

# LIGO Technical Report

**TR-89-1**

## **DRAFT OF SITE CHAPTER FOR LIGO DESIGN HANDBOOK**

Peter R. Saulson

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# Draft of Site Chapter for LIGO Design Handbook

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“XX” indicates numbers which are TBD.

## 1 Number of Sites

**SPECIFICATION**      The initial LIGO will comprise two widely separated sites within the United States.

### RATIONALE

Of the three classes of source—impulsive, stochastic, and periodic—impulsive sources offer the best chance for early detection. Coincident detection at widely separated sites is the only way to demonstrate unequivocally that an impulsive signal is not due to local phenomena. Equivalent signals from separated antennas give the strongest discrimination both against external noise sources and internal glitches. Multiple interferometers at the same location provide some discrimination, and interferometers of different length at the same location will tag the gravitational wave signature of effect proportional to arm length. But interferometers at one location will never be able to answer every skeptic's questions in the same way that spatially separated interferometers will.

Detection of stochastic sources requires signals from two antennas whose only plausible cross-correlation is due to gravitational waves. Again geographical separation is the key, although here with a price, since as interferometers move farther apart the resulting phase shifts make the cross-correlation experiment sensitive over a smaller region of the sky.

Detection of steady monochromatic gravitational waves is the only sort of measurement which could be plausibly made with a single site. The modulation of source frequency and amplitude due to the Earth's motion will be an unambiguous signature of a genuine gravitational wave, given sufficient signal-to-noise ratio.

The requirement for two sites could conceivably be satisfied with one detector in the United States and another in Europe or elsewhere. However, practical considerations demand two separated sites operated under common management. The construction and operation of two sites as a single national observatory assures that

- At least two sites will exist.
- The receivers will be built on the same time-scale.
- They will have matched sensitivity and frequency range.
- They will be operated on the same schedule for coincident data collection.

Detection of gravitational waves requires only two sites, but full extraction of astrophysical information from the signals requires at least three. Three sites allows determination of the location of the source on the sky, and the reconstruction of the complete waveform including the polarization. See Tinto and Gursel. (There is some limited ability to do this with two sites if there is locally determined polarization information from "orthogonal" 45 degree antennas. See site committee report.) For this reason, construction of additional antennas elsewhere in the world would be extremely advantageous scientifically. If no interferometers were built elsewhere, (or if built but not well coordinated in a network) it would be necessary to build a third American antenna to move beyond the detection stage to the era of doing mature science.

## 2 Size

### SPECIFICATION

1. The LIGO vacuum system will be in the shape of an "L" with two arms of length 4.0 kilometers (two arms equal to a precision = 2.5 cm).
2. The angle between the two arms will be 90 degrees (absolute tolerance = 15 degrees, two sites equal to a tolerance = 15 degrees).
3. The land under control of the LIGO project shall consist, at a minimum, of a strip XX feet wide along the line-of-march of the vacuum pipe, plus:
  - (a) XX square feet at the vertex of the L
  - (b) XX square feet at each end of the L
  - (c) XX square feet near the mid-point of each arm of the L (absolute tolerance 0.5 km, two arms equal to 2.5 cm, two sites need not be closer than the absolute tolerance spec.) (The absolute tolerance spec is a new number and needs to be thought over carefully in light of new quantitative analyses of mid-station concept.)
  - (d) XX square feet, at least XX feet from each of a., b., and c., for use as a power substation and cooling plant.

(The numbers not yet specified in the foregoing section are to be determined by the LIGO engineering staff, with the exception of the distance spec, which will be determined by the project after a recommendation of the Vibration Isolation Working Group.)

4. At least one of the sites shall be suitable (see section 6, below) for the addition of another L oriented 45 degrees (tolerance = 10 degrees) to the first L, with the land for such an addition either under control of the LIGO project or, in the judgement of the project, likely to be made available on reasonable terms.

## RATIONALE

### 2.1 Arm Length

The size 4.0 kilometers can't be determined precisely by scientific arguments, but there are several arguments that show it is about right. An astrophysical signal of strength  $h$  moves test masses separated by  $L$  an amount  $\Delta x = hL$ . Interferometer length  $L$  affects sensitivity relative to two classes of noise. First, most noise enters as forces on the test masses. This type of displacement noise is independent of  $L$ , while the signal  $\Delta x$  increases with  $L$ ; therefore the sensitivity of detectors dominated by test mass noise is proportional to  $L$ . Because the largest sources of test mass noise—seismic noise and thermal noise on the suspensions—fall steeply with frequency, these noise terms dominate at low frequency.

The accompanying graph shows the projected noise spectrum of the LIGO Mark I receiver at the design length of 4 kilometers, and at several shorter lengths. (The noise model is that presented to the NSF site visit committee in February, 1988.) The degradation in sensitivity due to test mass noise over the key 100 Hz to 1 kHz band is evident at shorter lengths. Clearly, even longer arms would improve the situation. A natural maximum comes when the round trip time in the arms reaches half the period of the gravitational wave signal. For 1 kHz, this is a length of 150 km, while for 100 Hz it corresponds to 1500 km.

A second class of noise, shot noise in the optical system, limits sensitivity at high frequency. The shot noise sensitivity is independent of  $L$  in a non-recycled system, but is proportional to  $\sqrt{L}$  in a system with standard recycling, and proportional to  $L$  with periodic recycling (in a simple model where the recycling gain is limited by reflectivity of the mirrors).

The curvature of the earth sets a length beyond which some noise will grow. As explained in more detail in the section on slope, below, pendulum suspensions are used to reduce the thermal noise in the horizontal direction. When the optic axis is not accurately horizontal, the larger vertical noise is sensed by the interferometer. Since the earth is curved, a long interferometer can not be level everywhere. If the vertical thermal noise is 300 times larger than the horizontal, then the arms can not be increased

beyond 40 km without also increasing the thermal noise contribution. More details are given below.

So why choose 4 km instead of 40 km? Mostly for practical reasons. One has to do with the availability of sites. In the United States there is probably no place other than the Great Salt Lake Desert which is flat enough to accommodate a very long interferometer (greater than, say, 5 km) without an inordinate amount of earthwork. The Columbia site can barely accommodate 4 km.

Cost is itself the strongest reason for restricting the length. The total cost of a 40 km antenna would be prohibitive. A cost model which distinguishes between costs which grow in proportion to antenna length and those which have to be paid independent of the length gives a natural economic length to the LIGO. A vacuum system which is so short that the cost is dominated by the length-independent costs is too short. A vacuum system which is so long that the length-independent costs are negligible, so that it costs an extra factor of two to buy an additional factor of two in low frequency sensitivity, is probably too long. Although the details of the argument depend on the specific LIGO design and cost estimate, we have consistently found that 4 km falls in between the two extremes.

