LIGO-T890021-00-R

44a

List of 3/1/89

FILE: Franzgrote 21689.tex

FROM: RW Feb 16 1989

TO: E. Franzgrote and W.Althouse

CONCERNING: The ingredients of the choice of 43 inches for a LIGO delay line at 1.06μ

The statement of 43 inches as a maximum size for a LIGO delay line interferometer does not come from rigid criteria as can be seen from curve 8a in the document on delay line scaling. The steps involved in the specific choice, using information available in 1986 and 1987, were the following.

1) An assessment of the mirror coating technology for 1 meter class mirrors was made. It seems reasonable to assume that coating houses could achieve reflectivities of .9997 to .9998 on large mirrors. This gives coating losses due to scattering and absorption between 1.5×10^{-2} to 1.0×10^{-2} for a 50 beam system.

2) Discussions with the optics industry indicated that mirror radius tolerances of $\Delta \approx 5 \times 10^{-3}$ were still within reasonable bounds of then current technology. This again indicated that 50 beams was a reasonable value to shoot for.

3) The choice for the mirror loss was made on the basis that there was no strong argument to make the mirror diffraction losses smaller than the coating loss, hence the choice of 10^{-2} diffraction loss mirrors. Figure 8a indicates that such a mirror, used with 1.06μ light, has a diameter of approximately 110 cm (43 inches).

Although the foregoing may sound logical (and it is on the basis of what the technology is able to do), it clearly would change if there are substantial developments in coating, grinding and testing techniques. At the time I chose parameters which could be backed by experience in industry. Another consideration was (and is?) that I did not want to propose more speculative things than necessary, in particular the concept of servoing the mirror radii, which would reduce the demands on the intrinsic mirror accuracy, and the astigmatic geometry, which could alter the mirror size for a fixed number of beam passes, had never been tried.

FILE:delayline.tex

TO: Franzgrote and Althouse

FROM: RW Feb 8,1989

CONCERNING: Delay line scaling

The purpose of this note is to give the scaling of a multipass delay line for the LIGO at the longest wavelength, 1.06μ, being contemplated. The delay line storage geometry at this wavelength sets the maximum scale for the diameter of the LIGO beam tubes when they are used for a single interferometer.

The note is organized as follows

1) An introduction describing the properties of the delay line
2) A paraxial ray analysis of the Herriott delay line
3) Criteria to establish mirror size and beam envelope
4) The gravitational wave transfer function of a delay line
5) Mirror materials
6) Mirror tolerances

INTRODUCTION

The delay line is one of a class of optical storage geometries (White cells, multiple mirror systems, tagged beams, etc) in which the individual light beams, that reflect between the storage cavity mirrors, retain their identity. The light beams do not have to satisfy a resonance boundary condition at the mirrors, as a consequence the intensity at the mirror surface is that of a single beam. Furthermore, in those geometries in which the beam separation is accomplished geometrically (all but the compact tagged beam system), there is no requirement that beams be transmitted through transparent mirror substrates. The frequency stability required of the light source in a system not using recycling is given by

$$\nu(f) < \frac{\nu_0}{2} h(f) \frac{\delta L}{L}.$$

L is the cavity length, δL is the difference in length of the two cavities, ν_0 is the frequency of the light and $h(f)$ is the limiting amplitude spectral density of gravitational wave strain.

Optical phase noise at the recombined output of the interferometer can occur in these geometries due to the interference of the main beam with scattered beams that have taken different times to arrive at the output of the interferometer. Besides the use of low scatter mirrors (which to date have not been applied to delay lines), two techniques have been developed to control the noise due to this scattering. One is to control the frequency of the light to better precision than that given above so that the phase difference between the light having taken a different time to arrive at the output than the main beam is held fixed. This technique reduces the scattering noise due to phase fluctuations of the scattered beams providing the scattering sources themselves are stationary. The other technique is to randomize the phase of all the beams by phase modulation of the input light. This technique reduces the scattering noise for both stationary and moving scattering sources. The (untried) tagged beam system accomplishes the same end by frequency shifting each beam.

The Herriott delay line geometry

Two delay line geometries have been employed in gravitational wave antenna prototypes, the Herriott delay line using spherical mirrors and a circular spot pattern; which was used in the MIT 1.5 meter system and in the German 3 meter and 30 meter system. The other is a White cell tried in the early days at Glasgow. The scaling relations will be given for the Herriott delay line geometry since there is substantial experience with it; and should a delay line be used in the LIGO, it will most likely have the Herriott geometry because of this heritage.

The paraxial geometric analysis of the Herriott delay line

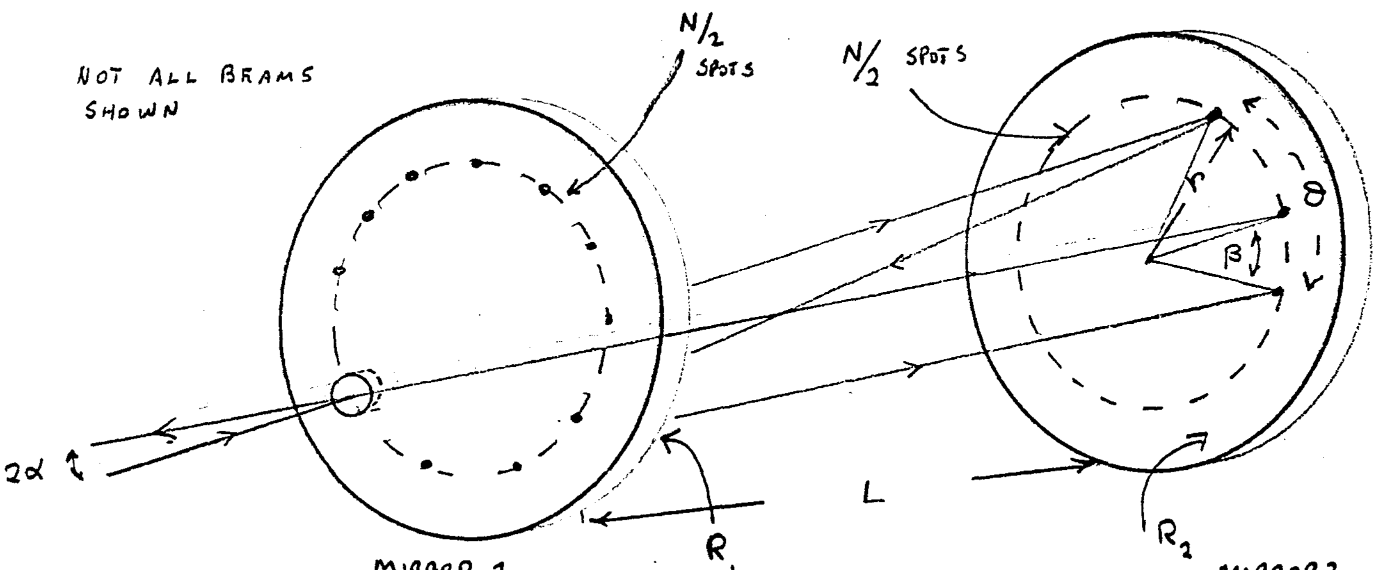


FIGURE 1

As shown in the figure above, light enters the delay line cavity through a hole in mirror 1 bounces back and forth between the mirrors 1 and 2 and exits by the same or another hole. The ray optics is most easily carried out in matrix formulation using the paraxial approximation (angles small enough so that $sin x = x$ and $cos x = 1$) which is appropriate for both the LIGO and the prototypes. The small effects due to spherical aberration, next order terms in $sin x$, are insignificant. The ray vector is defined as

