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Memo on scattering in LIGO vacuum pipes

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The intent of this memo is to give an account of my ideas on the physics of the noise contribution due to scattering off the pipe enclosing the Fabry-Perot interferometer. The theoretical background for these ideas is given in my longer contribution "Internal Scattering in Fabry-Perot Interferometers" which I will refer to as A.

Scattering off mirrors

The source of scattered light are imperfections on mirrors (microroughness) which divert some of the light resonating in the TE00 mode toward the walls of the vacuum tank. The details of the distribution of scattered light depend on the details of microroughness distribution. In his notes on Scattering in a Michelson interferometer, Kip Thorne used the following expression for the scattering probability in solid angle $d\Omega$ enclosing an angle θ ($\theta << 1$) with respect to the angle of specular reflection:

$$P(\theta)d\Omega = \frac{\beta(1-R_E)}{\theta^2}d\Omega \tag{1}$$

This expression is a reasonable fit to Michele's measurements of scattering off a particular (nonsuper)mirror and the value of β was 3×10^{-3} and the coefficient $(1-R_E)$ was not given. Elson and Bennett (1979)have published results of scattering measurements off silvered polished and superpolished mirrors. Superpolished mirrors were found to be about two orders of magnitude better then ordinary mirrors. Combining Michele's and Elson Bennett's results, it seems reasonable to assume that the coefficient $\beta(1-R_E)$ is for supermirrors at most of the order 3×10^{-5}. Elson and Bennett's calculations actually indicate that it could be as low as 1.5×10^{-6}. The discrepancy could be due to scattering off plasmons in the silvered coating of the mirrors. This scattering is not present with dielectric coating and it is possible that dielectric supermirrors are as good as the calculation indicates. In further estimates I use the value $\beta(1-R_E) = 10^{-5}$.

Scattered light as a noise source

The light scattered off the mirror reaches the vacuum pipe which is moving due to seismic motion and due to all other noise forces acting on the pipe (wind, thermal drifts,...). The light reflected or scattered off the pipe is phase modulated by this motion and if it hits the photodiode, where interference for detecting the gravity waves takes place, it introduces noise proportional to the amplitude of the phase modulated light. In A I call this the direct contribution. Another coupling mechanism (described in A as "phase modulation of the main beam") is due to the part of scattered or reflected light off the pipe that takes part in the interfering path. In other words, the interferometer with microrough mirrors and a vacuum enclosure is not quite an ideal Fabry-Perot interferometer with precisely the gaussian TE00 mode resonating in it, but it also contains a parasitic interferometer of very low finesse , which circulates a certain fraction of scattered light. The observed mode is the sum of the ideal gaussian TE00 and of whatever mode forms in the parasitic interferometer. Since the parameters of the parasitic interferometer change with changing boundary conditions (moving walls), the phase of the complete mode can change slightly, coupling the wall motion to the gravity wave signal.

The direct contribution - the upper limit

I have shown in A that the phase noise in the interferometer, which is due to a small contribution of scattered field ψ, depends on the ratio of the amplitude in the perturbing field ψ and the amplitude in the main TE00 mode Ψ_0 in the following way:

$$\Delta\Phi = \frac{\int \psi^* \Psi_0 dS}{\int \mid \Psi_0 \mid^2 dS} \tag{2}$$

The integrations extend over the surface where the detection takes place - i.e. on the surface of the photodiode. In A I assumed that this surface coincides with the exit mirror and I will continue to do so with no other reason then convenience. Note that mode cleaners at the output may reduce some components of the perturbing field ψ, but I will not pursue this issue here.

In A I gave an expression for the time dependent perturbed field ψ as a function of the wall displacement $\vec{N}(\vec{r},t)$ as follows *:

$$\psi(\vec{r},t) = \int (\vec{N}.\vec{n}) \frac{\partial\Psi_0(\vec{r}',t)}{\partial n'} \nabla' G(\vec{r}',\vec{r}).d\vec{S}' \tag{3}$$

* The equation (3) looks different then eq. (19a) in A, however realizing that $\nabla\Psi_0$ is orthogonal to the surface due to boundary conditions, they become the same

Here $\vec{n}$ is the normal to the pipe surface, $G(\vec{r}',\vec{r})$ is the Green's function for the stationary wave equation inside the cavity formed by the pipe walls and the mirrors.