From the flavor of the preceding argument, it is clear that the precision with which we specify the absolute length of 4.0 kilometers is not given by scientific arguments alone. Rather we want to choose the largest installation which is feasible (however feasible is determined). Also, we must guard against being pushed down the slippery slope of trading a bit of length at a time to solve budget problems as they arise.

The tolerance in arm length match at a single site is supposed to represent what is readily achievable with present-day standard laser distance measurements used by surveyors. (If this number is incorrect, please change it.)

If we match the distance between the vacuum chambers to a tolerance of 2.5 cm, then we will be able to match the distance between the mirrors of the optical cavities to high precision (better than a wavelength of light) by moving the masses within the chambers to eliminate any residual differences. This adjustment is readily accomplished with motorized stages. Mechanical designs calling for adjustment range much larger than 2.5 cm are more complicated.

Equality of arm length is a significant advantage for some interferometer designs, though it is not critical for early LIGO interferometers. A length mismatch can couple laser frequency noise to the interferometer output; interferometers made with arms of matched length and equal number of bounces are insensitive to such noise. More precisely, the noise is

$$h(f) = \frac{\Delta\nu(f)}{\nu_0} \frac{\Delta\tau_{\text{stor}}}{\tau_{\text{stor}}} \frac{1}{S},$$

where  $\Delta\nu(f)$  is the frequency noise spectral density,  $\nu_0$  is the frequency of the light,  $\tau_{\text{stor}}$  is the average optical storage time of the cavities,  $\Delta\tau_{\text{stor}}$  is the difference in the storage times of the two cavities, and  $S$  is a suppression factor resulting from common-mode electronic or mechanical subtraction of the frequency noise. So far the 40-m prototype has achieved  $\frac{\Delta\nu(f)}{\nu_0} = 8 \cdot 10^{-21} \text{ Hz}^{-1/2}$ , and  $S = 500$ , indicating that  $\frac{\Delta\tau_{\text{stor}}}{\tau_{\text{stor}}}$  can be of order unity without compromising the sensitivity of early LIGO detectors.

The tolerance on the match of the arm length at the two sites has been set no tighter than the overall design tolerance. (Perhaps there is an engineering reason to match them more closely.)

## 2.2 Opening Angle

We use two arms because we reject frequency noise by “subtracting” the outputs of the two arms, either electronically or preferably by interference (“recombination”). Two arms at 90 degrees give maximum sensitivity to gravitational waves, because their characteristic tensor polarization causes opposite effects in orthogonal directions. The signal is proportional to the sine of the angle between the arms (so parallel arms give zero sensitivity to gravitational waves). More explicitly, we have

$$S = \left( \frac{1}{2} (1 + \cos^2 \theta) \sin 2\phi \cos 2\psi + \cos \theta \cos 2\phi \sin 2\psi \right) \sin \alpha,$$

where  $\theta$  is the source’s zenith angle,  $\phi$  is the source’s azimuth measured from the bisector between the two arms,  $\psi$  is an angle specifying the orientation of the source’s polarization, and  $\alpha$  is the opening angle between the two arms.

The tolerance of 15 degrees on the perpendicularity of the arms is determined by our (somewhat arbitrary) decision not to sacrifice more than 3.4 percent of sensitivity to this effect. (If we used up all of the site committee's 10 percent performance margin in angle alone, which wouldn't be advisable, we could tolerate plus or minus 25 degrees. At 5 percent, we have plus or minus 18 degrees.)

If there is some engineering reason we can think of, we can set a tolerance on the match in this angle between the two sites.

### 2.3 Area of Land Required

The width of the strip along the antenna line is primarily determined by consideration of the need for room during construction. If we want a permanent road along the antenna, this should also be factored in.

The size of the lots near the vertex, ends, and middles of arms is to provide room for buildings.

The question of a minimum distance between the power plant area and the instrument buildings is still TBD. Also not determined is the extent to which we want to pad the size of the lots around the instrument buildings to create a noise buffer, as opposed to assuming that we will take care of external noise by proper siting in the first place.

### 2.4 Expansion

The idea behind considering future addition of a second antenna at 45 degrees to the first is to determine complete gravitational waveforms, which requires polarization information. The site committee report pointed out the intimate connection between waveform solution and source position determination. The site committee compared simultaneous solutions from two sites, both of which had two "orthogonal" antennas, with solutions obtained from three sites, one of which had an extra antenna. Although the two site method works to certain degree, it is far inferior in precision and robustness. For this reason, the committee felt it was an undue burden on the project to require the capability to build orthogonal antennas at both sites. It seemed reasonable to expect that either a third antenna will be built in Europe or elsewhere anyway, or that the successful discov-



ery of gravitational waves by the two site LIGO would generate enough enthusiasm so that construction of a third site would be accomplished.

Subsequent work by Tinto and Gursel has shown how to get a complete solution from three sites each with a single L.

### 3 Location

#### SPECIFICATION

- The two LIGO sites should be separated by at least 300 km, preferably by a distance between 2500 and 4500 miles.
- The locations of the two LIGO sites should be such that, in combination with an antenna in Europe, they give the best time delay discrimination in two orthogonal directions, as expressed by an Area Factor (defined in Site Committee Report) of greater than 0.12.

#### RATIONALE

##### 3.1 Site separation

At a minimum, the two LIGO sites should be far enough apart so that there is negligible probability of correlated external noise. The site committee report lists 300 km as a safe lower limit on the distance, but this was a poorly determined number based on library research concerning correlation distances for possible external disturbances. If the question needs to be seriously answered (if, for example, we were to think about a third site to add sensitivity for stochastic sources instead of to maximize position information) then this number would have to be carefully reviewed.

The preferred separation range is based on a desire to maximize the information to be obtained from time delays. The top end of the range is the distance at which Earth curvature effects have made the two sites' average sensitivity decline by 5 percent. (The average sensitivity declines because the two interferometers are unavoidably misaligned because the local verticals point in different directions at different points on the earth. Since the

interferometers need to be level to about 1 mrad, they can be treated as perfectly level for the purposes of this argument, which is concerned with differences of local vertical of order 1 radian.) Below the bottom end of the range, there is reduction in time delay information with no substantial increase in sensitivity. The number is sensitive to assumptions about the polarization of the source and the signal-to-noise ratio. Details of the argument are given in the site committee report, section I.B.3.

