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Technical Note **LIGO-T070303-01-D** Date: 2007/12/20

Arm Cavity Finesse for Advanced LIGO

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1 Summary

In this note we re-evaluate the choice of finesse for the arm cavities in Advanced LIGO, examining the various issues that are affected by this parameter. The outcome of this evaluation is to recommend an arm cavity finesse of **450**, or a transmission of the input test masses of 1.4%. This is a factor of 2.8 reduction in finesse compared to the previously used value.

2 Basis of original choice of finesse

It is important to appreciate that in the power- and signal-recycled interferometer, if the optical losses in the recycling cavities are small compared to the arm cavity loss (which is generally the case), the quantum noise is independent of the arm finesse. The quantum noise depends fundamentally on the power stored in the arms, which in turn depends on the input power and the arm cavity loss. For the same stored power, a given quantum noise spectrum can be achieved with different arm finesses, by suitable tuning of the signal recycling.

The original choice of arm cavity finesse ($F = 1250, T_{\mathrm{ITM}} = 0.5\%$) was motivated by the large absorption characteristic of sapphire, when it was the leading candidate for test mass material. The goal was to limit thermal distortions by reducing the ITM substrate power as much as possible. Thus the arm finesse was increased, up to the point where SRC losses would become significant. With the choice of fused silica for the test mass material, substrate absorption became less of an issue. Yet substrate absorption was not insignificant, and the high arm finesse value was retained, but with the intention of re-evaluating the choice as the interferometer design progressed.

3 Arm loss and stored power

The target for the round trip loss in each arm cavity is 75 ppm. It is uncertain whether we will be able to reach this target; for comparison, the initial LIGO arm cavities exhibit loss up to about 150 ppm round trip. As stated above, in the limit that recycling cavity losses are negligible, only the arm cavity loss matters, not the arm finesse. Here we show that explicitly, and include some recycling cavity loss to make it more realistic; see Fig. 1. For the range of loss and arm finesse shown, the power gain dependence on finesse is 1.8% or less.

4 Arm finesse and recycling cavity loss

Here we consider the effect of non-negligible loss in the recycling cavities. Loss in the two recycling cavities pull in different directions for the arm finesse: power recycling cavity loss favors higher arm finesse (smaller recycling gain), and signal recycling cavity loss favors lower arm finesse. Note that a deficient power recycling gain can be compensated by higher input power, to reach a target arm stored power level. Signal recycling cavity loss, however,

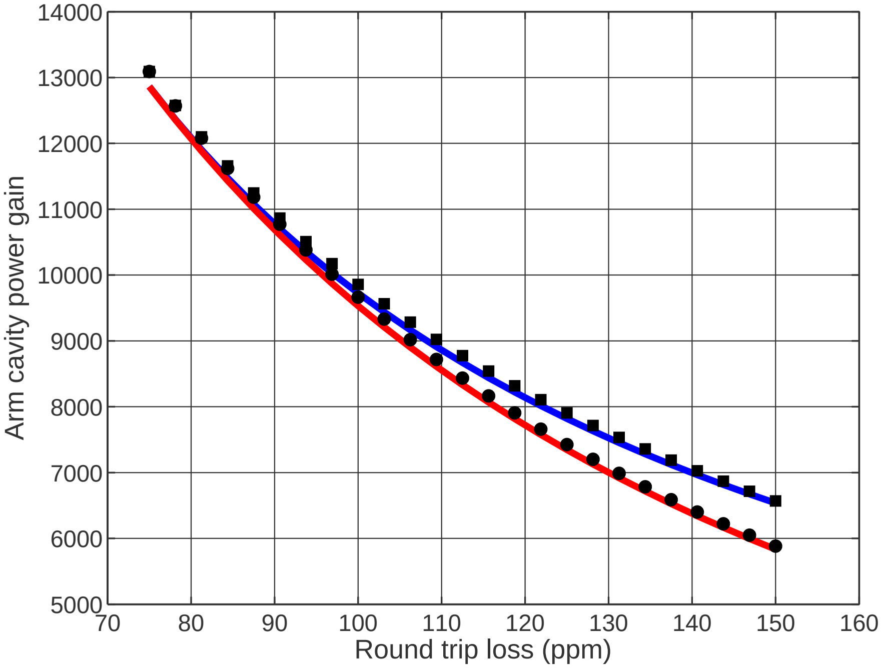

Figure 1: Power gain in the arm cavities, for 1 W incident on the interferometer, as a function of the round trip loss in the arms. A fixed power recycling mirror loss of 10^{-3} is included. The solid curves (red and blue) correspond to an arm finesse of 625, and the symbols correspond to a finesse of 1250. For the upper set (blue curve and black squares), the PRM transmission is optimized at each loss value. For the lower set (red curve and black circles), the PRM transmission is fixed at the optimum value for 75 ppm loss.

would need to be compensated by a higher stored power level, which brings with it increased thermal distortion and radiation pressure troubles. So for equal loss in the two recycling cavities, we would favor a lower arm finesse to minimize the effect of the SRC loss.