The above expression is generally quite complicated to evaluate for any given geometry because of the generally complicated form of the Green's function. However the physical meaning is easy to grasp. Suppose we have one scattering center on one mirror which produces a point source of light on this mirror. Due to reflection and diffraction off the pipe some light in addition to the line of sight contribution will be present at the other mirror. The interference of the line of sight field and the scattered and reflected field produces a hologram of the pipe wall which can be seen with the reference beam emanating from our scattering center on the first mirror. In cases of our interest the scattering centers are very far from the hologram plane (the second mirror) and the hologram is very small with respect to this distance. Therefore, the line of sight field is essentially a plane wave perpendicular to the hologram plane and the hologram intensity is proportional to the amplitude of the field $\psi(\vec{r},t)$. The expression (3) tells how much the hologram changes if pipe walls move by the amount specified by the field $\vec{N}(\vec{r},t)$.

With this picture in mind we can set some upper limits. I imagine that the worst happens if all scattered light is focused into a diffraction pattern closely resembling the main beam Ψ_0 and this image overlaps with the exit pupil of the interferometer. A way of arranging this worst case would be to make the pipe an elliptical mirror with the foci on both cavity mirrors. I will now discuss this configuration in some detail in order to set the upper limit of the noise coupling from scattered light.

Consider now the pipe which is an extremely elongated elliptical mirror whose foci are on both cavity mirrors. With such a pipe any distribution of light at one focus is (appart from diffraction effects) reproduced as an image at the other focus. When the pipe axis shakes with respect to the axis connecting the two mirrors, the image of the scattering center moves with respect to the main beam. This produces a time dependent interference on the exit pupil, which is a source of noise.

The circulating power in scattered light is:

$$P_{scatt} = P_0\beta \int \frac{d\Omega}{\theta^2} \tag{4}$$

$$= 2\pi P_0\beta \int \frac{d\theta}{\theta} = 2\pi P_0\beta \; ln(\frac{\pi L}{2R})$$

For LIGO that is:

$$P_{scatt} \sim 6\times 10^{-4} P_0 \equiv \epsilon P_0 \tag{4'}$$

The integration extends over the solid angle through which the pipe is seen at one mirror, i.e. the angle θ goes from $rR/2L$ to $\pi/2$. Here L is the length of the pipe and 2R is its diameter.

According to the worst case scenario I suppose that the scattered light focuses on the other mirror into a Gaussian spot with diameter σ_s, while the main beam is a Gaussian with diameter σ_0. The spot of the main beam at the exit pupil is thus described by:

$$\Psi_0 = \frac{\sqrt{P_0}}{\sqrt{2\pi}\sigma_0} e^{-\frac{r^2}{4\sigma^2}} e^{ikL} \tag{5a}$$

and the scattered spot by:

$$\psi_s = \frac{\sqrt{\epsilon P_0}}{\sqrt{2\pi}\sigma_s} e^{-\frac{(\vec{r}-\vec{\delta})^2}{4\sigma^2}} e^{ikF} \tag{5b}$$

Here $\vec{\delta}$ is the separation of the centers of the two spots and F is the mayor axis of the elliptical enclosure $(F > L)$.

Suppose that one end of the pipe stays fixed and the other one shifts by $\vec{N}$. The image of the scattering center, i.e. the spot produced by the scattered light, shifts by $\vec{N}$ and the perturbed field ψ is:

$$\psi = \vec{N}.\nabla\psi_s = \frac{\vec{N}.(\vec{r}-\vec{\delta})}{2\sigma_s^2} \frac{\sqrt{\epsilon P_0}}{\sqrt{2\pi}\sigma_s} e^{-\frac{(\vec{r}-\vec{\delta})^2}{4\sigma^2}} e^{ikF} \tag{6}$$

Using this in (1), I get the following expression for the interferometer phase noise:

$$\Delta\Phi_{lateral} = \sqrt{\epsilon}\frac{\sigma_s\sigma_0}{(\sigma_s^2+\sigma_0^2)^2}\vec{N}.\vec{\delta}e^{-\frac{\delta^2}{4(\sigma_s^2+\sigma_0^2)}} sin[\frac{2\pi}{\lambda}(L-F)] \tag{7}$$

It can be seen from eq. (7) that the phase noise due to scattered light still depends critically on two parameters - the distance (δ) between the spot produced by scattered light and the main beam, and the size of the scattered light spot σ_s.I take the absolutely worst case where $\sigma_s = \sigma_0$ and $\vec{\delta} = \sigma_0\frac{\vec{N}}{|\vec{N}|}$ and $sin[\frac{2\pi}{\lambda}(L-F)] = 1$. In this way the upper limit for noise due to lateral motion of the pipe ($N \equiv |\vec{N}|$) is obtained:

$$\Delta\Phi_{lateral\ max} = \sqrt{\epsilon}\frac{N}{\sigma}e^{-1/8} \approx \sqrt{\epsilon}\frac{N}{\sigma} \approx \sqrt{\epsilon}\frac{N}{\sqrt{\lambda L}} \tag{8}$$

