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CONCERNING: Mechanical noise from optical contacting

This note is about some very preliminary and unfortunately qualitative ideas concerning the mechanical noise that might be generated in optically contacted surfaces. I contend that the interface between optical contacted surfaces must be a source of noise until the mating surfaces have come to a minimum energy and that the noise is generated as the mating surfaces make either "viscous" or discontinuous transitions to the minimum energy configuration. The transitions are thermally driven. The phenomena is similar to stress induced creep. The outline of a theory is presented with a very preliminary estimate of the power spectrum of the surface motions. I am not optimistic that should more time be spent on developing a theory, it will give results with real predictive power. The best way to settle the issue is experimentally.

The basis of the model is that the contacted surfaces will always be rough on atomic scales and that the mating surfaces will be described by a two dimensional distribution function of hills and valleys. One way to characterize the mating surfaces is as a two dimensional distribution of sinusoidal gratings placed between two reference planes in the solid objects being contacted. The distribution function could be represented as

$$z(k_x, k_y, t) = A(k_x, t)A(k_y, t) \sin(k_x x) \sin(k_y y).$$

The initial amplitude coefficients are determined by the surface grinding procedure and in principle could be measured by optical scattering from the surfaces to be mated before contacting. The time dependence of the distribution function is determined by the thermally driven relaxation. The noise we are sensitive to is the spectrum of  $z(f)$  as the distribution function relaxes toward the minimum energy configuration. The relaxation process must head toward minimizing the area surrounding voids in the grating since the molecules at open surfaces are less well bound than those in the solid. I imagine the process in the region of the grating as always being in static equilibrium, the attraction of neighboring non contacting voids being balanced by the repulsion of the contacted sections. These specific states of static equilibrium are not absolute potential minima since a reorganization of the grating can reduce the total energy stored in the grating further. The grating distribution function will tend to larger amplitudes at small  $k$  values to reduce the surface energy until the only  $k$  value represented in the distribution is 0. (The mechanism is similar to the description of Barkhausen noise in magnets and electrets.) The phonon spectrum (temperature) in the bulk material will drive the grating from one configuration to the next. The transitions could be "viscous" or discontinuous. I believe they will be discontinuous because of Coulomb friction in the grating, the thermal energy required to overcome the friction will give a relaxation time for each discrete grating configuration. I have no good idea at the moment of how to calculate these relaxation times, but as the distribution function maximum heads toward smaller  $k$  the relaxation times will probably increase generating a typical  $\frac{1}{f}$  behaviour.

If there is any truth in this model it would seem that, aside from using monolithic test masses and mirrors, one could improve the situation for optical contacting by doing the following.

- 1) Reduction in the initial surface roughness.
- 2) Elevated temperature annealing to increase both the thermal energy available for grating reorganization as well as to increase thermal diffusion of individual molecules into voids in the grating to reduce  $k$ .
- 3) Ultrasonic excitation, which is used in diffusion bonding to enhance contacting, might be helpful in encouraging initial grating reorganizations.

### BEGINNINGS OF A THEORY

The potential function for the individual molecules is attractive through Van der Waals (induced dipole/induced dipole) interactions and becomes strongly repulsive when atomic electron cores come into "contact". The potential function is often given as

$$U(r) = \frac{C}{r^{12}} - \frac{B}{r^6}$$

where  $C$  is the amplitude of the repulsive term and  $B$  the amplitude of the attractive one. The potential minimum occurs at

$$r_0 = \left( \frac{2C}{B} \right)^{1/6}$$

and the second derivative of the potential or "spring constant" at  $r_0$  is given by

$$\frac{d^2U}{dr^2} = k = \frac{1710B}{r_0^8}$$

To relate the potential to bulk measurements one has to integrate the interaction over adjacent planes of molecules in the solid, which yields the potential energy per unit area of plane separated by distance  $h$  as

$$U(h) = \frac{\alpha}{h^{10}} - \frac{\beta}{h^4}.$$

The intermolecular potential constants are related to  $\alpha$  and  $\beta$  as

$$C = \frac{10\alpha}{2\pi\sigma^2} \quad B = \frac{4\beta}{2\pi\sigma^2}$$

where  $\sigma$  is the two dimensional particle density in molecules/cm<sup>2</sup>. The equilibrium spacing of the planes is

$$h_0 = \left( \frac{2.5\alpha}{\beta} \right)^{1/6}.$$

The stress on the planes as a function of the separation,  $z$ , from the equilibrium spacing is

$$S(z) = \frac{24\beta z}{h_0^6}.$$

The internal stress between planes is given by

$$S(h) = 4\beta \left( \frac{25h_0^6}{h^{11}} - \frac{1}{h^5} \right)$$

Young's modulus,  $Y$ , of the material gives an estimate of  $\beta$

$$\beta = \frac{h_0^5 Y}{24}.$$

$h_0$  is determined from direct measurements of the lattice spacing using X ray or neutron diffraction or can be estimated from the Debye temperature using specific heat data at low temperatures.