Quantitatively, the effect of SRC loss on the quantum noise has been calculated for an arm finesse of 1250[1], with the result that a round-trip SRC loss of 0.2% produces an increase in the quantum noise of about 5% at the two opto-mechanical resonances near 75 Hz and 200 Hz (for a NS-NS inspiral tuned interferometer). For the same degradation, a factor of 2 lower arm finesse would allow a factor of 2 higher SRC loss.

5 Substrate Absorption

Table 1 gives the estimated absorbed power in the ITMs for different parameters.

Substrate glass type	*Absorption coeff.*	*Surface absorption*	*Bulk absorption*
Suprasil 311	4 ppm/cm	0.24 – 0.4 W	$0.2 \cdot (1000/F)$
Suprasil 3001	0.25 ppm/cm	0.24 – 0.4 W	$< 0.0125 \cdot (1000/F)$

Table 1: Absorbed power in an ITM, with 800 kW stored in the arm cavity. The surface numbers are given for an absorption range of $0.3-0.5$ ppm, where the upper end corresponds to the nominal assumed value, and the lower end has been observed over a large fraction of the LASTI test mass coated at LMA-Lyon in spring 2007. F is the arm cavity finesse.

With the ultra-low absorption fused silica (Suprasil 3001, formerly known as 311SV), bulk absorption is just not an issue. The concern with this glass type has been that it might not have sufficient homogeneity to used for an ITM. However this has recently been tested with a full size sample of the glass, and the homogeneity was found to be acceptable (see LIGO-T070258-00). Thus, there is no longer a motivation to keep the power low in the ITM substrates (and beamsplitter and compensation plates), and this factor is removed from the arm cavity finesse choice.

6 Coating thermal noise

The lower the arm finesse, the fewer dielectric coating layers are needed on the ITM, reducing thermal noise due to coating mechanical loss. The effect is small, however, since the total amount of coating material is dominated by the ETM coating. Table 2 shows the estimated thermal noise impact for different values of finesse. Coating thermal noise thus rather weakly favors lower finesse. The effect would be even weaker for the newly favored asymmetric cavity design, where the ITM beam size is a bit smaller and the ETM beam size a bit bigger.

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[1] http://ilog.ligo-wa.caltech.edu:7285/advligo/SignalRecyclingCavityLoss

Finesse	T_{ITM}	*No. ITM layers*	*Total no. layers*	*Rel. thermal noise*
450	1.5%	6	24	0.97
625	1.0%	7	25	0.98
1250	0.5%	8	26	1

Table 2: Dependence of coating layers and thermal noise on arm finesse. The number of layers (each layer being a pair of low- and high-index materials) and the coating thermal noise are calculated with the Bench formulae. The number of ETM coating layers is 18 (5 ppm transmission), and the beam size is the same on the ITM and ETM (6 cm radius). The thermal noise (at 100 Hz) is given relative to the high finesse case, and scales approximately as the square root of the number of layers.

7 Coupling of auxiliary degrees-of-freedom

Noise in the Michelson couples into the GW channel proportionally to the inverse of the arm finesse, so this argues for higher finesse. Thermal noise of the beamsplitter suspension is the limiting Michelson noise, and we can use this to set a minimum arm finesse. Beamsplitter displacement noise (along its optic axis) is less important than test mass noise by a factor $\sqrt{2}F/\pi$. According to Bench, the test mass differential TM suspension thermal noise is $1.0 \times 10^{-21}\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 100 Hz. From M Barton's model, the beamsplitter suspension thermal noise (wire suspension, horizontal noise only, since the wedge is horizontal) at 100 Hz is $1.5 \times 10^{-20}\mathrm{m}/\sqrt{\mathrm{Hz}}$. So if we require the beamsplitter thermal noise to be 10 times below the test mass thermal noise, we need:

$$\begin{aligned} 1.5 \times 10^{-20} &\leq \frac{10^{-21}}{10}\frac{\sqrt{2}F}{\pi} \\ F &\geq 350\,. \end{aligned}$$

Of course, a finesse higher than this minimum would provide added margin for excess Michelson noise (e.g., higher than anticipated suspension thermal noise, or Michelson sensing noise that cannot be sufficiently removed from the GW channel).