The interferometer is also sensitive to changes of the pipe diameter through the changes in the focal distance F. Going through similar steps as above, I arrive at the following upper limit for the sensitivity to diameter changes:

$$\Delta\Phi_{radial} \approx 8\pi\sqrt{\epsilon}\frac{NR}{\sigma_0^2} \tag{9}$$

The interferometer is thus more sensitive to radial motion of the pipe then to lateral displacements by a factor:

$$\frac{\Delta\Phi_{radial}}{\Delta\Phi_{lateral}} \approx 8\pi\frac{R}{\sigma} \tag{10}$$

For LIGO this is:

$$\approx 560$$

In order to get an estimate for the strain noise in the interferometer, one needs to know the expected lateral and radial amplitudes. Assume that both amplitudes are typical seismic noise amplitudes, which taken from the MIT report of 1983 is typically:

$$x_{seismic} \approx 10^{-7} m/\sqrt{Hz}(\frac{1Hz}{f})^2 \tag{11}$$

With these numbers and assuming that the finesse (Q) of LIGO is 100, the upper limits for direct rms noise contribution due to radial and lateral displacements of the pipe are (the notation and the definition of rms noise are that of the LIGO proposal 1987 eq. A.19):

$$\Delta h_{radial} \approx 6 \times 10^{-21} \times (\frac{100Hz}{f})^{3/2} \tag{12}$$

and

$$\Delta h_{lateral} \approx 10^{-23} \times (\frac{100Hz}{f})^{3/2}$$

It is obvious that the pipe is not an ellipsoid centered on the two mirrors, however the central part of the pipe corresponding to the same Fresnel zone cannot be distinguished from an ellipsoidal mirror. The size of the central Fresnel zone is given by (for $L >> R$):

$$\Delta x_{Fresnel} = \sqrt{L\lambda/2}\frac{L}{R} \tag{13}$$

For LIGO this is:

$$\approx 250m \qquad ()$$

If the central Fresnel zone of a mirror pipe should move coherently, one can compute the upper noise limits along the same lines as before. The only difference are the limits of integration in (4). The coefficient ϵ is smaller in this case, i.e.:

$$\epsilon_{Fresnel} \approx 4\pi\beta\frac{\sqrt{L\lambda/2}}{r} \tag{13}$$

So that the ratio $\epsilon/\epsilon_{Fresnel}$ for LIGO is about 70, and the upper limit for the noise estimate reduces by only $\sqrt{70} \approx 8$ times. Should it be possible to reach the upper noise limit, the scattering noise would be a limiting factor for advanced LIGOs. Before discussing the probabilities that the upper limit would be reached in LIGO and discussing means to avoid this noise coupling to a high degree, I will describe the other noise coupling source mentioned in the introduction.

Phase modulation of the main beam - the upper limit

In A I have shown that the amplitude of the mode Ψ_0, resonating in a cavity, is perturbed if the walls of the cavity move. The perturbation obeys the forced harmonic oscillator equation with the force term proportional to the integral of the stray field at the wall folded with the wall displacement:

$$\frac{d^2}{dt^2}\alpha_0 e^{i\omega_0 t} + \frac{2}{\tau}\frac{d}{dt}\alpha_0 e^{i\omega_0 t} + \omega_0^2 \alpha_0 e^{i\omega_0 t} = -\frac{c^2}{2} e^{i\omega_0 t}\{\frac{\int(\vec{N}.\nabla\Psi_0^*)(\nabla\Psi_0.d\vec{S})}{\int \mid \Psi_0 \mid^2 dV}\} \tag{14}$$

It is difficult to calculate the gradient of the distorted mode at the wall for any given cavity, so that the physical intuition has to be gained from simple examples. Consider again the elliptical mirror pipe (or the central Fresnel zone) as discussed in the previous paragraph. The assumption was, that the scattering center on one mirror is imaged by the wall onto the other mirror. The path mirror - pipe - mirror - pipe - mirror closes a parallel Fabry-Perot path. Supposing that the finesse of this interferometer is low, the field at the wall can be thought of as a sum of the incoming field from the scattering center on the mirror, plus a reflected field. For the low finesse path the incoming field is well approximated by the field due to scattering off the mirror if there was no pipe, multiplied by the finesse of this cavity (less then 1). The reflected field is generated from the incoming field by changing the sign of the propagation vector along the normal, so that the normal derivative of the field is simply twice the normal derivative of the incoming field. The right hand side of eq. (14) becomes therefore:

$$c^2 e^{i\omega_0 t}\frac{\int(\vec{n}.\vec{N}) \mid \frac{\partial\Psi_0}{\partial n} \mid^2 dS}{\int \mid \Psi_0 \mid^2 dV} \tag{15}$$