### 3.2 Triangle Area

If three sites are nearly in a line, there is almost no information added beyond what the two most distant sites provide. A measure of the amount of information added is given by the area of the triangle defined by the three sites. The site committee expressed this in normalized form as the Area Factor. A table of values for selected sites is reproduced below:

Sites	Area Factor
Edwards-Columbia-France	0.43
Edwards-INEL-France	0.15
Edwards-LSU-France	0.44
Columbia-INEL-France	0.37
Columbia-LSU-France	0.18
INEL-LSU-France	0.43
Edwards-Columbia-Japan	0.85
Edwards-INEL-Japan	0.21
Edwards-LSU-Japan	0.44
Columbia-INEL-Japan	0.66
Columbia-LSU-Japan	0.60
INEL-LSU-Japan	0.56

### 3.3 Latitude

*Weak Preference:* Sites closer to the equator are marginally better (at the 10 percent level across the U.S.) at detecting sources in the Virgo Cluster.

## 4 Orientation of Antennas

### SPECIFICATION

- There is no requirement on the absolute orientation of the antennas.
- The orientation of the two antennas with respect to one another should be, to a precision of plus or minus 13 degrees, the average of two alignments:
  - the coincident projection alignment, in which the antennas are oriented so that, when projected onto the plane which bisects the line connecting the sites, the arms of the antennas are superposed.
  - the Virgo-optimized alignment, which gives the best match in signal strengths from sources in the Virgo cluster (i.e., near the celestial equator.)

The coincident-projection alignment is given by

$$\alpha = 2 \arctan \frac{\tan \gamma_- \sin \bar{\beta}}{\cos \beta_-}$$

where  $(\beta_1, \gamma_1)$  and  $(\beta_2, \gamma_2)$  are the latitudes and longitudes for two L-detectors,  $\bar{\beta} = (\beta_1 + \beta_2)/2$ ,  $\beta_- = (\beta_1 - \beta_2)/2$ ,  $\gamma_- = (\gamma_1 - \gamma_2)/2$ , and  $\alpha$  is the difference in orientation, with positive  $\alpha$  indicating that detector 2's bisector should be rotated counterclockwise on a conventional map relative to detector 1's by  $\alpha$ . For example, if  $(\beta_1, \gamma_1) = (44.67^\circ, -67.9^\circ)$  (Columbia),  $(\beta_2, \gamma_2) = (34.95^\circ, -117.78^\circ)$  (Edwards), then  $\alpha = 33.27^\circ$ . So if Columbia's bisector is oriented  $20^\circ$  East of North, then the coincident-projection alignment for Edwards is  $13.27^\circ$  West of North, modulo  $90^\circ$ .

The Virgo-optimized alignment has to be calculated with a computer program. Schutz and Tinto have done this for Edwards- Columbia, among other pairs.

## RATIONALE

### 4.1 Absolute Orientation

Schutz and Tinto (1987) showed that for a pair of antennas separated by 4200 km the averaged sensitivity only varies at the 2 percent level as the orientations of the antennas are rotated (together). That is, there is always a relative orientation of two antennas at two sites which gives the same number of coincidences (within a few percent), irrespective of the absolute orientation of the antennas. Earlier unpublished work by Whitcomb (from the same calculation partially reproduced in the site committee report as Appendix B) had given similar results.

### 4.2 Relative Orientation

As summarized in the site committee report, a guiding principle in designing the LIGO is to maximize the probability of detecting gravitational waves. In so far as it doesn't conflict with this objective (by more than a negligible amount), a second principle in design choices is to maximize the information to be obtained from the gravitational signals which are detected.

Choice of interferometer alignment is a case where these two objectives conflict, so we have to stick to the priority of making the detection. Aligning the detectors maximizes the chances that, if the signal is big at one site, it will be big at the other. This increases the chances that we will have statistically significant coincident detections.

Note that we have paid a price for this choice, assuming that we are successful in detecting gravitational waves. Coincident alignment throws away most of the information about the orthogonal polarization component. Furthermore, coincident alignment also causes many signals to have low amplitude at both antennas. So we have reduced the sky coverage of the pair, although only if the waves are not linearly polarized would we have as many statistical coincidences without alignment as with it.

Schutz (quoted in site committee report) has argued that most gravitational waves are likely to be elliptically polarized, and therefore that coincident alignment won't increase the number of signals which are strong at

both sites. Then, he argues, it makes more sense to get extra information by deliberately misaligning the antennas. This is a perfectly respectable argument, but it is too much of a risk to take, in the judgement of the site committee.

The difference between coincident projection alignment and Virgo-optimal alignment is small (around 10 degrees for Edwards- Columbia). Specifying the average of the two as the best choice is a judgement call, reflecting uncertainty about the distribution on the sky of the first gravitational wave sources to be detected. Specifically, supernovae may be strong enough to be the first sources to be seen. They are abundant enough that the strongest signals will clump in the nearest cluster of galaxies, the Virgo Cluster. On the other hand, if supernovae are weak, it may be that other objects are the first sources we see. If they are less abundant, then they will only be seen at reasonable event rates (several per year) if we can see them substantially beyond the Virgo Cluster. One such source is the collapse of neutron star binaries, but note that they will give elliptically polarized radiation, so their existence does not argue for any particular relative alignment of interferometers. (An extensive discussion of gravitational wave source models, with references to the published literature, is given in Chapter II of the Blue Book.) At this distance, the distribution of galaxies begins to look rather isotropic. No one can say with any confidence which scenario is more likely. Anyway, we don't pay much of a price for straddling, so we split the difference on this issue.

## 5 Slope

**SPECIFICATION**      The LIGO arms should be level to within 3 milliradians.

### **RATIONALE**

Suspensions for the test masses in gravitational wave interferometers have always been based on the use of a low frequency pendulum as the innermost stage. A pendulum is typically quite anisotropic in its isolation (vertical motions not nearly so well isolated as horizontal.) It nevertheless has a key advantage – its mechanical Q can be substantially greater than

that of an oscillator with a spring made of the same material as the pendulum wire. This is because in a properly designed pendulum by far the largest part of the energy storage is in the gravitational field, which has no mechanical losses. This high  $Q$  is valuable since thermal noise (Brownian motion) is inversely proportional to the square root of  $Q$ .

Anisotropic isolation is tolerable because the interferometer is sensitive (in first order) only to the degree of freedom of the mass which is parallel to the optic axis. Thus, the natural arrangement which has always been used is to have the optic axes of both arms horizontal.