The handbook values for various of the relevant properties of fused quartz and  $\alpha$  crystal quartz ( $\text{SiO}_2$ ) are:  $Y(\text{fused}) = 7.84 \times 10^{11}$  dynes/cm<sup>2</sup>,  $h_0 = 4.9 \rightarrow 5.4 \times 10^{-8}$  cm, the Debye temperature of crystal quartz  $\theta_d = 470$  K, the melting temperature of crystal quartz  $T_m = 1740$  K, the activation temperature for evaporation of  $\text{SiO}_2 = 3.4 \times 10^4$  K, the activation temperature for the diffusion of  $\text{O}_2$  in quartz =  $1.35 \times 10^4$  K and the activation temperature for diffusion of Si in quartz =  $6.96 \times 10^4$  K.

Using Young's modulus and the lattice spacing the stress between planes away from equilibrium is

$$S(z) = -1.6 \times 10^{19} z \text{ dynes/cm}^2.$$

Other relations derived from the handbook values are:  $\beta = 1.02 \times 10^{-26}$  ergs cm<sup>2</sup> with  $\sigma = \frac{1}{h_0^2} = 4 \times 10^{14}$  per cm<sup>2</sup>, the attractive potential constant  $B = 4.06 \times 10^{-56}$  erg cm<sup>6</sup> and the repulsive one  $C = 3.17 \times 10^{-100}$  erg cm<sup>12</sup>. The binding energy per molecule at equilibrium separation is  $1.3 \times 10^{-12}$  ergs which corresponds to .8 eV or 9400 K which is not far from the simple statistical mechanics result that the binding energy should be near 5 times the melting energy. At the atomic level crystal and fused quartz are very similar.

Following a suggestion by Peter Saulson, one can make a very crude estimate of the noise by looking at the energetics of the relaxation process. Suppose the grating is far from its final state where the spacing of molecular planes is  $h_0$ . The energy released in the total relaxation is approximately

$$\Delta E = \frac{AYh_0}{24}$$

providing that  $h > h_0$ .  $A$  is the geometric area of the mating surfaces. As the grating relaxes, the average mechanical power dissipated in the system can be expressed as

$$\langle P \rangle = \frac{\Delta E}{\tau_{relax}} = F \cdot v = \int_{\frac{1}{2\pi\tau_{relax}}}^{f_{max}} \frac{F^2(f)}{m\omega} df$$

where  $\tau_{relax}$  is the grating relaxation time,  $f_{max}$  is a maximum frequency represented in the force power spectrum and  $m$  is the mirror mass. If the force spectrum is white upto  $f_{max}$  the integration over frequency gives

$$F^2(f) = \frac{2\pi \Delta E m}{\tau_{relax} \ln(2\pi f_{max} \tau_{relax})} \text{ dynes}^2/\text{Hz}$$

The motion of the mirror surface at frequencies,  $f$ , below the drum head flexural mode or the first longitudinal acoustic mode,  $f_0$ , (using which ever is lower) is given by

$$x^2(f) = \frac{F^2(f)}{(2\pi)^4 m^2 f_0^4}$$

Collecting terms, the displacement amplitude spectrum is

$$x(f) = \left(\frac{1}{f_0^2}\right) \left(\frac{AY h_0}{24m\tau_{relax}(2\pi)^3 \ln(2\pi f_{max}\tau_{relax})}\right)^{\frac{1}{2}}$$

Using numbers such as:  $\tau_{relax} = 1 \text{ year}$ ,  $f_{max} = \frac{c_s}{h} = 10^{11} \text{ Hz}$ ,  $m = 25 \text{ grams}$  (typical 1.5" mirror),  $A = 6 \text{ cm}^2$ ,  $f_0 = 70 \text{ kHz}$ , the estimated noise amplitude becomes

$$x(f) \approx 10^{-15} \text{ cm/Hz}^{1/2}$$

The model estimate is insensitive to the choice of  $f_{max}$

The estimate says that it is energetically possible to make a noise that could be important but it does not prove that it really is so. Another model ( not written up), gives an estimate for  $x(f)$  10 to 100 times smaller. This model begins with an assumed creep rate of 10's of angstroms per year to define a viscosity which is then used in a standard Nyquist force thermal noise calculation.

Should one continue this kind of calculation further, a more organized program would include:

- 1) Determine the energy associated with white grating distributions as a function of  $k_{max}$  while keeping the total number of molecules in the grating fixed.
- 2) Try to make an exponential model of the relaxation times using the surface free energy. The tough question remains of how to incorporate the friction or quantization.