Here Ψ_0 is the field at the wall as if there was no wall and $\mid \partial\Psi_0/\partial n \mid^2$ is the intensity of light at the wall multiplied by the cosine square of the angle between the normal to the surface and the direction of the incoming light, and also multiplied by k^2 ($k = 2\pi/\lambda$) (see also A). Due to the term $(\vec{n}.\vec{N})$ in the integrand, the integral is large only if the displacement field $\vec{N}$ is radial and is much smaller (zero if the main beam is on the axis of the pipe) if $\vec{N}$ is a constant lateral displacement. With $\vec{N}$ being a position independent radial displacement, the integral in (15) becomes trivial, and after inserting it in (13), I obtain α_0. Remembering that the imaginary part of α_0 is he phase change, the result is:

$$\Delta\Phi_{main} = \frac{\pi^3}{\lambda} N\beta Q \tag{16}$$

Note that for our data this is the most important noise source - i.e.:

$$\frac{\Delta\Phi_{main}}{\Delta\Phi_{radial}} = \frac{\pi^2}{8}\sqrt{\frac{\beta}{2\pi ln(\pi L/2R)}\frac{QL}{R}} = 420 \tag{16}$$

The corresponding strain noise is:

$$\Delta h_{main} = 2.5 \times 10^{-18} (\frac{100Hz}{f})^{3/2} \tag{17}$$

A comment about this estimate might be in place, since Kip Thorne finds in his notes a much smaller contribution due to what he calls the double scattering contribution which is a different name for the same effect (I didn't change the name because of reference to A, where the particular technique used in deriving (14) justifies the name.). The difference arises from the difference in initial conditions. While I have assumed the worst case where all the light hitting the wall reaches the mirror and is thus recirculated (with the finesse 1) in the interfering path, Kip Thorne was studying the more realistic case with baffles which block most of the light reaching the wall from ever reaching the mirror and even less reaching the mirror again. In my language Kip Thorne's parallel interferometer has a much much lower finesse then 1. Note also that I have studied the phase modulation of the main beam for the best case of random walls and there this effect is much smaller then the direct contribution.

Remarks on direct contribution

So far I have discussed the conditions and estimates of the worst case coupling to the wall motion. What can one do to decrease the coupling? The worst case happens if the image of one mirror, i.e. the spot produced by scattered light, almost coincides with the other mirror. However, according to eq. (7), the coupling to noise decreases exponentially with the distance ($\vec{\delta}$) between the image and the mirror. For example if δ is only $3\sigma_0$, the coupling decreases by a factor 10^3, bringing the seismic noise coupling (eq. 7) below the expected sensitivity level of advanced detectors.

Two components of the derivation are crucial for obtaining this result. The first is that the pipe acts as an imaging device from one end to the other. The validity of this calculation is best illustrated in fig. 1, where I have photographed the image of a pinhole through different diameter and different caliber pipes. The experiment shoved that the imaging capacity of the pipe depends only weakly on the quality of the surface - the pipe used to make pictures in fig. 1a had a reasonably clean surface and the pipe in 1c was quite dirty and corroded. It appears also that larger caliber pipes produce sharper image. I conclude that (at least for beams close to the cylinder axis) the first assumption is reasonably well justified.

The other component of the derivation of (7) and (9) is that the pipe moves as a whole displacing or changing the phase of the image. This assumption is not well justified, especially for LIGO, since the acoustic wavelength of disturbances along the pipe is much less then the length of LIGO. Instead of moving as a whole, the pipe would be seen as an ocean surface excited by the wind, i.e. it would have hills and valleys constantly changing with time. The image of one mirror on the other end of the pipe is, therefore, not expected to simply move, but to be distorted by the random perturbation of the wall. One expects the noise coupling to be less, since a contribution of a hill and a valley generally cancel and only the excess of hills or valleys contributes.

I believe that the interference hologram picture is a powerful tool to understand this contribution, therefore, I illustrate the physics of eq. (3) with a simple, calculable example.