$$\psi = \begin{pmatrix} r \\ dr/dz \end{pmatrix}$$

where r is the radial position of the beam relative to the z axis which is chosen to lie along the optical axis, the line connecting the centers of curvature of the two mirrors. dr/dz is the angle of the beam propagation with the z axis. The particular formulation assumes cylindrical symmetry about the z axis, the ray vectors and operator matrices can be written in x and y coordinates, if needed, to account for astigmatic mirrors and skew geometries. To determine most of the interesting results the additional complexity of defining 4 x 1 ray vectors and 4 x 4 operators is not needed. The geometric problem is defined by determining the transformation of the input ray vector into the output ray vector through operations of the optical elements

$$\psi(out) = (OPERATOR)\psi(in).$$

The basic operations involved are beam propagation and reflection given by the following operator matrices

$$P = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad \Re_i = \begin{pmatrix} 1 & 0 \\ -2/R_i & 1 \end{pmatrix}$$

where L is the length of the cavity and R_i is the radius of curvature of the spherical mirror i. The process of beam transformation from the input to the output of the delay line is written as

$$\psi(out) = P\Re_2(P\Re_1 P\Re_2)^{N/2} P\psi(in).$$

The operations read from right to left. The product in the brackets is repeated $N/2$ times where N is the number of passes of the beam in the cavity. The stability of the delay line geometry is determined by the product of operations in the brackets. After each back and forth traversal of the beam, the beam should be left in the same ray state except for a multiplicative constant, call it λ. The eigenvalue equation is

$$\psi(out) = (P\Re_1 P\Re_2)\psi(in) = \lambda\psi(in).$$

With the specific operations of the delay line the eigenvalue equation becomes

$$\lambda^2 - 2\left(1 - \frac{2L(R_2 + R_1)}{R_1 R_2} + \frac{2L^2}{R_1 R_2}\right)\lambda + 1 = 0.$$

This has bounded solutions for λ providing that

$$\left|\left(1 - \frac{2L(R_2 + R_1)}{R_1 R_2} + \frac{2L^2}{R_1 R_2}\right)\right| < 1$$

It is useful to define

$$cos(\theta) = \left(1 - \frac{2L(R_2 + R_1)}{R_1 R_2} + \frac{2L^2}{R_1 R_2}\right)$$

where θ is the advance angle of the optical beam around the beam spot circle, centered on the optical axis of the cavity, on mirror 1. The stability conditions are identical with those for the Fabry - Perot which is a degenerate case of the cavity geometry corresponding to injection and extraction of the stored light along the optical axis of the cavity.

The Herriott delay lines used in the gravitational wave prototypes, up to now, have a single hole in the input mirror through which both the input and output beams enter and leave the cavity. In the prototype systems the separation of the input and output beams is accomplished by injecting the input beam at an angle to the optic axis. The injection angle is

$$\alpha = r/L$$

where r is the radius of the circular set of spots on the mirrors and L is the cavity length. The output beam emerges from the hole at an angle $-\alpha$. The angle between the input and output beams is therefor 2α with the vertex of the angle lying on the curved surface of the input mirror at the center of the hole in the mirror. This particular configuration is called the reentrant delay line and has significant virtues. In particular, the ray paths outside of the cavity are independent of the position and angle of the far mirror so long as the beams still hit the far mirror. Another important property is that the optical phase of the output beam relative to the input beam varies only in second order with motions of either mirror transverse to the cavity optic axis; this is so both for rotations and translations of the mirrors. As a consequence, the Herriott delay line is not difficult to align initially nor is it difficult to maintain its alignment in operation. The reentrant geometry, imposes a further constraint

$$\theta N/2 = 2\pi m$$

where m is an integer. This constraint which is really a statement that besides cavity stability one also demands that the final beam traversal of the cavity hit mirror 1 at a specified location, sets the relation between L, R_1 and R_2 and establishes the mirror tolerances to be discussed later in this note.

Should a delay line be used in the LIGO, it will most likely not be a reentrant geometry since the input and output beam angular separation at the small angles in the LIGO would require long sections of auxiliary beam pipes to separate the input and output beams. The alternatives are to use polarization separation like that being planned for the Fabry-Perot which carries the additional hazard of scattering and power limitation, or to use a separate input and an output hole. The concept of using two holes in a LIGO scale delay line interferometer has been analyzed by Burka. There is a small sacrifice in the insensitivity to transverse motions of the mirrors but otherwise the system behaves much as the reentrant geometry. The dual hole system is proposed as the delay line design in the November 1987 Caltech/MIT proposal.

It is convenient, though not essential since the light storage does not depend on interference in the cavity, to mode match the optical beams to the delay line cavity - to set the optical beam Gaussian wavefront radius equal to the spherical mirror radius when the beam encounters the mirror. Mode matching is required at the exit pupil of the entire interferometer where the interference takes place, this can be accomplished with auxiliary optics after the light emerges from the cavities.

For Gaussian beams in the TE_{00} mode, the Gaussian beam propagation equations are

$$\omega^2(z) = \omega_0{}^2\left(1 + (z/z_0)^2\right)$$

$$R(z) = z\left(1 + (z_0/z)^2\right)$$

where $z_0 = \pi{\omega_0}^2/\lambda$, ω_0 is the $(1/e)^2$ radius of the beam at the place where the wavefront curvature, $R(0) = \infty$, the Gaussian focus. The value of ω_0 depends on the wavelength, λ, and the specific cavity geometry. The Herriott delay line that has been used in the prototypes is symmetric, spherical mirrors of equal radius at the two ends of the cavity. For this configuration the Gaussian focus occurs in the middle of the cavity, $z = 0$, and the mode matched condition is that $R(\pm L/2) = R_m$. The spot radius at the Gaussian focus is then

$$\omega_0 = \frac{1}{2}\left(\frac{\lambda}{\pi}\right)^{1/2}\left(\frac{2{R_m}^2 L}{(R_m - L/2)}\right)^{1/4}.$$

The spot radius on the mirrors is $\omega(\pm L/2) = \sqrt{2}\omega_0$. The spot radius for the 4 km LIGO is shown in figure 2 for both 1.06 and 0.53 μ as a function of R_m/L.