Coupling of the signal recycling cavity length to the GW channel has been investigated with Optickle, with the finding that the coupling can be parametrized as:

$$K = 0.02 \cdot \left(\frac{10\,\mathrm{Hz}}{f}\right)^2 \left(\frac{P_{arm}}{750\,\mathrm{kW}}\right)\left(\frac{0.01}{T_{ITM}}\right)\left(\frac{DARM_{off}}{12\,\mathrm{pm}}\right).$$

The coupling K is equivalent differential arm (DARM) displacement per signal recycling mirror displacement (meter/meter); P_{arm} is the power stored in each arm cavity; $DARM_{off}$ is the offset applied for the DC readout. Thermal noise of the SRM suspension is the limiting SRC noise. At 20 Hz, it is estimated to be $1.5 \times 10^{-18}\mathrm{m}/\sqrt{\mathrm{Hz}}$, for a wire suspension. For the parameters in the expression above (finesse of 625), the equivalent DARM noise would be $7.5 \times 10^{-21}\mathrm{m}/\sqrt{\mathrm{Hz}}$ (at 20 Hz), which is a factor of 5 below the test mass suspension thermal noise at 20 Hz. At 10 Hz, the SRM thermal noise would be closer to the TM thermal noise, but this is with high power operation, where quantum radiation pressure noise is higher than TM thermal noise anyway. In summary, $F = 625$ is a reasonable upper limit to the arm finesse, based on SRC noise coupling.

8 Arm loss and DC readout

For DC readout of the GW channel, part of the local oscillator carrier field comes from a loss difference between the arms. How does this depend on the arm finesse? Assume there is a given loss difference, dL, independent of finesse. As a fraction of the beamsplitter power, the carrier power at the output due to loss difference scales as: $(dL/T_{\rm ITM})^2$. The beamsplitter power scales as $T_{\rm ITM}$ (higher power recycling gain with higher $T_{\rm ITM}$), so as a fraction of input power, the output carrier power scales as: $dL^2/T_{\rm ITM} \propto dL^2 F$. To set the scale, here are power levels for an arm finesse of 1250, and an input power of 125 W:

	Avg. arm loss: 70 ppm $dL = 40\,ppm$	*Avg. arm loss: 140 ppm* $dL = 80\,ppm$
BS power	2.25 kW	1.11 kW
Output power	2.2 mW	4.3 mW

The DARM offset is applied to then significantly increase the output carrier power, to achieve the desired homodyne detection phase (or close to it). So in principle a larger carrier power due to loss difference will lead to a larger total detected power. Fig. 2 show how this works out quantitatively. We see that there is very little gain in increasing the carrier power more than 10 times the zero-offset level. Furthermore, for the high-finesse (1250), high-dLoss (80 ppm) case, the resulting detected power is still only $\approx 50\,\mathrm{mW}$.

9 Lock acquisition

In the most simple acquisition paradigm the arm cavity is freely swinging and the time during which the standard PDH signal just goes down with Finesse: $\tau = \lambda/(2Fv_{fringe})$

where v_{fringe} is the arm fringe velocity. Without an SPI or any other exotic lock acquisition schemes, $v_{fringe} \simeq 0.3\mu m/s$ RMS, mostly dominated by microseismic fluctuations of the SEI platform. RMS fringe crossing times are listed below:

	$v_{fringe} : 0.1\mu m/s$	$v_{fringe} : 0.3\mu m/s$
$F = 1250$	4.2 ms	1.4 ms
$F = 450$	11.8 ms	3.9 ms

In addition to the direct effect of having more time to stop the mirror, the lower finesse allows for fringe linearization algorithms to work over a wider range.