Consider the perturbations of a plain mirror (i.e. as if the cavity was in the right hand half space defined by a plane mirror). The Green's function in the right half space is:

$$G(\vec{r},\vec{r}') = \frac{1}{4\pi}\left\{\frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} - \frac{e^{ik|\vec{r}^{*}-\vec{r}'|}}{|\vec{r}^{*}-\vec{r}'|}\right\} \quad (18)$$

Here $\vec{r}^{*}$ is the image of the point $\vec{r}$ in the plane mirror. (Note that $|\vec{r}^{*}-\vec{r}'|=|\vec{r}-\vec{r}'^{*}|$, so that the Green's function is symmetrical.) The displacement $\vec{N}(\vec{r},t)$ satisfying the hermiticity condition (see A) is normal to the mirror plane ($\vec{N}(\vec{r},t) = N(\vec{r},t).\vec{n}$). Suppose that N has a compact support and calculate $\psi(\vec{r},t)$ in the region far from the domain where $N \neq 0$ (i.e. $|\vec{r}| \gg |\vec{r}'|$). The result is:

$$\psi(\vec{r},t) \approx i\frac{\vec{r}.\vec{n}}{\lambda r^2}e^{ikr}\int N(\vec{r}',t)\frac{\partial\Psi_0}{\partial n'}e^{-ik\vec{r}.\vec{r}'/r}\,dS' \quad (19)$$

Let Ψ_0 (which would be the scattered field at the wall) be a plane wave reflected off the mirror, so that $\frac{\partial\Psi_0}{\partial n} \propto A_0 \times e^{i\vec{\kappa}.\vec{r}'}$; where $\vec{\kappa} = \vec{k} - (\vec{n}.\vec{k})\vec{n}$ is the component of the wave vector in the plane of the mirror. One can see that $\psi(\vec{r},t)$ is the 2-dimensional Fourier transform of N:

$$\psi(\vec{r},t) = iA_0\frac{\vec{r}.\vec{n}}{r^2}e^{ikr}\int \frac{1}{\lambda}N(\vec{r}',t)e^{-i(\vec{r}-L\vec{\kappa}/k).k\vec{r}'/L}\,dS' \quad (20)$$

*

To complete the example take N to be a gaussian bump with the width $\bar{l}$. The beam $\psi(\vec{r},t)$ is seen (at least at far distance) as an elliptical gaussian beam pointing in the direction of the reflected beam and its waist coincides with the waist of the gaussian bump (See fig.2).

There are two important aspects of this result which, I believe, are independent of the simplicity of the particular geometry of this example:

i)the perturbed field peaks in the direction of the reflected beam

ii)the angular spread ($\delta\theta$) of the perturbed field is inversely proportional to the size of the disturbance:

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* It is interesting to note that if $\psi(\vec{r},t)$ is interferred with Ψ_0 as in deformation holography, one obtains the intensity field which is the Fourier transform of N. In deformation holography one wants to get N which is obtained by Fourier transforming the intensity field in the focal plane of the lens.

$$\delta\theta_{\perp} \approx \frac{\lambda}{2\pi\bar{l}} \tag{21a}$$

and

$$\delta\theta_{\parallel} \approx \frac{\lambda}{2\pi\bar{l}cos(\theta)} \tag{21b}$$

Now return to the picture of hills and valleys on the LIGO vacuum pipe. Suppose that the hills and valleys are gaussian and their size ($\bar{l}$) is correlated with the characteristic frequency attributed to them by $\bar{l} \approx c_{sound}/f$. According to the previous result, hills and valleys generate the perturbed field ψ in the direction of the image. However the spread of the scattered field depends on their size. Therefore, the image of the perturbed field is blurred with respect to the image produced by the mirror wall. According to (21b) a single gaussian bump with the width $\bar{l}$ blurs the beam in the sagittal plane by:

$$\sigma_{sagittal} = \frac{\lambda}{2\pi\bar{l}cos(\theta)}x \tag{22}$$

$$= \frac{\lambda}{2\pi\bar{l}}\frac{x(L-x)}{R}$$

this is for LIGO:

$$= .64m \times \frac{1m}{\bar{l}} \times \{4\frac{x}{L}(1-\frac{x}{L})\}$$

Here x is the distance from the gaussian bump to the image plane.

Clearly the bluring is important only if $\sigma_{sagittal}$ is comparable or larger then the width of the main beam (Ψ_0) on the mirror. For $\bar{l} \geq 20m$ the bluring is unimportant and for $\bar{l} \leq 1m$ the beam reflected off a gaussian bump approximately fills the end of the pipe. Using the wavelength-frequency relation for the LIGO pipe derived from Peter Saulsen's report on "Wind-induced Motion of Unprotected LIGO Vacuum Pipes", I conclude that hills corresponding to frequencies below 10Hz do not significantly diffuse reflected light and hills corresponding to frequencies above 500 Hz completely blur the image of the mirror at the other end of the pipe producing a more or less uniform distribution of phase modulated scattered light at the end of the pipe.