If the beam is not mode matched to the cavity, the spot size on the mirrors oscillates around the mode matched size with an amplitude bounded by $\frac{\Delta\omega}{\omega}$ where $\Delta\omega$ is the difference between the matched and unmatched spot size. The oscillation period and amplitude is the same for all deviations from the perfect matched case; for example, if the input beam is matched to one mirror but the mirrors have different radii of curvature, the oscillation of the spot size is bounded by

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta R_m}{R_m}.$$

The number of cavity beam passes needed to go through one period of the beam size oscillation, if mismatched, is given by

$$n = \frac{N\theta}{2\left(1 - \frac{2L(R_2+R_1)}{R_1 R_2} + \frac{2L^2}{R_1 R_2}\right)}.$$

Criteria to establish mirror size and beam envelope

1) Power lost by a TE_{00} Gaussian beam in passing through an aperature of finite size.

The intensity of the light as a function of radius, r, is given by

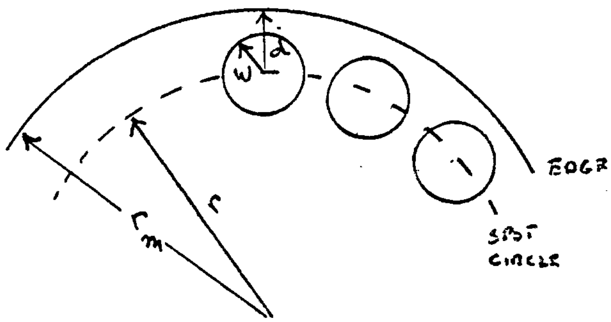
$$I(r) = \left(\frac{2P_T}{\pi\omega^2}\right)e^{-2(\frac{r}{\omega})^2}$$

where ω is the Gaussian spot radius and P_T is the total power in the beam. The power lost is that cut off by the edge of the hole, the integral of the intensity over the area lost

$$\frac{P(r > R)}{P_T} = 2\int_{\sqrt{2}R/\omega}^{\infty} xe^{-x^2}\,dx = e^{-2(R/\omega)^2} \qquad x = \sqrt{2}r/\omega$$

R is the radius of the hole. The fractional loss of power vs normalized hole radius is shown in figure 3.

2) Power lost at the mirror edge



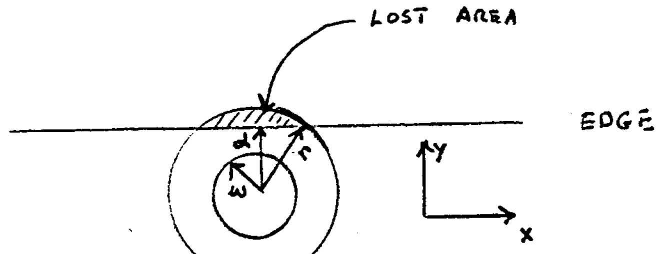


FIGURE 4

The power lost at the delay line mirror edge in the geometry shown above (fig 4) is most easily calculated in rectangular coordinates in the approximation that the beam is small enough so that the mirror edge can be approximated as flat. The fractional power lost over the mirror edge which is a distance, d, from the center of the Gaussian beam is

$$\frac{P(lost)}{P(total)} = \left(\frac{4}{\pi\omega^2}\right) \int_d^{\infty} \int_0^{\infty} e^{\frac{-2(x^2+y^2)}{\omega^2}} dxdy = \left(\frac{2}{\pi}\right)^{1/2} \int_{d/\omega}^{\infty} e^{-2z^2} dz \qquad z = y/\omega$$

Figure 5 shows the integral as a function of the normalized distance d/ω.

3) Power leaked through a hole in the mirror from a neighboring beam spot on the mirror.

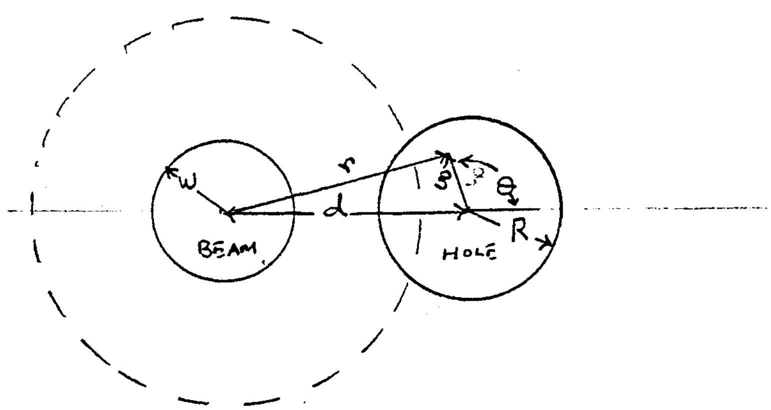


FIGURE 6

The geometry is shown in figure 6 above. The distance between the beam spot center and the center of the neighboring hole is d. The hole size is R while the beam has a Gaussian spot radius ω . The integral over the hole is easiest done by defining

$$r^2 = d^2 \left(1 - 2\frac{\rho}{d}cos(\theta) + (\frac{\rho}{d})^2\right).$$

The fractional power leaked by the hole is

$$\frac{P(hole)}{P(total)} = \frac{2}{\pi} e^{-2y^2} \int_0^{2\pi} \int_0^{R/\omega} e^{-2x^2} e^{4xycos(\theta)} xdxd\theta \qquad x = \rho/\omega \qquad y = d/\omega$$

The integral is a misery. The results of a numerical integration for 5 values of R/ω which correspond to holes with a transmission loss on beam axis (figure 3) of 10^{-1} (R/ω = 1.08), 10^{-2} (1.52), 10^{-3} (1.86), 10^{-4} (2.15), 10^{-5} (2.40) is shown in figure 7.

The mirror size and beam spot envelope is determined by the loss and leakage that can be tolerated and the storage time (number of beam passes) desired. In the circular beam spot pattern, the geometric constraint is given by

$$R_{mirror} = \omega_{mir}\left(\frac{Nf}{4\pi} + g\right)$$

R_{mirror} is the radius of the mirror and therfore also the envelope of the beams at the ends of the beam pipes. The beam envelope becomes close to $\omega_{mir}/\sqrt{2}$ at the center of the cavity where all the beams cross. N is the total number of beams in the cavity, there are $N/2$ spots on each mirror. f is the hole to nearest spot spacing (center to center) in units of ω which is chosen for a specified leakage and g is the spot center distance from the mirror edge in units of ω determined by the amount of acceptable edge loss. This loss given by figure 5 per beam must be multiplied by the number of beams to determine the cavity loss. Various cases are shown in figure 8 for both 1.06 and 0.53 μ.

