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Contrast, Throughput, and Storage Time of Two-Mirror Cavities

Robert E. Spero

March 2, 1989

1 Quantities of Interest

Consider the standard reflection-locking scheme for a two-mirror cavity. Let $I =$ intensity of light on photodiode, $A =$ amplitude of component on diode leaking out of cavity, $K =$ contrast, and $M =$ fraction of light correctly mode-matched. Normalize to incident intensity $= 1$. Then

$$\begin{aligned} K &= 1 - I \\ &= M\left(1 - |1-A|^2\right) \end{aligned} \tag{1}$$

2 The Leakage Field

On resonance (and with zero modulation),

$$\begin{aligned} A &= t_1^2 r_2 \left(1 + r_1 r_2 + (r_1 r_2)^2 + \ldots\right) \\ &= \frac{T_1\sqrt{R_2}}{1-\sqrt{R_1 R_2}} \end{aligned} \tag{2}$$

where $r_1, t_1, \ldots$ are amplitude reflectivities and transmissions, and $R_1, T_1, \ldots$ are energy reflectivities and transmissions.

Now $R_2 \approx 1$, and $R_{1,2} + T_{1,2} + L_{1,2} = 1$ where $L_{1,2} =$ absorption and scattering loss. Then,

$$1 - \sqrt{R_1 R_2} \approx (L/2)(1 + \gamma_1 + \gamma_2) \tag{3}$$

where $L = L_1 + L_2$ and $\gamma_{1,2} = T_{1,2}/L$. Therefore,

$$A = \frac{2\gamma_1}{1 + \gamma_1 + \gamma_2} \tag{4}$$

3 Contrast and Throughput

3.1 General Cavity

Combining (1) and (4),

$$K = \frac{4M\gamma_1(1+\gamma_2)}{(1+\gamma_1+\gamma_2)^2} \tag{5}$$

The contrast has a maximum value of M when the transmission of the input mirror matches the sum of the scattering and absorption losses of both mirrors plus the transmission loss of the end mirror: $\gamma_1 = \gamma_2 + 1$. The throughput, or power transmissivity of the cavity η is related to the cavity leakage field by $\eta = M|A|^2(\gamma_2/\gamma_1)$. From Equation (4),

$$\eta = \frac{4M\gamma_1\gamma_2}{(1+\gamma_1+\gamma_2)^2} \tag{6}$$

The throughput is unchanged if the cavity mirrors are interchanged (assuming the mode matching is the same). The throughput is always less than the contrast, and their ratio ψ is independent of the transmission of the input mirror:

$$\psi \equiv \eta/K = \frac{\gamma_2}{1+\gamma_2} \tag{7}$$

3.2 Mode Cleaner

A high throughput cavity such as a mode cleaner requires large γ_1 and γ_2. If $\gamma_1 = \gamma_2 = \gamma$, then

$$K = \frac{4M\gamma(1+\gamma)}{(1+2\gamma)^2} \tag{8}$$

$$\stackrel{\gamma \gg 1}{=} M\left[1-(2\gamma)^{-2}+\ldots\right] \tag{9}$$

$$\eta = \frac{4M\gamma^2}{(1+2\gamma)^2} \tag{10}$$

$$\stackrel{\gamma \gg 1}{=} M(1-\gamma^{-1}+\ldots) \tag{11}$$

A measurement of contrast, throughput, and storage time gives the reflectivity, loss, and transmission. Define the bounce number n

$$n = c \cdot \text{energy storage time/cavity length.}$$

Then

$$R_{1,2} = 1 - 1/n \tag{12}$$

$$T_{1,2} = \left(\frac{1}{n}\right)\frac{2\psi}{1+\psi} \tag{13}$$

$$L_{1,2} = \left(\frac{1}{n}\right)\frac{1-\psi}{1+\psi} \tag{14}$$

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