Concluding the discussion of the direct contribution, I would like to remark the following:

i) The expression (12) represents the strongest possible noise coupling to the wall and yet it is "only" about three orders of magnitude above the target for noise in advanced LIGOs.

ii) It is quite unlikely that the upper limit could ever be reached in a LIGO, since the conditions for (12) to apply are so restrictive that they may never materialize. Remember that the two mirrors of the interferometer must be placed so that they cast the image of one onto the other and all the parts of the pipe must add coherently.

iii) Two mechanisms provide a large reduction of noise coupling with respect to (12). At low frequencies (below 10Hz or so) the perturbed field (ψ) is identical to the field of light reflected off the wall and is thus reasonably well focused in the image of the mirror. It has been shown that by placing the exit pupil sufficiently far from this image, the noise coupling can be significantly reduced - a reduction by factor of 1000 seems quite reasonable. At higher frequencies (500Hz) diffraction smears the modulated reflected light over the whole end of the pipe and the amplitude of modulated light is no longer so sharply reduced. The reduction factor with respect to (12) is probably $\frac{size\ of\ smeared\ beam}{size\ of\ main\ mode}$, which is only about 10 at 500Hz. However a statistic cancellation takes place. Assuming that the pipe is homogeneously (?) excited, one expects approximately $2\pi RL/\bar{l}^2$ hills and valleys with the dimension $\bar{l}$. The contribution of all but $\sqrt{2\pi RL/\bar{l}^2} \approx 300$ (for $\bar{l}$ corresponding to 500Hz) hills (or valleys) cancels. Therefore, at 500Hz, (12) is too large by at least a factor of 3000, not counting the fact that the remaining field will in general not be matched to the main mode.

iv) I would like to emphasize that the equation (3), which describes the source of the perturbation is quite general. It can be a good description of perturbations of mirror surfaces, of rough surfaces or of baffled surfaces. I have only used it for mirror surfaces, because I was able to analytically compute the generated field only for this case. However, the identification of the field ψ with the intensity field in a deformation hologram makes the physical picture clear. According to this picture the noise generating field ψ can be viewed as a maze of gaussian beams emanating from hills and valleys in the deformation function of the wall (N). Note that the effective displacement field entering eq.(3) is the projection of the displacement onto the local normal to the pipe wall ($\vec{N}.\vec{n}$). This means that corrugations in the wall (such as baffles) modulate hills and valleys ($\vec{N}$ changes slowly, but $\vec{n}$ switches the direction as one moves from one side of the baffle to the other). If the corrugations are small, they can spread (by diffraction) scattered light in large solid angles and make it's contribution small. * The other effect of the large number of diffuse

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* the meaning of this effect can be observed in deformation holography. A typical deformation hologram is observed in the light of the reference beam through the hologram of the undeformed object. When the object is deformed, it's image, as seen through the hologram, develops interference fringes corresponding to the motion of the surface of the object with respect to the previous wave front. It has been observed that objects with different kinds of surfaces produce different fringe patterns. In a mirror surface the fringe corresponding to the motion of a particular part of the wall will be seen only in the reflected light off this part of the wall, i.e. the light forming the fringe is very directional. A diffuse wall will on the other hand show fringes from all points of view where they are observable. The intensity of a fringe observed on a diffuse wall is proportional to the projection of the displacement of the wall on the line of sight (assuming that this displacement is small compared to the wavelength of light), so that in general the intensity of a fringe corresponding to the same point on the wall is different if looked at from different directions. I was arguing before that the perturbation field ψ is essentially the Fourier

beams emanating from displaced corrugation on the walls is that a typical beam which fills the end of the pipe produces a gaussian beam which changes the phase many times in the plane of the exit pupil. Therefore, the sum of all the interfering beams has a widely varying phase on the exit pupil diminishing the overlap integral (2). On the basis of this picture one expects a large noise reduction with respect to (12) if light is scattered off the wall by a number (P) of corrugations (with dimensions $\bar{l}$) instead of being reflected off a mirror wall. The noise reduction factor ρ is the product of the factor describing the lesser amplitude of the spread beam: $s \approx \frac{\sigma_0/L}{\lambda/(2\pi\bar{l}\sqrt{cos(\theta)})} \approx 1.5\times10^{-3}\times(\bar{l}/1mm)$ and the factor describing the fact that a large number of interfering beams has a varying phase across the exit pupil: $v \approx 1/\sqrt{P}$. Obviously it is not difficult to obtain a large noise reduction factor ($\rho = v \times s$) by making baffles with small corrugations - their size in the transverse direction should be of the order $\approx 1mm$ and in the radial direction of the order of a few cm.

v) I have not studied in detail the contribution of the phase modulation of the main beam except for the perfectly randomized case in A. I believe that these results are correct. It is worth pointing out that this contribution is proportional to the energy flux from the wall to the mirror and decreases much faster then the direct contribution which is proportional to the field intensity, i.e. to the square root of the energy flux.