The gravitational wave transfer function of an antenna composed of two cavities is given in terms of the optical phase shift derived per gravitational wave strain amplitude as a function of gravitational wave frequency, f. The transfer function for the delay line cavity is

$$\frac{\phi(f)}{h(f)} = 2\pi N\frac{L}{\lambda}\left(\frac{sin(\frac{\pi fNL}{c})}{\frac{\pi fNL}{c}}\right)$$

The magnitude of the transfer function divided by L/λ is shown in figure 9 for a 4 km length as a function of gravity wave frequency for $N = 20, 40, 60, 80$.

The mirror size to achieve a specific storage time using a delay line system is larger than for a Fabry Perot. The circular spot pattern, that has been used in the prototypes and is analysed above, is not the most efficient pattern if long storage times are desired. The number of beams stored in the delay line can be increased for fixed mirror and tube size by making the mirrors astigmatic - by breaking the cylindrical symmetry of the mirror and giving the mirrors different radii of curvature for the x and y dimensions. The result of doing this is that the beam advance angles in x and y are no longer the same so that the spot pattern becomes a Lissajous figure on the mirrors bounded by the injection angles in x and y providing that the stability condition is separately satisfied for both x and y beam motions. Delay lines of this kind have been demonstrated but not used in the prototypes. The reason why this concept has not been further developed is twofold; first, there was no need to increase the complexity of the delay line prototypes and, second, the large mirrors with astigmatic surfaces for a large baseline system would be more difficult to grind and figure than spherical ones. If delay lines are ultimately to be used in the LIGO and longer storage times are desired, this concept should be revisited.

Mirror materials

The fact that the mirror substrate need not be transparent removes one constraint in the choice of the allowed materials. The design tradeoff still include the following.

1) Material must be able to polished to a microroughness of an Angstrom or less to reduce scattering.

2) The substrate should have a thermal expansion coefficient that matches hard coatings such as SiO_2 and TiO_2 that have been applied to make low loss and low scatter surfaces.

3) The substrate material should have high transverse and longitudinal sound speeds and low internal dissipation so that the resonant modes of the mirror fall outside of the gravitational wave detection band and that the off resonance thermal noise lies below the other noise terms in the antenna noise budget.

4) The substrate material should have a high thermal conductivity and as low as a thermal expansion coefficient as possible commensurate with 2) to reduce thermal wavefront distortion due to coating absorption.

Besides fused quartz, Beryllium and Silicon (amorphous and single crystal) are two substrate materials that look promising. There is experience in the optics industry with both Silicon and Beryllium in grinding and testing the almost flat mirrors on the meter scales that would be needed for a LIGO delay line.

Mirror tolerances

The most restrictive geometric conditions on the delay line mirrors come from the desire to match the two cavity storage times while simultaneously requiring the beams to emerge through the output hole of mirror 1. The storage time equality could be relaxed but this puts a larger burden on laser frequency stabilization and restricts the type of phase modulation that can be applied to the input light to control scattering. The analysis below evaluates the mirror tolerances that are permitted for monolithic passive mirrors (not actively controlled) so that it is still possible, by adjustment of cavity length and beam input and output optics, to bring $\Delta L_{opt}/L_{opt}$, the fractional difference in optical path length of the two cavities, to 0.

The total optical path, s, in the delay line cavity is

$$s \approx LN(1 + (r/L)^2(1 - cos(\theta/2))$$

Neglecting terms of order r/L the change in optical path with L is

$$\frac{\Delta s}{\Delta L} = N$$

The spot pattern on mirror 1 are $N/2$ beams equally spaced around the circle of radius, r, separated by an angle, $\beta = 4\pi/N$. The beam advance angle around the circle is θ , which depends on R_m/L, and is usually larger than, β, the angle between neighboring beams on the spot circle. For example, in a delay line with $N = 34$ and choosing R_m/L of .7851; the advance angle is 211.76 degrees while the angle between neighboring spots on mirror 1 is

21.17 degrees. The spots that neighbor the input/output hole are the 10th and 24th pass of the cavity.

The critical parameter that sets the allowed tolerances on the mirror is R_m once L and N have been chosen. Suppose that one cavity has been aligned and set to have length L with N passes using mirrors with average radius $R = (R_1 + R_2)/2$. What are the allowed tolerences of the mirrors in the second cavity so that $\Delta L = 0$ by adjusting only the input injection angle?

Let the two mirrors have radii

$$R_1 = R(1 + \Delta) \qquad \text{and} \qquad R_2 = R(1 - \Delta)$$

The change in the advance angle with Δ is

$$\delta\theta = \left(\frac{4L}{Rsin(\theta)}\right)\left(2 - \frac{L}{R}\right)\Delta.$$

To ease the algebra with no significant loss of information it is useful to approximate, for estimation purposes, that $L/R \approx 1$ and that one picks cases so that $sin(\theta) \approx 1$.

$$\delta\theta \approx 4\Delta$$

The allowed range in the offset angle determined by the input injection conditions is 2β before N is changed, the tolerance in R that can be accomodated to still maintain $\Delta L = 0$ is given by

$$\frac{N\delta\theta}{2} < 2\beta \qquad \text{or} \qquad \Delta < 4\pi/N^2$$

A convenient means of relating the mirror tolerance to optical shop practice is to give it in terms of permitted uncertainty in the mirror sagitta. The mirror sagitta is

$$h = \frac{r^2}{2R} \qquad \text{so that} \qquad \frac{\delta h}{h} = \frac{-\delta R}{R} = -\Delta$$

The sagitta of a mirror with $R = 4$ km and $r = 50$ cm is 3×10^{-3} cm or about 30 wavelengths at 1.06 μ. The allowed uncertainty in sagitta for, say, $N = 30$ is about 0.4 wavelengths. The optics industry (Perkin Elmer, Kodak and Itek), when consulted about this requirement in 1986, saw no problem in this specification. It is nevertheless uncomfortable to have to get something "right" even when it is not pushing the state of the art. For this reason, should delay lines become candidates for use in the LIGO, serious consideration will again be given to techniques to allow small perturbations to be made to the mirror radius by servo control of mirror temperature or application of electrostatic forces, this would become especially important if N becomes larger than 100.