In laboratory-scale apparatus it is possible to adjust the level of the optic axes. It is a different matter in the case of LIGO with its 4 km long arms. Even at sites chosen especially for flatness, substantial extra expense might have to be incurred to make the elevation of the test masses equal to closer than 20 feet or so. (This is the case both at Edwards and Columbia.) Thus the question arises, "How close to truly level do the arms have to be?"

Surprisingly, this is a question which actually has a well-defined answer. The reason is that there is a characteristic angle to the problem, namely the angle subtended at the center of the earth by an interferometer arm. This angle is

$$\Theta = \frac{L}{R_{\oplus}} = \frac{4km}{6000km} = \frac{2}{3} mrad.$$

The significance of this angle is that, if the optic axis is precisely horizontal at one end of an arm, then the axis makes an angle  $\Theta$  with the horizontal at the other end of the arm. A straight line 4 km long can never be level everywhere along its length to better than  $\frac{\Theta}{2}$ , the value at both ends if the line is level at its middle. Thus one can never find an orientation of the arms which does much better than discount vertical motion by a factor of about  $3 \times 10^3$ .

Here are a few numbers to set the scale of the problem. If the earth were perfectly smooth, then a line 4 km long, set level at the middle and with both ends at the surface of the earth, will be buried about 1 foot below the surface at its midpoint. If we make the line level at one end, and place that end at the surface of the earth, its other end will be about 4 feet above the surface.

The argument given in the previous paragraph says that nothing in the

installation of the LIGO can save us from having to face misalignments of the beam with the horizontal at about the  $10^{-3}$  level. We can turn the question around and ask, "How large a misalignment can the interferometers tolerate?"

The question comes down to how much anisotropy have we allowed up until now in our suspension designs, and how isotropic could we make them if we paid enough attention to the problem. We need to consider both transmitted seismic noise, and the thermal noise in the suspension.

Calculations of the vertical and horizontal isolation of model suspensions show that, without some attention to suspension anisotropy, we might find ourselves with a surprisingly large contribution from vertical seismic motion. (Several examples are given in the memo "Draft Specification for the Slope of LIGO Arms".) The more encouraging note is that measures which are not very heroic can tame the problem. Thus, with proper care, isolation anisotropy should not limit performance of interferometers which are level to several parts in  $10^3$ .

One other aspect of suspensions which we have to investigate is thermal noise, the very feature which led to the choice of anisotropic suspensions in the first place. For frequencies above the resonance, the thermal noise spectral density is given by

$$x_{thermal}(f) = \frac{1}{f^2} \sqrt{\frac{kTf_0}{2\pi^3mQ}}$$

(a slightly recast version of equation (5), page V-21 in the Blue Book). If the vertical mode is in the vicinity of 10 Hz instead of 1 Hz, and if its  $Q$  is lower by about  $10^4$ , then the vertical thermal noise is 300 times larger than in the horizontal direction. Thus as long as the arms are level to 1 part in 300 or better, then the noise we expected from the horizontal motion is still the dominant effect.

## 6 Characteristics of an Ideal Site

- Available for free or cheap, sale or long-term lease
- Flat enough to allow line-of-sight with min cut and fill

- Soils and drainage suitable for construction, minimize blasting
- Environmental concerns easily met
- Seismically and acoustically quiet
- Little human activity, wind and trees, ocean?
- Low probability of future development
- Convenient to transportation, construction labor source, technically trained staff, maintenance
- Convenient to home institutions, outside visitors
- Security
- Mild climate

How should we weigh the different site criteria? There is no natural point system. Instead, judgement is required to weigh risks of being stopped cold (as in environmental impact, security regulations), versus costs in dollars (e.g. need for blasting rock) costs in sensitivity (seismic noise), or costs in inconvenience (distance to sites from home institutions.)

## 7 Description of Specific Sites

(This section to be filled in by LIGO engineering staff for selected sites and, perhaps, for one or more alternate sites.)

- Location
- Map
- Summary of topography, rms, cut and fill along antenna line
- Size of available parcel, orientations allowed, ability to add (in future) length, arms
- Soils and drainage, rock and clay from drilling, seismic refraction