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transform of the interference pattern as seen on the holographic image. The picture just described, therefore, confirms that the information about the motion of a particular part of the wall goes very precisely in the direction of the reflected beam if the surface of the object is mirror-like and is diffused in all directions for diffuse surfaces.

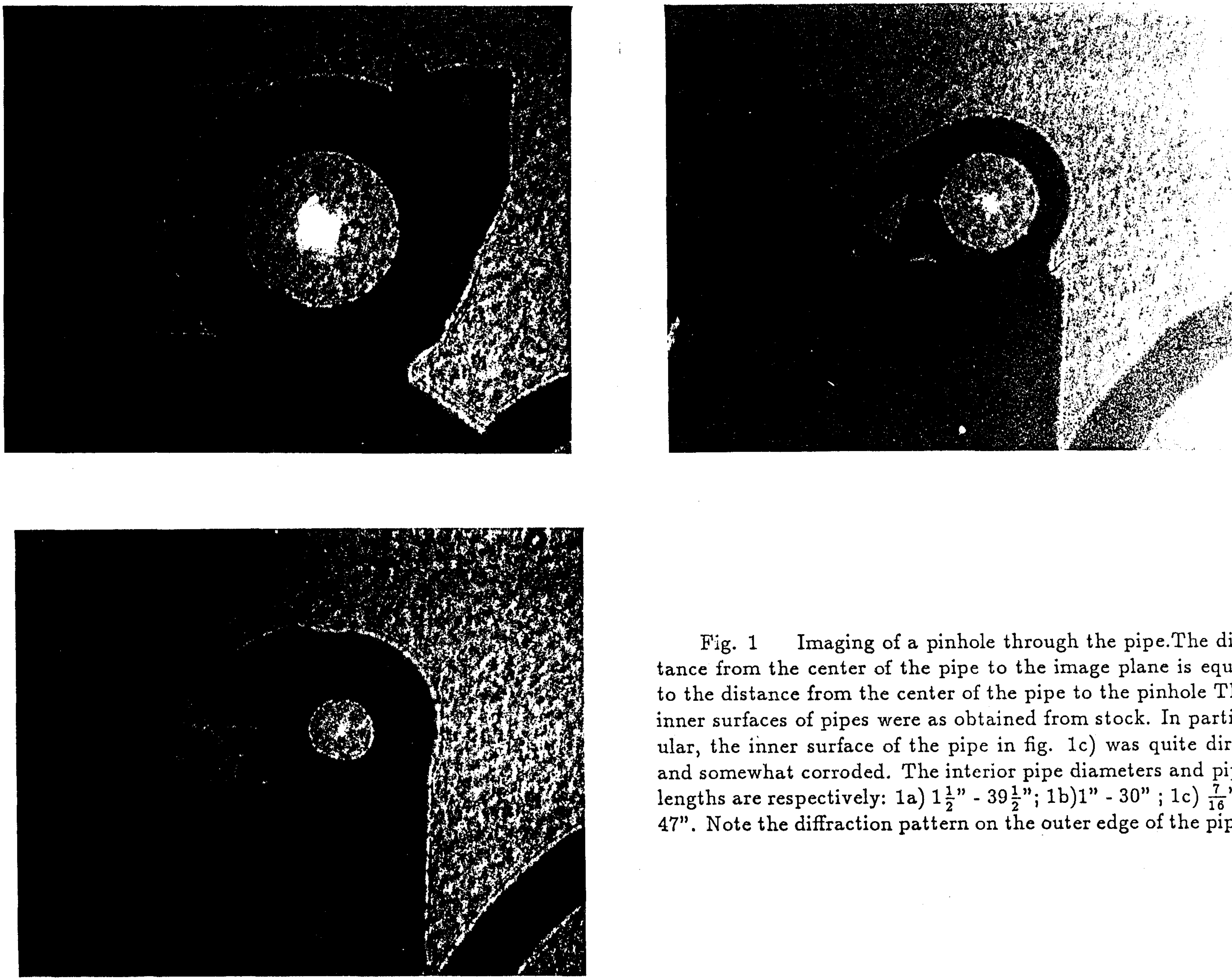
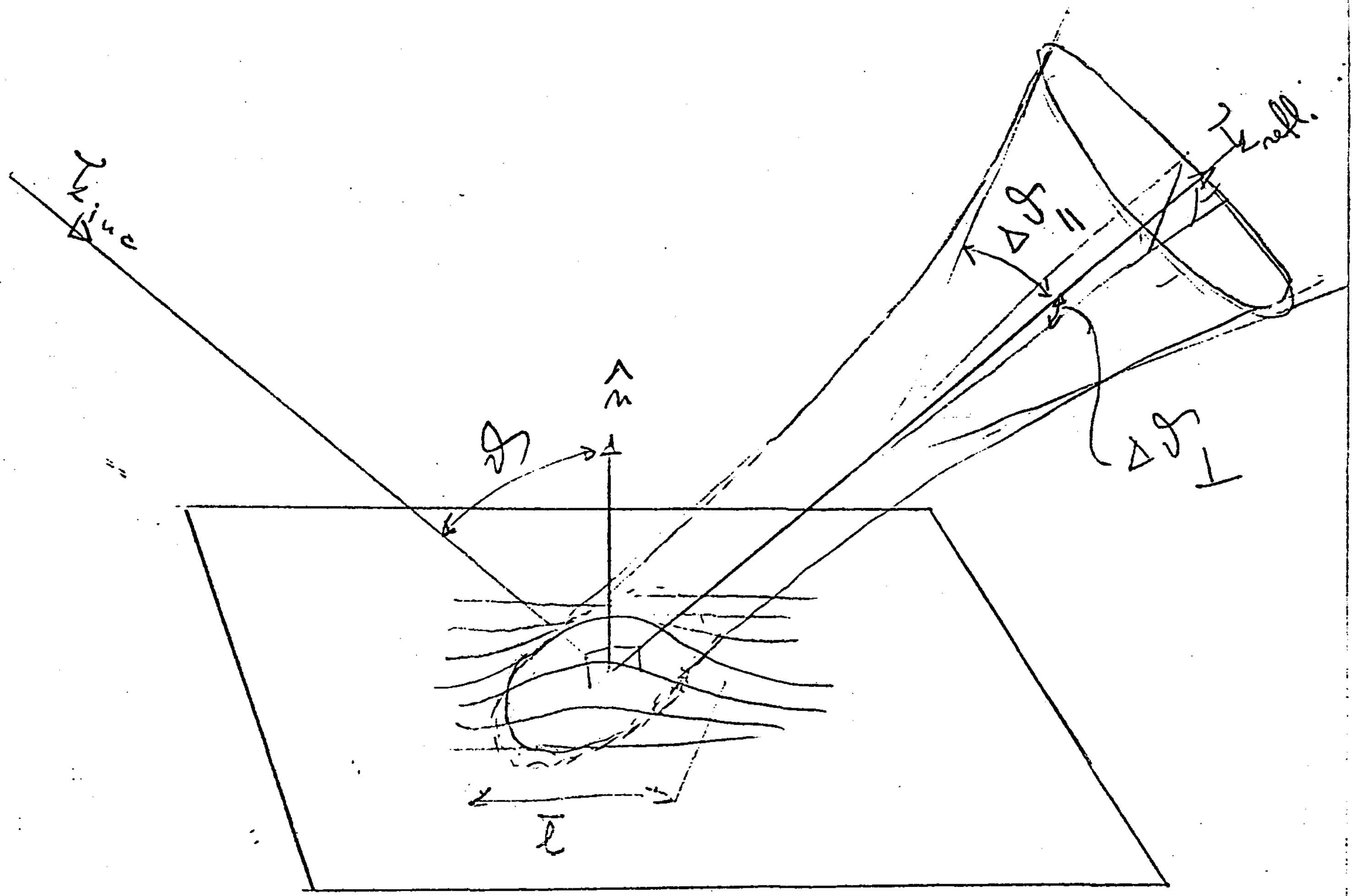


Fig. 1   Imaging of a pinhole through the pipe.The distance from the center of the pipe to the image plane is equal to the distance from the center of the pipe to the pinhole The inner surfaces of pipes were as obtained from stock. In particular, the inner surface of the pipe in fig. 1c) was quite dirty and somewhat corroded. The interior pipe diameters and pipe lengths are respectively: 1a) $1\frac{1}{2}$" - $39\frac{1}{2}$"; 1b)1" - 30" ; 1c) $\frac{7}{16}$" - 47". Note the diffraction pattern on the outer edge of the pipe.

Fig. 2



I_{inc}
$I_{refl.}$
$\Delta\vartheta_{\parallel}$
$\Delta\vartheta_{\perp}$
$\hat{n}$
ϑ
l