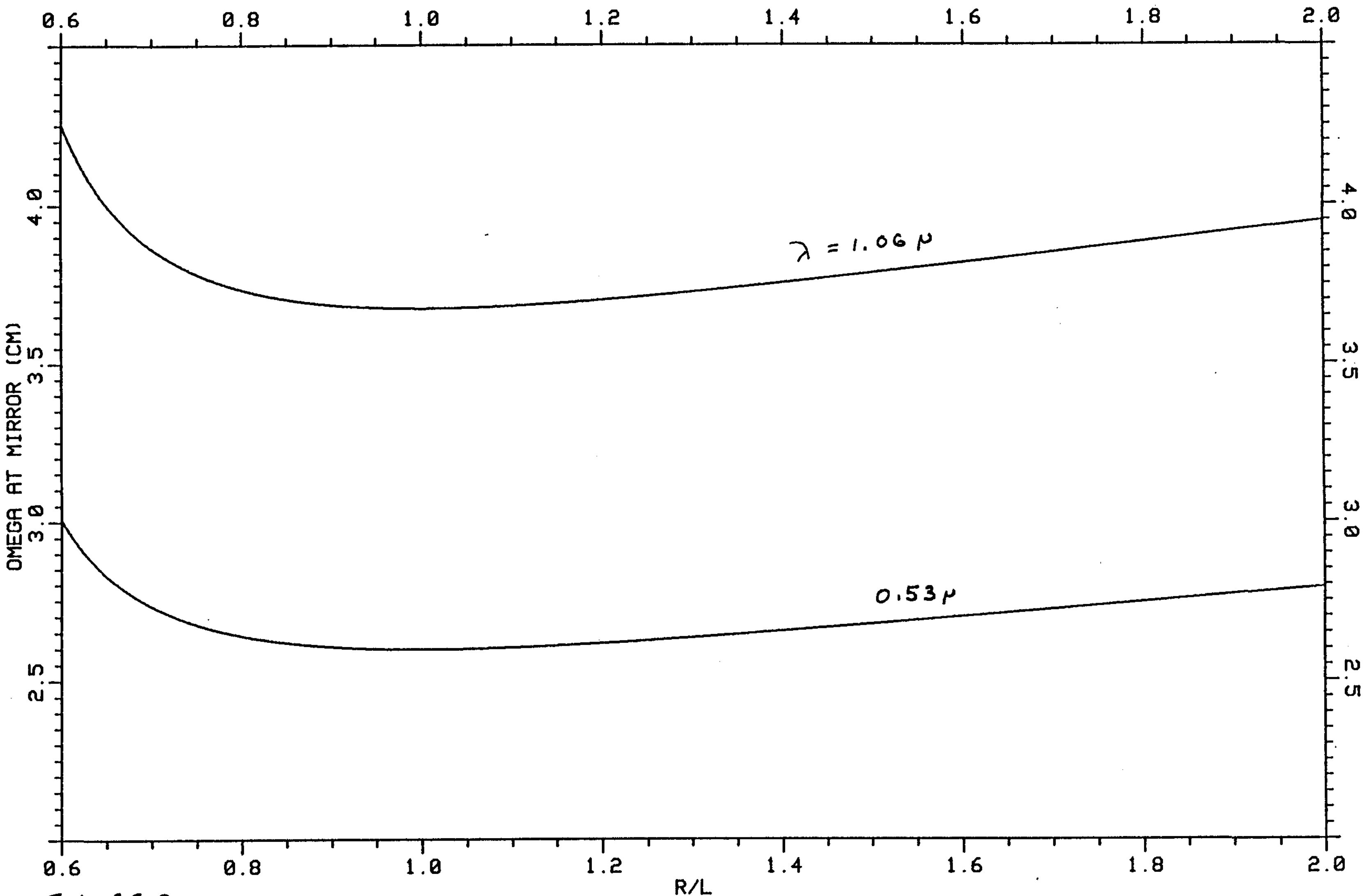


FIGURE 2

Spot size in a 4 km symmetric cavity at 1.06 and 0.53 microns

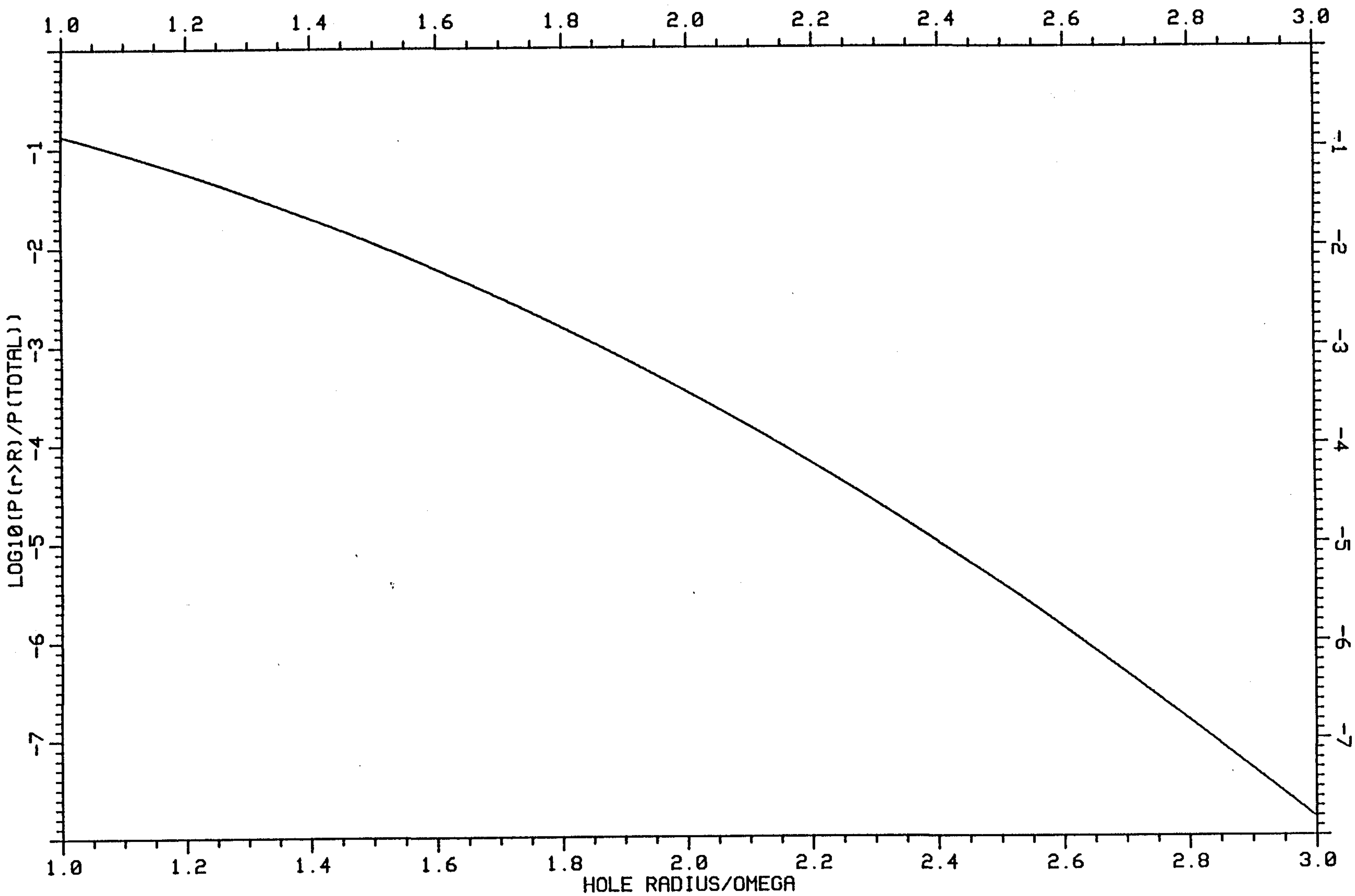


FIGURE 3