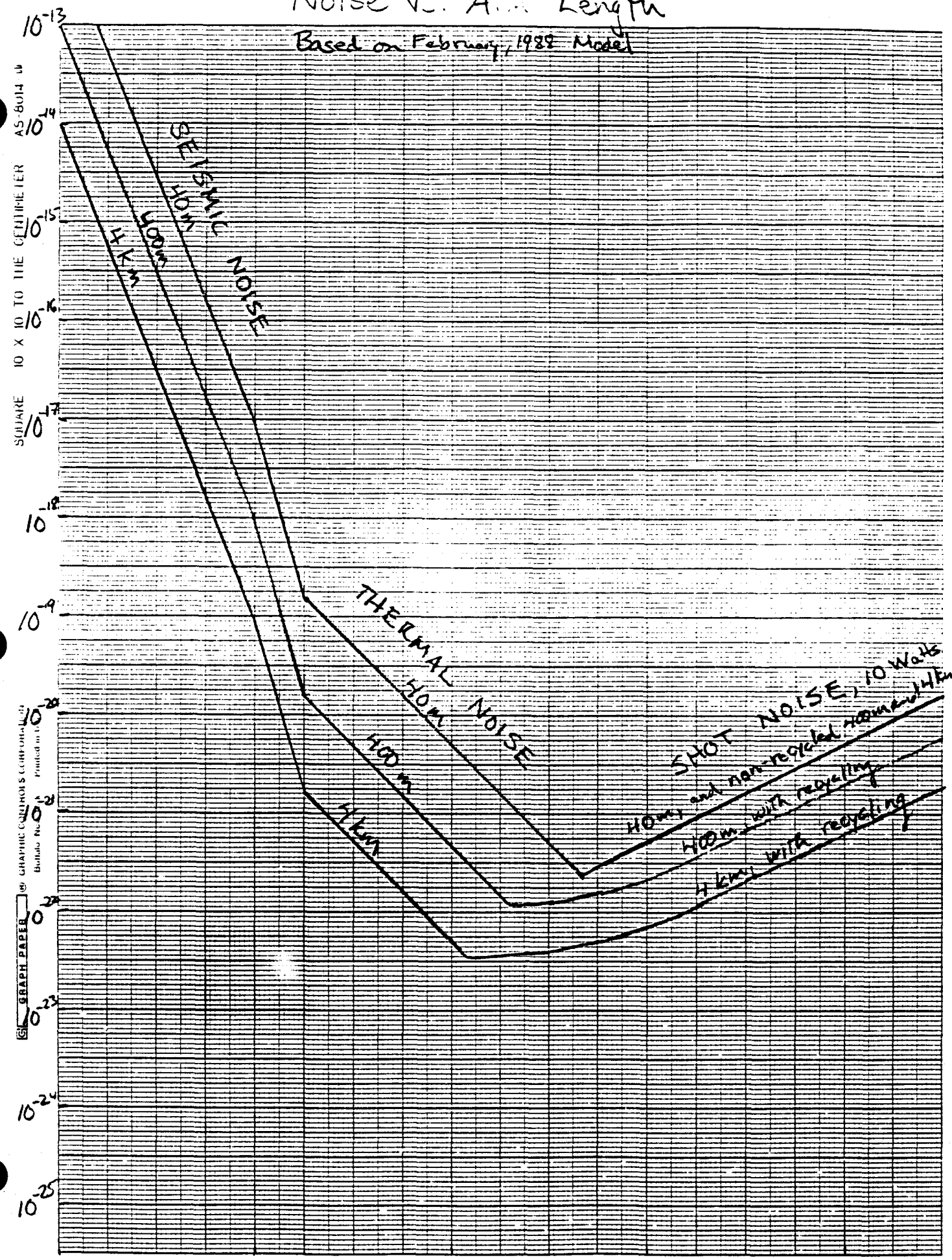


- Construction options available (above ground, below ground)
- Climate: temperature variations, precip., wind
- Access by road, rail, air, (distance to heavy routes for noise as well)
- Ownership
- Environmental impact statement—requirements and status
- Seismic noise (acoustic noise?)
- Construction labor cost factor
- Pre-existing infrastructure, security, etc.
- Population density in area, nearby industry, traffic on nearby routes, prospects for future growth
- Availability of electric power, water, other services
- Support from local government, universities
- Earthquake and flood risks
- RF environment
- Externally mandated restrictions (below gnd only, bldg heights,...?)

# Noise vs. Ant Length

Based on February, 1988 Model

$$h(f) [Hz^{-1/2}]$$



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1 Hz      10 Hz      f      100 Hz      1 kHz      10 kHz

BATCH  
START

---

STAPLE  
OR  
DIVIDER

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## 1 Number of Sites

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The accompanying graph shows the projected noise spectrum of the LIGO Mark I receiver at the design length of 4 kilometers, and at several shorter lengths. (The noise model is that presented to the NSF site visit committee in February, 1988.) The degradation in sensitivity due to test mass noise over the key 100 Hz to 1 kHz band is evident at shorter lengths. Clearly, even longer arms would improve the situation. A natural maximum comes when the round trip time in the arms reaches half the period of the gravitational wave signal. For 1 kHz, this is a length of 150 km, while for 100 Hz it corresponds to 1500 km.

A second class of noise, shot noise in the optical system, limits sensitivity at high frequency. The shot noise sensitivity is independent of  $L$  in a non-recycled system, but is proportional to  $\sqrt{L}$  in a system with standard recycling, and proportional to  $L$  with periodic recycling (in a simple model where the recycling gain is limited by reflectivity of the mirrors).

The curvature of the earth sets a length beyond which some noise will grow. As explained in more detail in the section on slope, below, pendulum suspensions are used to reduce the thermal noise in the horizontal direction. When the optic axis is not accurately horizontal, the larger vertical noise is sensed by the interferometer. Since the earth is curved, a long interferometer can not be level everywhere. If the vertical thermal noise is 300 times larger than the horizontal, then the arms can not be increased

beyond 40 km without also increasing the thermal noise contribution. More details are given below.

So why choose 4 km instead of 40 km? Mostly for practical reasons. One has to do with the availability of sites. In the United States there is probably no place other than the Great Salt Lake Desert which is flat enough to accommodate a very long interferometer (greater than, say, 5 km) without an inordinate amount of earthwork. The Columbia site can barely accommodate 4 km.

Cost is itself the strongest reason for restricting the length. The total cost of a 40 km antenna would be prohibitive. A cost model which distinguishes between costs which grow in proportion to antenna length and those which have to be paid independent of the length gives a natural economic length to the LIGO. A vacuum system which is so short that the cost is dominated by the length-independent costs is too short. A vacuum system which is so long that the length-independent costs are negligible, so that it costs an extra factor of two to buy an additional factor of two in low frequency sensitivity, is probably too long. Although the details of the argument depend on the specific LIGO design and cost estimate, we have consistently found that 4 km falls in between the two extremes.

From the flavor of the preceding argument, it is clear that the precision with which we specify the absolute length of 4.0 kilometers is not given by scientific arguments alone. Rather we want to choose the largest installation which is feasible (however feasible is determined). Also, we must guard against being pushed down the slippery slope of trading a bit of length at a time to solve budget problems as they arise.

The tolerance in arm length match at a single site is supposed to represent what is readily achievable with present-day standard laser distance measurements used by surveyors. (If this number is incorrect, please change it.)

If we match the distance between the vacuum chambers to a tolerance of 2.5 cm, then we will be able to match the distance between the mirrors of the optical cavities to high precision (better than a wavelength of light) by moving the masses within the chambers to eliminate any residual differences. This adjustment is readily accomplished with motorized stages. Mechanical designs calling for adjustment range much larger than 2.5 cm are more complicated.



Equality of arm length is a significant advantage for some interferometer designs, though it is not critical for early LIGO interferometers. A length mismatch can couple laser frequency noise to the interferometer output; interferometers made with arms of matched length and equal number of bounces are insensitive to such noise. More precisely, the noise is

$$h(f) = \frac{\Delta\nu(f)}{\nu_0} \frac{\Delta\tau_{\text{stor}}}{\tau_{\text{stor}}} \frac{1}{S},$$

where  $\Delta\nu(f)$  is the frequency noise spectral density,  $\nu_0$  is the frequency of the light,  $\tau_{\text{stor}}$  is the average optical storage time of the cavities,  $\Delta\tau_{\text{stor}}$  is the difference in the storage times of the two cavities, and  $S$  is a suppression factor resulting from common-mode electronic or mechanical subtraction of the frequency noise. So far the 40-m prototype has achieved  $\frac{\Delta\nu(f)}{\nu_0} = 8 \cdot 10^{-21} \text{ Hz}^{-1/2}$ , and  $S = 500$ , indicating that  $\frac{\Delta\tau_{\text{stor}}}{\tau_{\text{stor}}}$  can be of order unity without compromising the sensitivity of early LIGO detectors.

