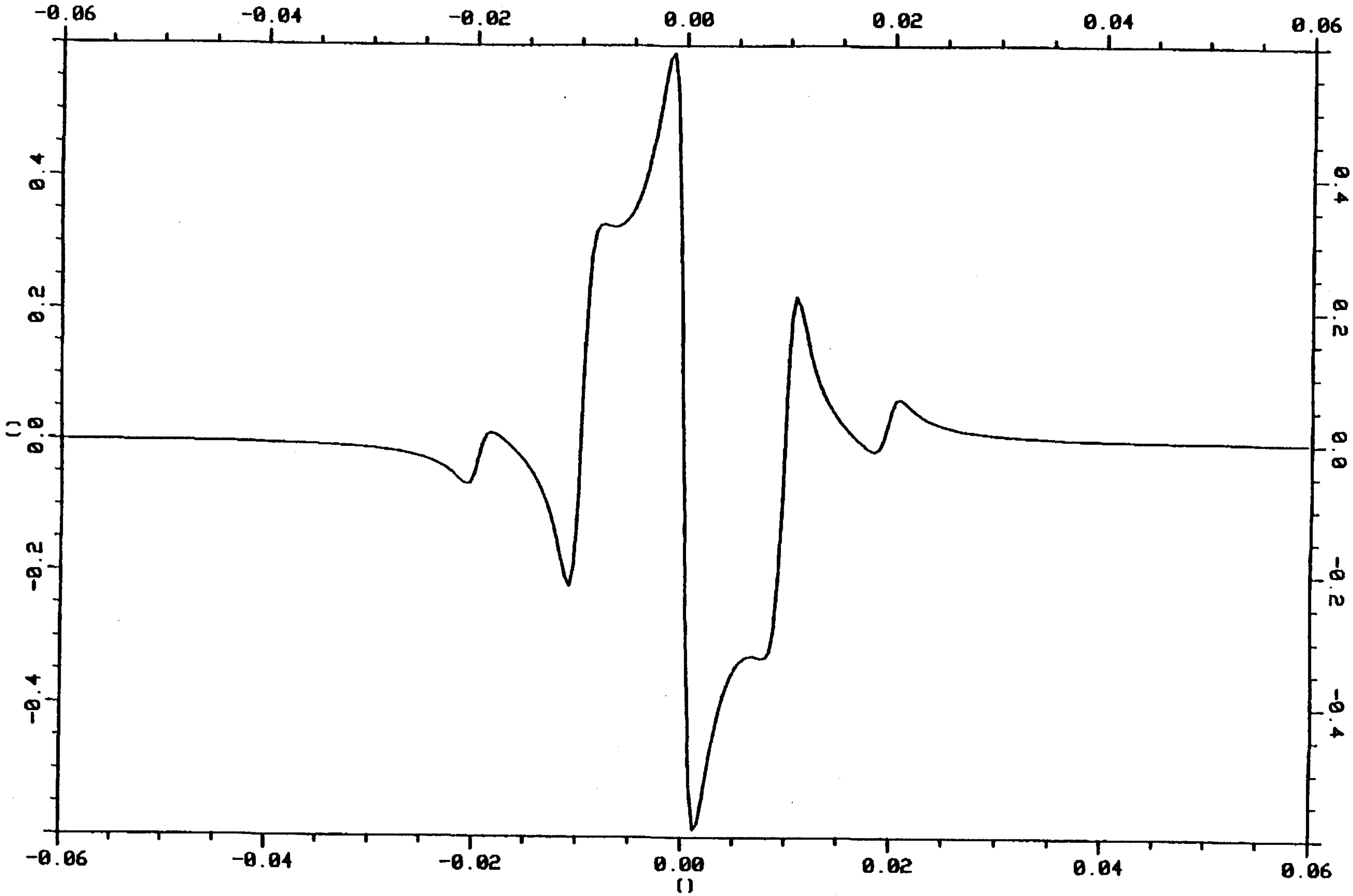
DISCRIMINANT AT GAMMA = .5 IN PHASE 2W

03/14/89 14:22 ☎617 253 7014 MIT GRAVITY GRP. →→→ LIGO ☑047

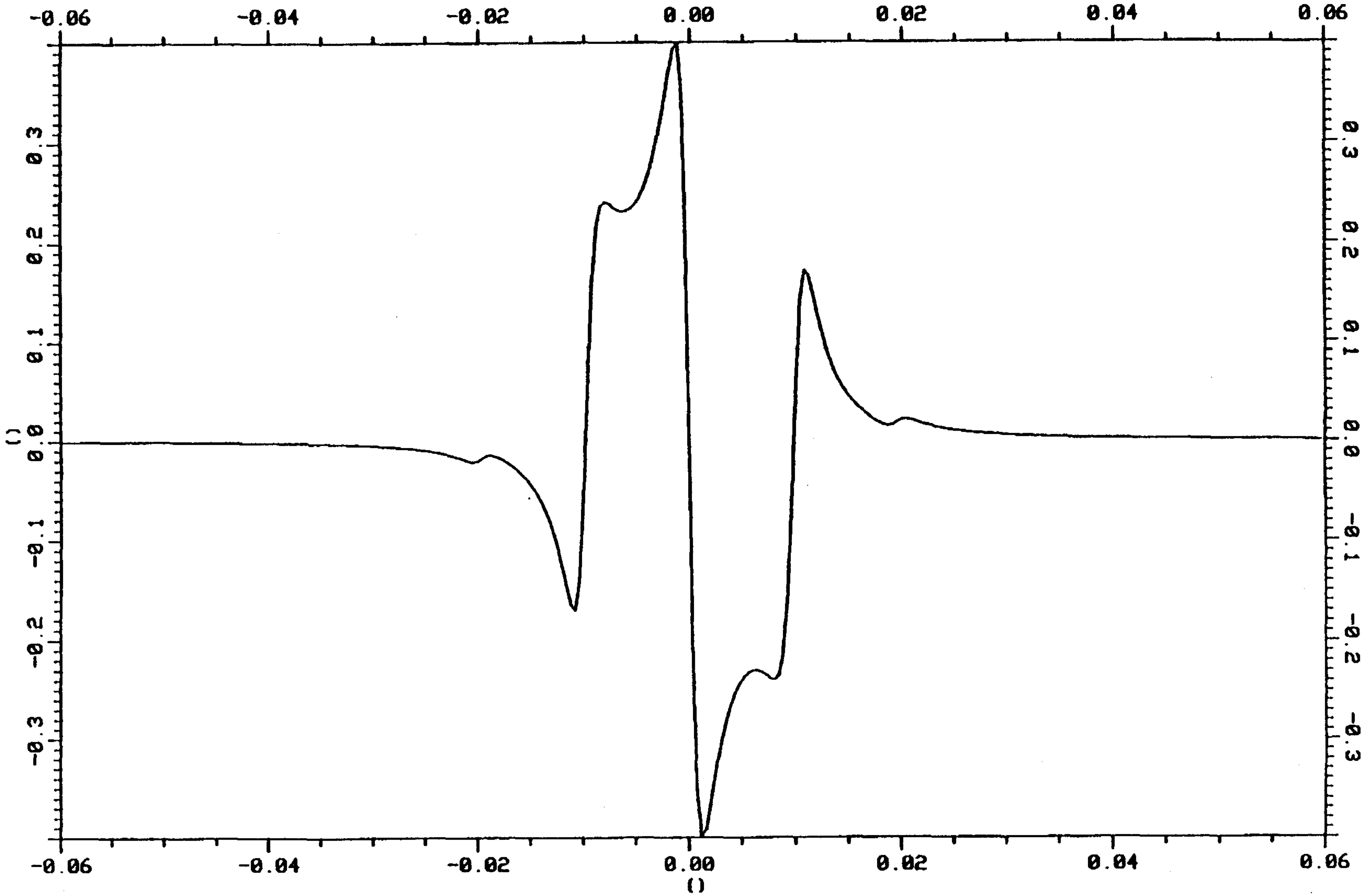


DISCRIMINANT GAMMA = .5 QUAD PHASE 2W

03/14/89 14:22 ☎617 253 7014 MIT GRAVITY GRP. → LIGO ☑048



ω in phase, $\Gamma = 1.0$



ω IN PHASE, $\Gamma = 0.5$

$= 5.38$ MHz

$A_1 = 1 \times 10^{-3}$

$T_1 = 2.85 \times 10^{-2}$

$A_2 = 2 \times 10^{-3}$

$\ell = 55$ cm