FRACTION OF POWER LOST DUE TO DIFFRACTION VS HOLE

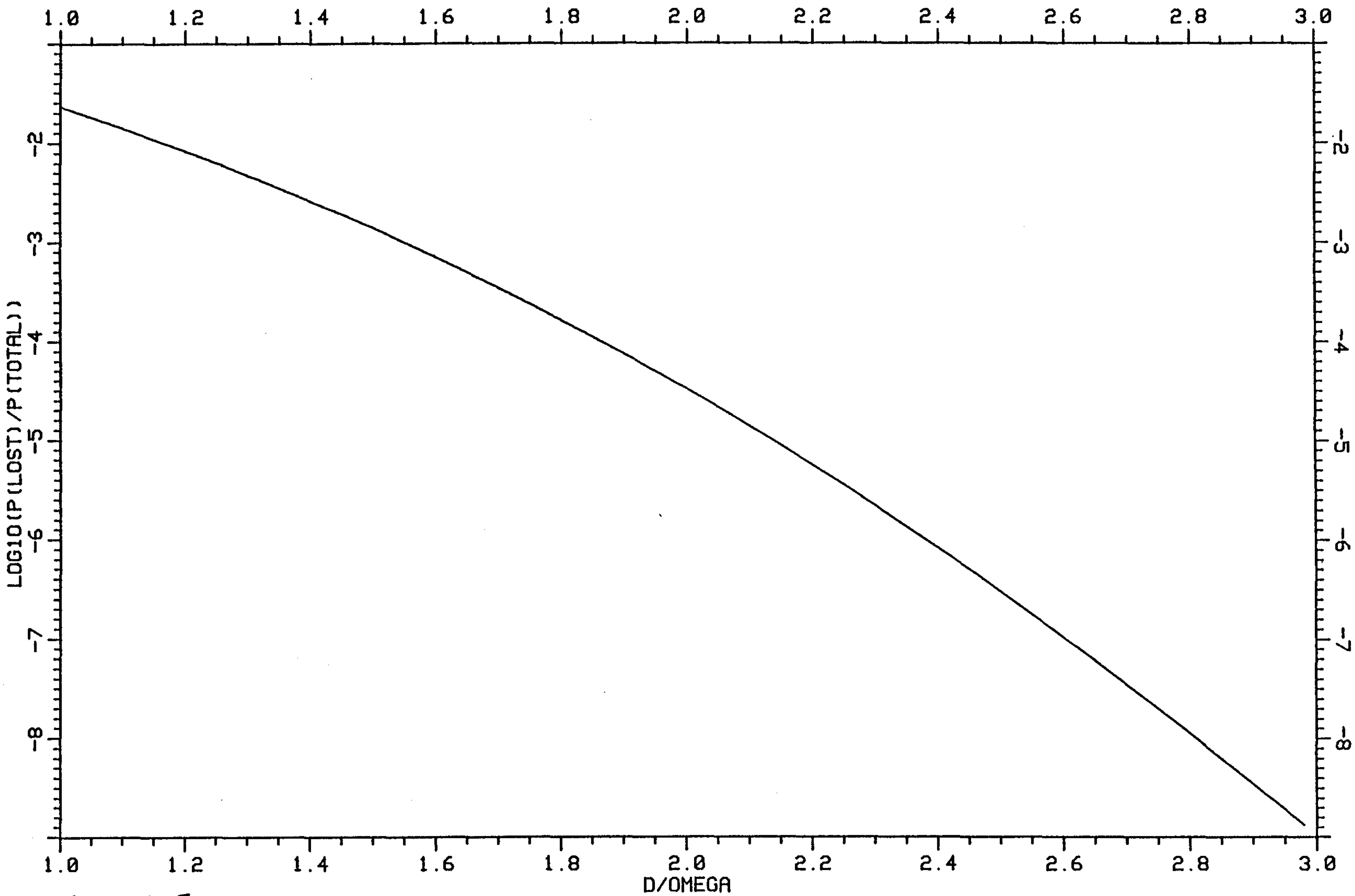


FIGURE 5

LOSS AT EDGE OF MIRROR

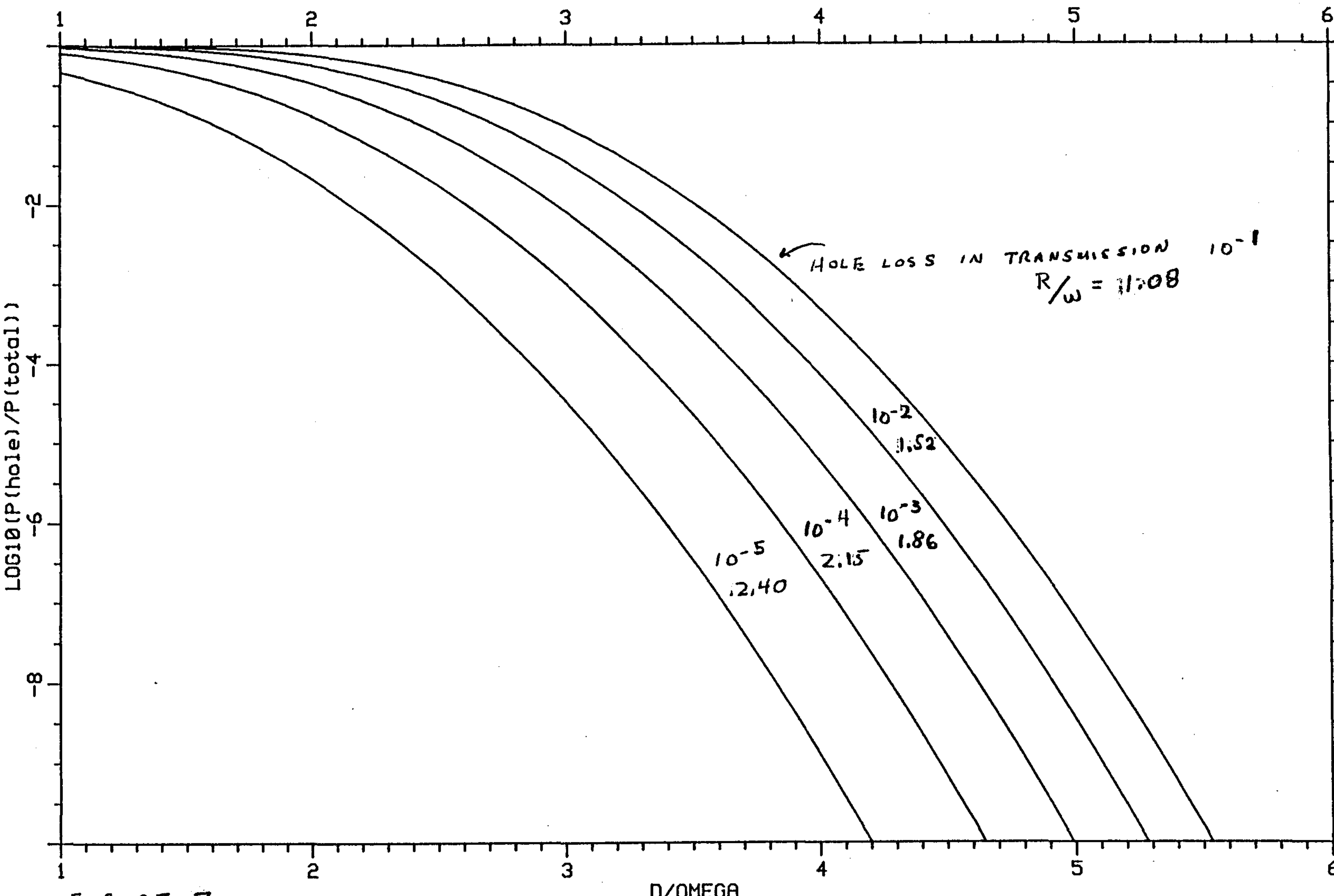


FIGURE 7

