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List of 3/1/89

Review of Ron Drever's Memo on Half-Length/Full-Length Interferometer System

Ron's memo gives a convincing quantitative demonstration of the value of multiple (that is, more than double) coincidences in reducing false alarms in the LIGO to a negligible level. The numbers he uses are plausible, in some cases even conservative.

It is worth pointing out that the calculation for coincidence rates assumes that all of the spurious pulses are uncorrelated. This is probably not a bad assumption for coincidences between interferometers at different sites. For colocated interferometers, the assumption will probably be true for some noise sources, but may fail for others. A complete observing strategy would almost certainly have to include auxiliary measurements (seismic and acoustic monitors, data on vacuum, and others) to help reject pulses due to external non-gravitational causes. It should be possible in principle to find (through enough hard work) a separate monitor for any mechanism which causes correlated noise in two interferometers. Conversely, those mechanisms least likely to reveal their existence on an external sensor are the least likely to cause correlated noise in two interferometers. In the process of, first, debugging a LIGO receiver and, second, collecting data for a pulse search, we will want to use every possible sort of information to understand the pulses we do see. Auxiliary measurements will give some of that information. Still, the case is quite clear that multiple coincidences are an important tool for rejecting spurious pulses.

There is an aspect of the question of multiple coincidences which I believe deserves further quantitative analysis – what is the best length ratio between the two sets of interferometers at one site? It is not immediately obvious that a ratio of one half is the best. In the past some sentiment favored multiple interferometers of equal length as the best way to generate multiple coincidences. I'd like to devote the rest of this review to examining the arguments about the length ratio.

The competing claims of length ratios of unity versus one half have to do with the competition between signal-to-noise ratio and distinctive signature. As the last paragraph of Ron's memo points out, marginal signals may be statistically significant in a full-length interferometer and insignificant in a half-length device. (Indeed, this argument is at the heart of our requirement that we build a 4 km LIGO instead of a 2 km version.) If, as is quite possible, the first signals that we detect are just at the limits of our sensitivity, then coincidences between full- and half-length interferometers may fail us just when we need them the most. In this case, it would be coincidences between identical interferometers that would be of the greatest value.

Beyond the demand that real gravitational wave pulses give temporally coincident signals from all interferometers, we can further distinguish real gravitational waves from various forms of interference by requiring that all the signals have the right amplitude. Because there is an angular dependence to the interferometer sensitivity which will not be easy to account for unless there are three or more sites, the only signal ratios which are simple to use as checks are the ratios from interferometers at the same site.

We require that a candidate gravitational wave give signals proportional to the length ratio of the two colocated interferometers. Then one figure of merit is the fractional precision with which the ratio can be determined. We can write

$$\frac{\sigma_r}{r} = \sqrt{\frac{\sigma_s{}^2}{s} + \frac{\sigma_l{}^2}{l}} \tag{1}$$

where s is the signal in the short interferometer, l is the signal in the long interferometer, and $r = s/l$ is the ratio of the signals (also the ratio of the lengths if the interferometers are otherwise matched.) If we assume that the noise in the two interferometers is equal, then we can simplify the previous expression to read

$$\frac{\sigma_r}{r} = \sigma\sqrt{1 + 1/r^2}. \tag{2}$$

Clearly, we minimize the fractional uncertainty by maximizing r. Since $r \leq 1$, then the optimum has $r = 1$. Thus if our requirement is to make the most precise determination of the signal ratio in the two interferometers, then we want the two to be the same length.

The virtue of half-length interferometers, as proposed by Ron, is supposed to be that they will demonstrate that a purported gravitational wave possesses the distinctive signature of signal strength proportional to length. It is hard to imagine any spurious signals which will mimic this property. The optimum we derived in the previous paragraph took no account of this. Using only signal-to-noise ratio as a criterion, that argument favored a signal ratio of unity, which could be mimicked by other effects. It would be nice to find an optimum sensitivity to a ratio other than unity. This would represent the most precise way for us to pick out the characteristic tidal signature of a gravitational wave. I propose that we minimize the figure of merit

$$\frac{\sigma_r}{r(1-r)} = \sigma\frac{\sqrt{1 + \frac{1}{r^2}}}{1-r}, \tag{3}$$

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which weights the precision by how far the ratio is from unity.

To find the minimum of this figure of merit, we can follow the standard procedure of finding where the derivative of the expression (with respect to r) is equal to zero. Some involved but boring algebra shows that this requirement is equivalent to the equation

$$r^3 + 2r - 1 = 0. \tag{4}$$

This cubic equation has its sole real root at $r = 0.453$. This is close to $r = 0.5$. In fact, direct evaluation of the expression for the figure of merit shows that the half-length case differs from the optimum by about one percent.

The preceding argument gives the quantitative justification for the use of half-length interferometers, and shows that half length is just about optimum for the purpose for which they were proposed.

Thus it appears clear that there is value in multiple coincidences both from colocated interferometers of equal length and from sets of half- and full-length interferometers. The advantages of equal length interferometers are optimum precision and equal sensitivity even in the case when signals are of barely significant size. The advantage of half-length interferometers is optimum ability to demonstrate the characteristic gravitational tidal signature.

These considerations lead one to wonder whether there isn't some way to have the best of both worlds. It seems likely that, in the early phase of receiver operation, tracking down the sources of spurious pulses will be of paramount importance. For that purpose auxiliary sensors will have a role, but clearly half-length interferometers give a characteristic discriminant which will be very valuable. At each site, correlated spurious pulses can be distinguished from gravitational waves to the extent that is is unlikely that spurious effects cause signals in the ratio of the arm lengths.

In the mature phase of the operation of a receiver system, one can hope that the number of correlated spurious pulses has been reduced substantially, or at least that the appropriate veto sensors have been found. In that case, it might make sense to shift to a complement of a half-length and full-length interferometer at one site, and the installation of two full length interferometers at the other site. This set of interferometers would give a full-sensitivity triple coincidence, which should be ample to reduce false alarms below a worrisome rate (as long as we can neglect correlations between interferometers, as we are assuming here.) The one half-length interferometer at one site is sufficient to demonstrate the length dependence of the gravitational waves which are strong enough for it to detect. Some waves strong enough for this signature to be demonstrated will help us prove to the world (and to ourselves) that we are detecting gravitational waves. But we are also likely to be somewhat starved for signals, and requiring that all waves be strong enough for the half-length interferometers means a reduction of a factor of eight in the event rate (in the case where we are detecting homogeneously distributed sources, a likely though not certain case.) If we are able to use three equal-length interferometers at the two sites without worrying about a substantial false coincidence rate, then the benefit in the number of detected signals could be substantial.

Adoption of the foregoing strategy would not mean that the facilities constructed at the two sites should be different. It is probably more sensible just to occupy different sets of tanks at the two sites. If we were able to use the strategy I've outlined, it would mean that there would sometimes be more need for tanks at the end stations as opposed to the mid-stations.

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