The tolerance on the match of the arm length at the two sites has been set no tighter than the overall design tolerance. (Perhaps there is an engineering reason to match them more closely.)

## 2.2 Opening Angle

We use two arms because we reject frequency noise by “subtracting” the outputs of the two arms, either electronically or preferably by interference (“recombination”). Two arms at 90 degrees give maximum sensitivity to gravitational waves, because their characteristic tensor polarization causes opposite effects in orthogonal directions. The signal is proportional to the sine of the angle between the arms (so parallel arms give zero sensitivity to gravitational waves). More explicitly, we have

$$S = \left(\frac{1}{2}(1 + \cos^2 \theta) \sin 2\phi \cos 2\psi + \cos \theta \cos 2\phi \sin 2\psi\right) \sin \alpha,$$

where  $\theta$  is the source’s zenith angle,  $\phi$  is the source’s azimuth measured from the bisector between the two arms,  $\psi$  is an angle specifying the orientation of the source’s polarization, and  $\alpha$  is the opening angle between the two arms.

The tolerance of 15 degrees on the perpendicularity of the arms is determined by our (somewhat arbitrary) decision not to sacrifice more than 3.4 percent of sensitivity to this effect. (If we used up all of the site committee's 10 percent performance margin in angle alone, which wouldn't be advisable, we could tolerate plus or minus 25 degrees. At 5 percent, we have plus or minus 18 degrees.)

If there is some engineering reason we can think of, we can set a tolerance on the match in this angle between the two sites.

### **2.3 Area of Land Required**

The width of the strip along the antenna line is primarily determined by consideration of the need for room during construction. If we want a permanent road along the antenna, this should also be factored in.

The size of the lots near the vertex, ends, and middles of arms is to provide room for buildings.

The question of a minimum distance between the power plant area and the instrument buildings is still TBD. Also not determined is the extent to which we want to pad the size of the lots around the instrument buildings to create a noise buffer, as opposed to assuming that we will take care of external noise by proper siting in the first place.

### **2.4 Expansion**

The idea behind considering future addition of a second antenna at 45 degrees to the first is to determine complete gravitational waveforms, which requires polarization information. The site committee report pointed out the intimate connection between waveform solution and source position determination. The site committee compared simultaneous solutions from two sites, both of which had two "orthogonal" antennas, with solutions obtained from three sites, one of which had an extra antenna. Although the two site method works to certain degree, it is far inferior in precision and robustness. For this reason, the committee felt it was an undue burden on the project to require the capability to build orthogonal antennas at both sites. It seemed reasonable to expect that either a third antenna will be built in Europe or elsewhere anyway, or that the successful discov-

ery of gravitational waves by the two site LIGO would generate enough enthusiasm so that construction of a third site would be accomplished.

Subsequent work by Tinto and Gursel has shown how to get a complete solution from three sites each with a single L.

### 3 Location

#### SPECIFICATION

- The two LIGO sites should be separated by at least 300 km, preferably by a distance between 2500 and 4500 miles.
- The locations of the two LIGO sites should be such that, in combination with an antenna in Europe, they give the best time delay discrimination in two orthogonal directions, as expressed by an Area Factor (defined in Site Committee Report) of greater than 0.12.

#### RATIONALE

##### 3.1 Site separation

At a minimum, the two LIGO sites should be far enough apart so that there is negligible probability of correlated external noise. The site committee report lists 300 km as a safe lower limit on the distance, but this was a poorly determined number based on library research concerning correlation distances for possible external disturbances. If the question needs to be seriously answered (if, for example, we were to think about a third site to add sensitivity for stochastic sources instead of to maximize position information) then this number would have to be carefully reviewed.

The preferred separation range is based on a desire to maximize the information to be obtained from time delays. The top end of the range is the distance at which Earth curvature effects have made the two sites' average sensitivity decline by 5 percent. (The average sensitivity declines because the two interferometers are unavoidably misaligned because the local verticals point in different directions at different points on the earth. Since the

interferometers need to be level to about 1 mrad, they can be treated as perfectly level for the purposes of this argument, which is concerned with differences of local vertical of order 1 radian.) Below the bottom end of the range, there is reduction in time delay information with no substantial increase in sensitivity. The number is sensitive to assumptions about the polarization of the source and the signal-to-noise ratio. Details of the argument are given in the site committee report, section I.B.3.

### 3.2 Triangle Area

If three sites are nearly in a line, there is almost no information added beyond what the two most distant sites provide. A measure of the amount of information added is given by the area of the triangle defined by the three sites. The site committee expressed this in normalized form as the Area Factor. A table of values for selected sites is reproduced below:

Sites	Area Factor
Edwards-Columbia-France	0.43
Edwards-INEL-France	0.15
Edwards-LSU-France	0.44
Columbia-INEL-France	0.37
Columbia-LSU-France	0.18
INEL-LSU-France	0.43
Edwards-Columbia-Japan	0.85
Edwards-INEL-Japan	0.21
Edwards-LSU-Japan	0.44
Columbia-INEL-Japan	0.66
Columbia-LSU-Japan	0.60
INEL-LSU-Japan	0.56

### 3.3 Latitude

*Weak Preference:* Sites closer to the equator are marginally better (at the 10 percent level across the U.S.) at detecting sources in the Virgo Cluster.

## 4 Orientation of Antennas

### SPECIFICATION

- There is no requirement on the absolute orientation of the antennas.
- The orientation of the two antennas with respect to one another should be, to a precision of plus or minus 13 degrees, the average of two alignments:
  - the coincident projection alignment, in which the antennas are oriented so that, when projected onto the plane which bisects the line connecting the sites, the arms of the antennas are superposed.
  - the Virgo-optimized alignment, which gives the best match in signal strengths from sources in the Virgo cluster (i.e., near the celestial equator.)

The coincident-projection alignment is given by

$$\alpha = 2 \arctan \frac{\tan \gamma_- \sin \bar{\beta}}{\cos \beta_-}$$

where  $(\beta_1, \gamma_1)$  and  $(\beta_2, \gamma_2)$  are the latitudes and longitudes for two L-detectors,  $\bar{\beta} = (\beta_1 + \beta_2)/2$ ,  $\beta_- = (\beta_1 - \beta_2)/2$ ,  $\gamma_- = (\gamma_1 - \gamma_2)/2$ , and  $\alpha$  is the difference in orientation, with positive  $\alpha$  indicating that detector 2's bisector should be rotated counterclockwise on a conventional map relative to detector 1's by  $\alpha$ . For example, if  
 $(\beta_1, \gamma_1) = (44.67^\circ, -67.9^\circ)$  (Columbia),  
 $(\beta_2, \gamma_2) = (34.95^\circ, -117.78^\circ)$  (Edwards),  
then  $\alpha = 33.27^\circ$ . So if Columbia's bisector is oriented  $20^\circ$  East of North, then the coincident-projection alignment for Edwards is  $13.27^\circ$  West of North, modulo  $90^\circ$ .