POWER LOST/TOTAL POWER IN HOLE RADIUS R/W(0) AT A

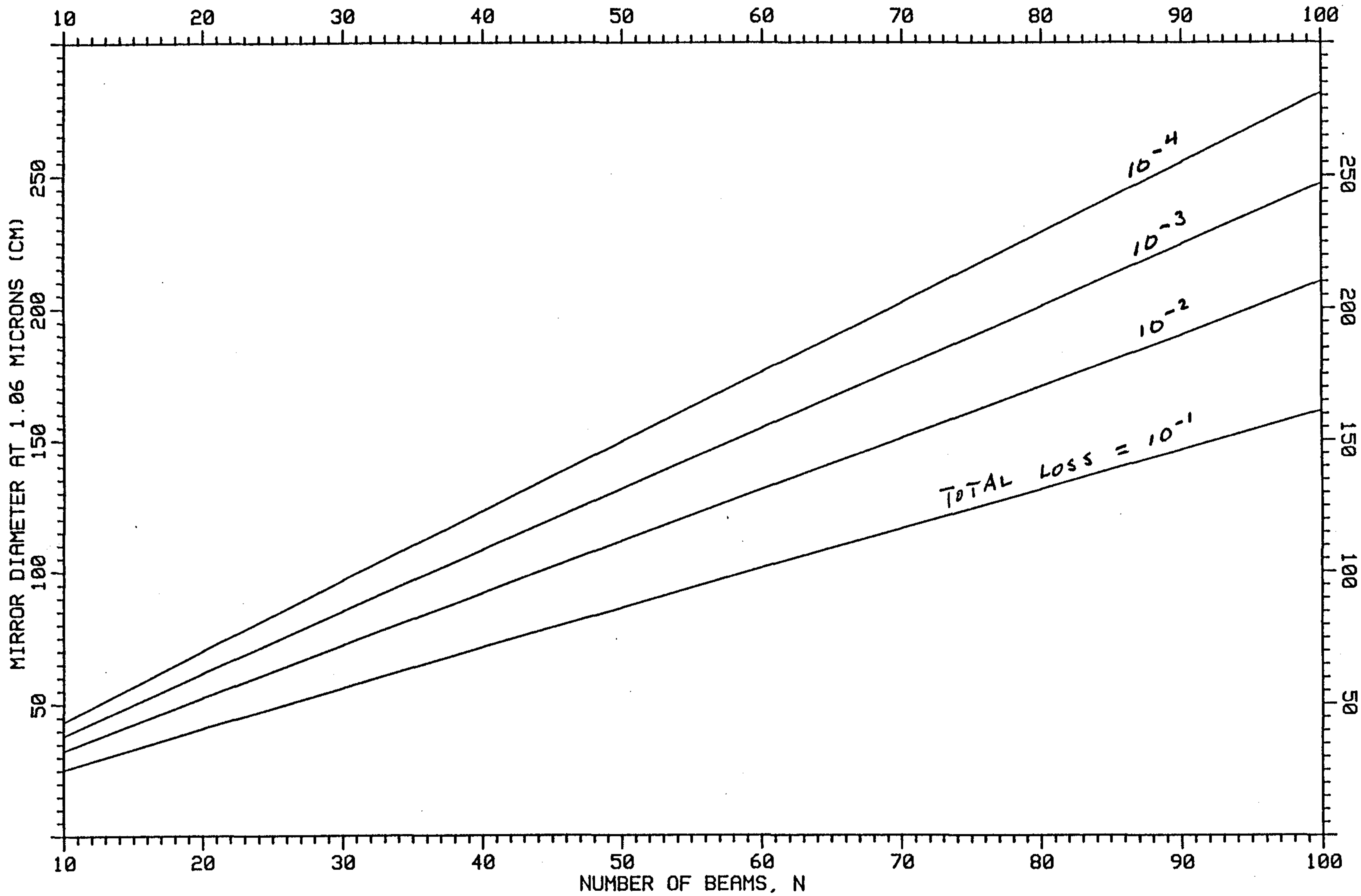


FIGURE 8a

MIRROR DIAMETER VS NUMBER OF BEAMS AT 1.06 MICRONS

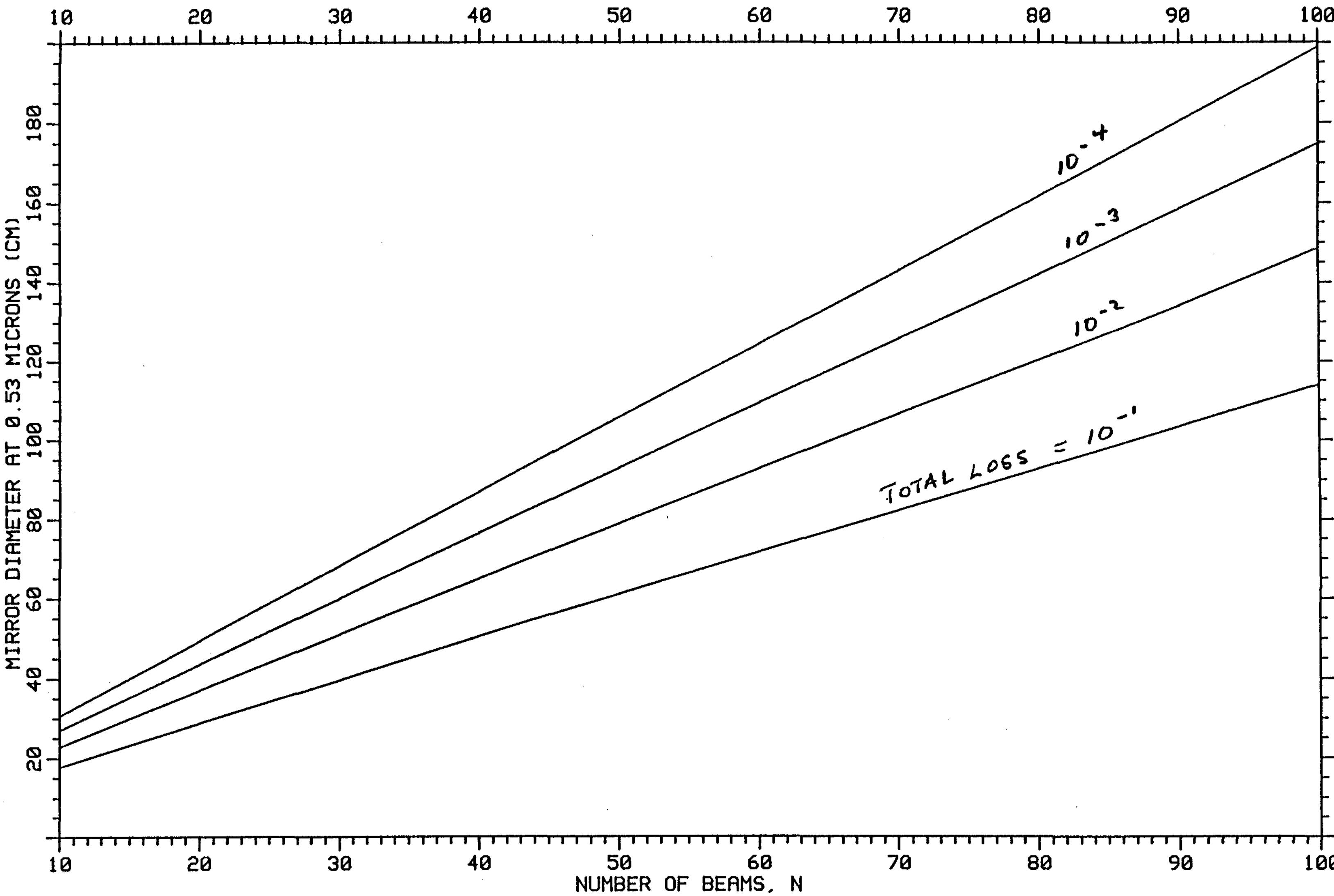


FIGURE 8b MIRROR DIAMETER VS NUMBER OF BEAMS, N AT 0.53 MICRONS

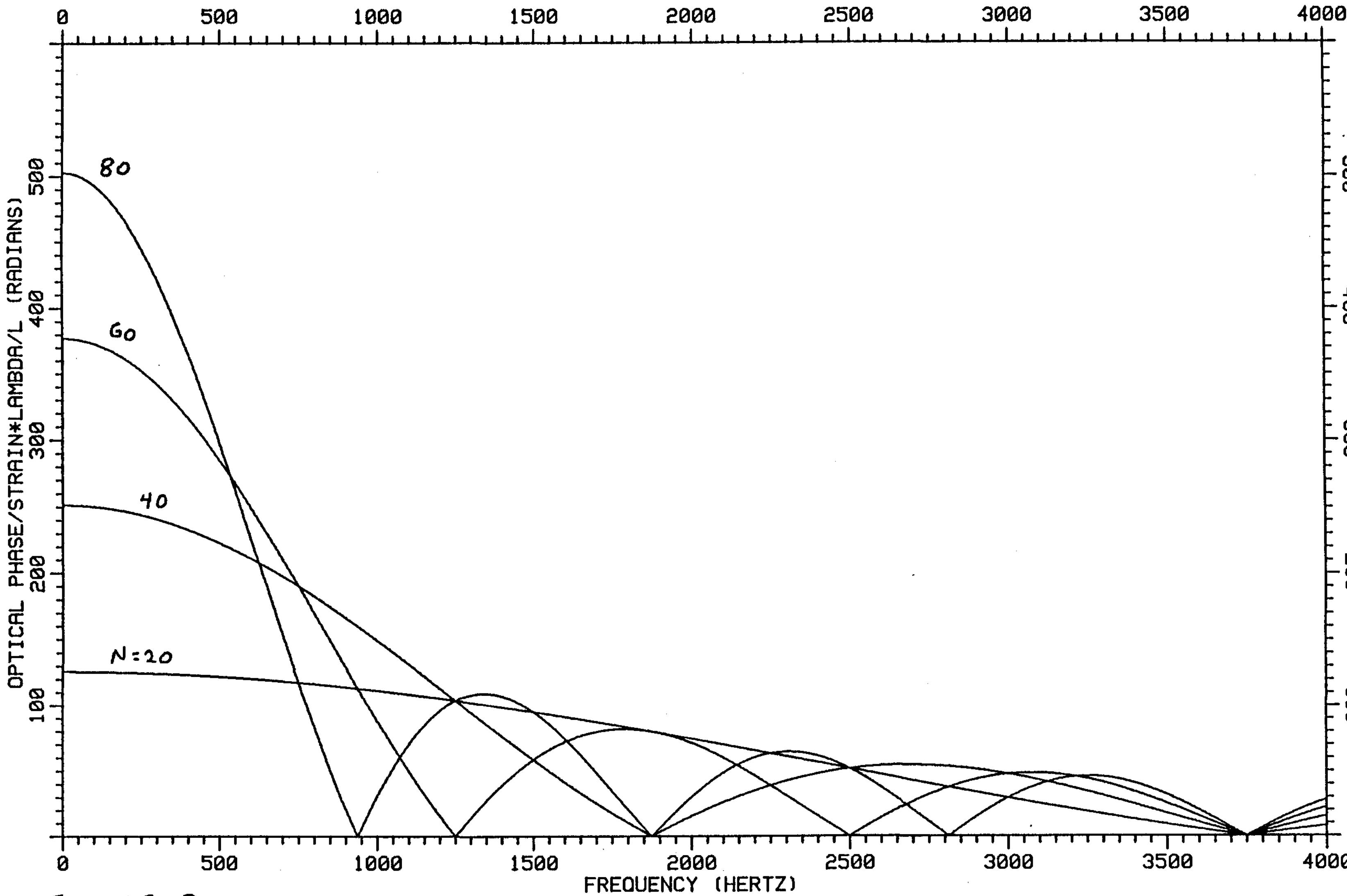


FIGURE 9

TRANSFER FUNCTION OF DELAY LINE