The Virgo-optimized alignment has to be calculated with a computer program. Schutz and Tinto have done this for Edwards- Columbia, among other pairs.

## RATIONALE

### 4.1 Absolute Orientation

Schutz and Tinto (1987) showed that for a pair of antennas separated by 4200 km the averaged sensitivity only varies at the 2 percent level as the orientations of the antennas are rotated (together). That is, there is always a relative orientation of two antennas at two sites which gives the same number of coincidences (within a few percent), irrespective of the absolute orientation of the antennas. Earlier unpublished work by Whitcomb (from the same calculation partially reproduced in the site committee report as Appendix B) had given similar results.

### 4.2 Relative Orientation

As summarized in the site committee report, a guiding principle in designing the LIGO is to maximize the probability of detecting gravitational waves. In so far as it doesn't conflict with this objective (by more than a negligible amount), a second principle in design choices is to maximize the information to be obtained from the gravitational signals which are detected.

Choice of interferometer alignment is a case where these two objectives conflict, so we have to stick to the priority of making the detection. Aligning the detectors maximizes the chances that, if the signal is big at one site, it will be big at the other. This increases the chances that we will have statistically significant coincident detections.

Note that we have paid a price for this choice, assuming that we are successful in detecting gravitational waves. Coincident alignment throws away most of the information about the orthogonal polarization component. Furthermore, coincident alignment also causes many signals to have low amplitude at both antennas. So we have reduced the sky coverage of the pair, although only if the waves are not linearly polarized would we have as many statistical coincidences without alignment as with it.

Schutz (quoted in site committee report) has argued that most gravitational waves are likely to be elliptically polarized, and therefore that coincident alignment won't increase the number of signals which are strong at

both sites. Then, he argues, it makes more sense to get extra information by deliberately misaligning the antennas. This is a perfectly respectable argument, but it is too much of a risk to take, in the judgement of the site committee.

The difference between coincident projection alignment and Virgo-optimal alignment is small (around 10 degrees for Edwards- Columbia). Specifying the average of the two as the best choice is a judgement call, reflecting uncertainty about the distribution on the sky of the first gravitational wave sources to be detected. Specifically, supernovae may be strong enough to be the first sources to be seen. They are abundant enough that the strongest signals will clump in the nearest cluster of galaxies, the Virgo Cluster. On the other hand, if supernovae are weak, it may be that other objects are the first sources we see. If they are less abundant, then they will only be seen at reasonable event rates (several per year) if we can see them substantially beyond the Virgo Cluster. One such source is the collapse of neutron star binaries, but note that they will give elliptically polarized radiation, so their existence does not argue for any particular relative alignment of interferometers. (An extensive discussion of gravitational wave source models, with references to the published literature, is given in Chapter II of the Blue Book.) At this distance, the distribution of galaxies begins to look rather isotropic. No one can say with any confidence which scenario is more likely. Anyway, we don't pay much of a price for straddling, so we split the difference on this issue.

## 5 Slope

**SPECIFICATION**      The LIGO arms should be level to within 3 milliradians.

### RATIONALE

Suspensions for the test masses in gravitational wave interferometers have always been based on the use of a low frequency pendulum as the innermost stage. A pendulum is typically quite anisotropic in its isolation (vertical motions not nearly so well isolated as horizontal.) It nevertheless has a key advantage – its mechanical Q can be substantially greater than

that of an oscillator with a spring made of the same material as the pendulum wire. This is because in a properly designed pendulum by far the largest part of the energy storage is in the gravitational field, which has no mechanical losses. This high  $Q$  is valuable since thermal noise (Brownian motion) is inversely proportional to the square root of  $Q$ .

Anisotropic isolation is tolerable because the interferometer is sensitive (in first order) only to the degree of freedom of the mass which is parallel to the optic axis. Thus, the natural arrangement which has always been used is to have the optic axes of both arms horizontal.

In laboratory-scale apparatus it is possible to adjust the level of the optic axes. It is a different matter in the case of LIGO with its 4 km long arms. Even at sites chosen especially for flatness, substantial extra expense might have to be incurred to make the elevation of the test masses equal to closer than 20 feet or so. (This is the case both at Edwards and Columbia.) Thus the question arises, "How close to truly level do the arms have to be?"

Surprisingly, this is a question which actually has a well-defined answer. The reason is that there is a characteristic angle to the problem, namely the angle subtended at the center of the earth by an interferometer arm. This angle is

$$\Theta = \frac{L}{R_{\oplus}} = \frac{4km}{6000km} = \frac{2}{3}mrad.$$

The significance of this angle is that, if the optic axis is precisely horizontal at one end of an arm, then the axis makes an angle  $\Theta$  with the horizontal at the other end of the arm. A straight line 4 km long can never be level everywhere along its length to better than  $\frac{\Theta}{2}$ , the value at both ends if the line is level at its middle. Thus one can never find an orientation of the arms which does much better than discount vertical motion by a factor of about  $3 \times 10^3$ .

Here are a few numbers to set the scale of the problem. If the earth were perfectly smooth, then a line 4 km long, set level at the middle and with both ends at the surface of the earth, will be buried about 1 foot below the surface at its midpoint. If we make the line level at one end, and place that end at the surface of the earth, its other end will be about 4 feet above the surface.

The argument given in the previous paragraph says that nothing in the



installation of the LIGO can save us from having to face misalignments of the beam with the horizontal at about the  $10^{-3}$  level. We can turn the question around and ask, "How large a misalignment can the interferometers tolerate?"

The question comes down to how much anisotropy have we allowed up until now in our suspension designs, and how isotropic could we make them if we paid enough attention to the problem. We need to consider both transmitted seismic noise, and the thermal noise in the suspension.

Calculations of the vertical and horizontal isolation of model suspensions show that, without some attention to suspension anisotropy, we might find ourselves with a surprisingly large contribution from vertical seismic motion. (Several examples are given in the memo "Draft Specification for the Slope of LIGO Arms".) The more encouraging note is that measures which are not very heroic can tame the problem. Thus, with proper care, isolation anisotropy should not limit performance of interferometers which are level to several parts in  $10^3$ .

One other aspect of suspensions which we have to investigate is thermal noise, the very feature which led to the choice of anisotropic suspensions in the first place. For frequencies above the resonance, the thermal noise spectral density is given by

$$x_{thermal}(f) = \frac{1}{f^2} \sqrt{\frac{kTf_0}{2\pi^3mQ}}$$

(a slightly recast version of equation (5), page V-21 in the Blue Book). If the vertical mode is in the vicinity of 10 Hz instead of 1 Hz, and if its Q is lower by about  $10^4$ , then the vertical thermal noise is 300 times larger than in the horizontal direction. Thus as long as the arms are level to 1 part in 300 or better, then the noise we expected from the horizontal motion is still the dominant effect.

## 6 Characteristics of an Ideal Site

- Available for free or cheap, sale or long-term lease
- Flat enough to allow line-of-sight with min cut and fill

- Soils and drainage suitable for construction, minimize blasting
- Environmental concerns easily met
- Seismically and acoustically quiet
- Little human activity, wind and trees, ocean?
- Low probability of future development
- Convenient to transportation, construction labor source, technically trained staff, maintenance
- Convenient to home institutions, outside visitors
- Security
- Mild climate

How should we weigh the different site criteria? There is no natural point system. Instead, judgement is required to weigh risks of being stopped cold (as in environmental impact, security regulations), versus costs in dollars (e.g. need for blasting rock) costs in sensitivity (seismic noise), or costs in inconvenience (distance to sites from home institutions.)

## 7 Description of Specific Sites

(This section to be filled in by LIGO engineering staff for selected sites and, perhaps, for one or more alternate sites.)

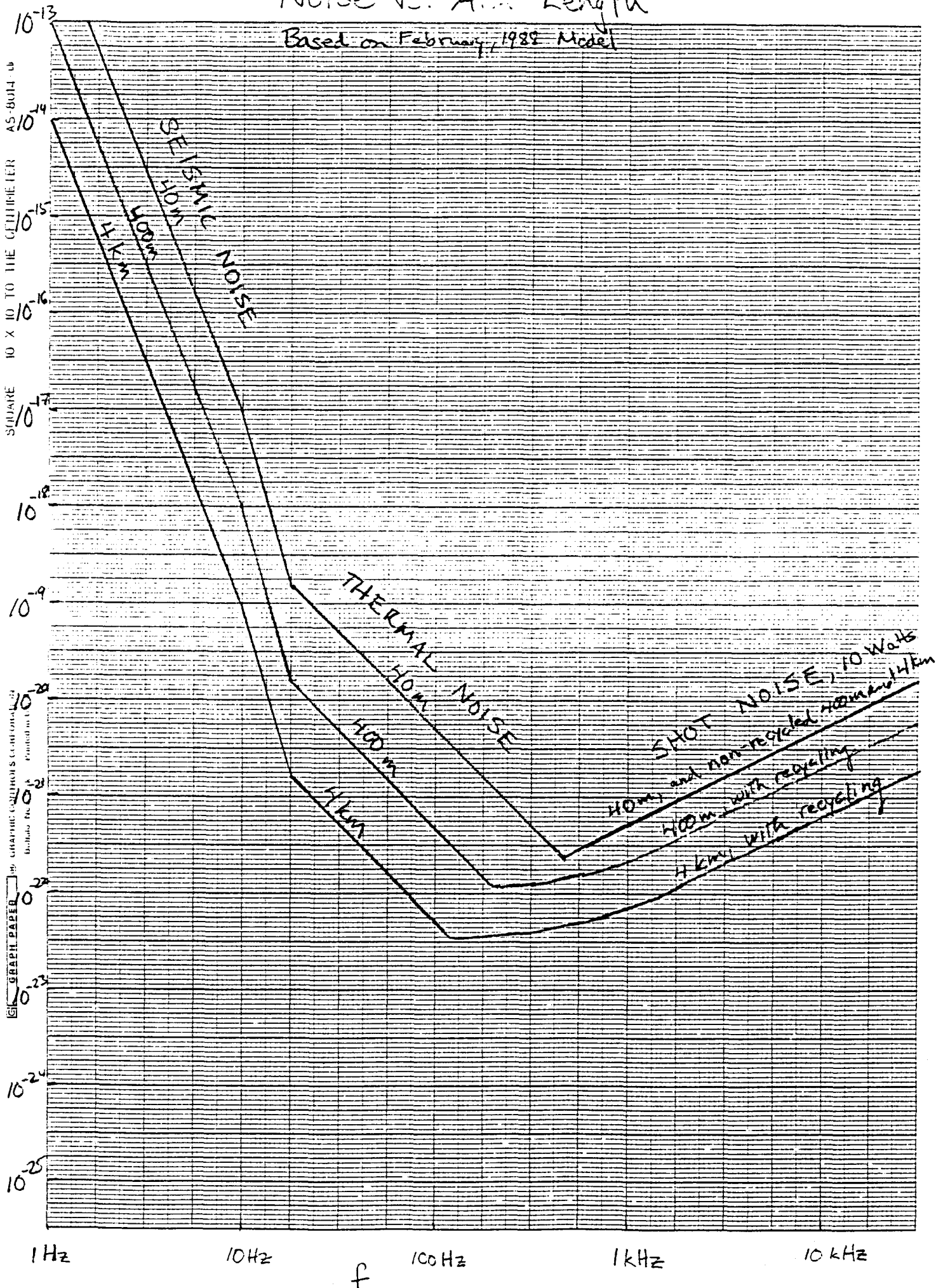
- Location
- Map
- Summary of topography, rms, cut and fill along antenna line
- Size of available parcel, orientations allowed, ability to add (in future) length, arms
- Soils and drainage, rock and clay from drilling, seismic refraction

- Construction options available (above ground, below ground)
- Climate: temperature variations, precip., wind
- Access by road, rail, air, (distance to heavy routes for noise as well)
- Ownership
- Environmental impact statement—requirements and status
- Seismic noise (acoustic noise?)
- Construction labor cost factor
- Pre-existing infrastructure, security, etc.
- Population density in area, nearby industry, traffic on nearby routes, prospects for future growth
- Availability of electric power, water, other services
- Support from local government, universities
- Earthquake and flood risks
- RF environment
- Externally mandated restrictions (below gnd only, bldg heights,...?)

# Noise vs. Antenna Length

Based on February, 1982 Model

$h(f) [Hz^{-1/2}]$



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