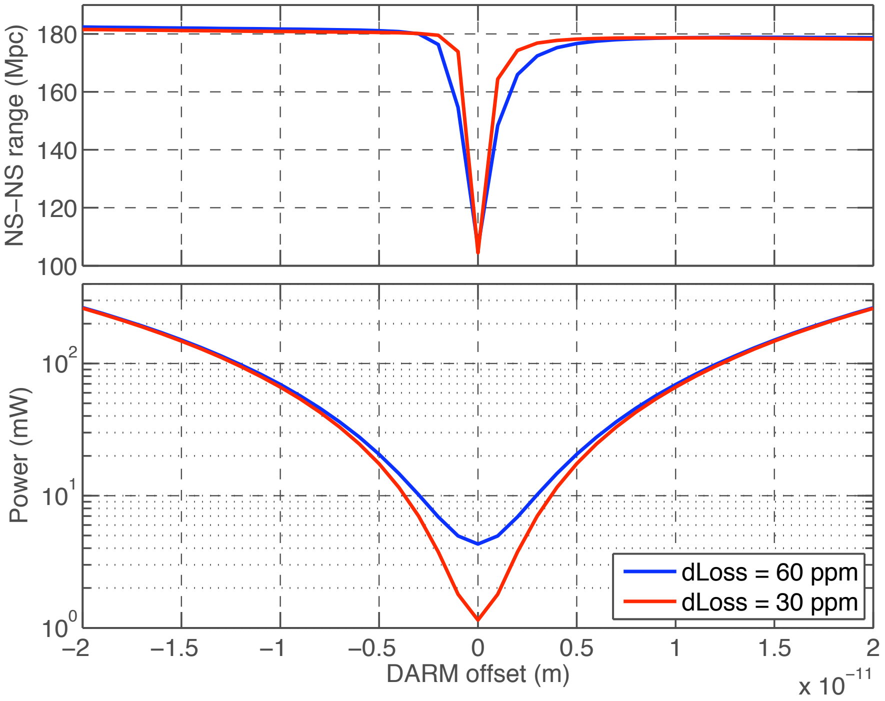

Figure 2: Detection port carrier power (lower) and NS-NS inspiral range (upper), as a function of DARM offset, for two values of differential arm loss. The arm finesse in both cases is 625, but since the zero-offset power scales as $dLoss^2F$, the two curves can also represent two values of finesse.

10 Non-Signal Recycled Mode

An attractive option is to start off commissioning Advanced LIGO without signal recycling, aiming for operation at moderated input power (10-25 W). The similarity of this configuration to initial LIGO could be expected to allow a relatively short commissioning period. So it is worth looking at how the arm finesse affects the potential sensitivity in such a mode; see Fig. 3. We see there is a modest preference for lower arm finesse.

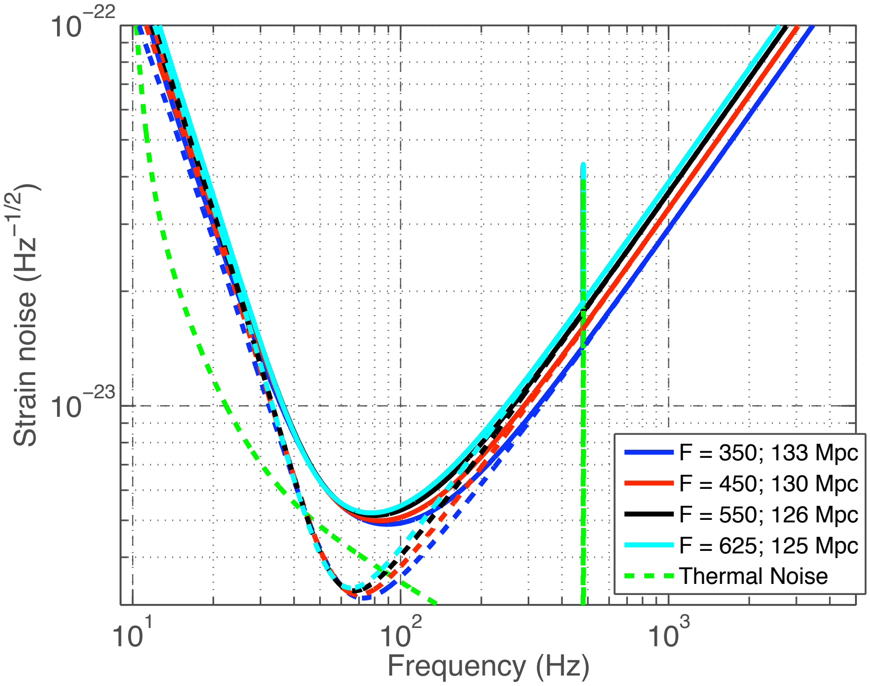

Figure 3: Strain noise with no signal recycling, for different values of arm cavity finesse, and an input power of 25 W. The dashed curves are quantum noise only, and the solid curves are total noise. The thermal noise curve is the sum of the suspension and test mass internal thermal noise. The range numbers are for NS-NS inspirals.

11 High Frequency Tuned Configuration

For the initial signal recycling mirror transmission (chosen for good broadband, or NS-NS inspiral response) the difference in quantum noise for the arm finesse range being considered is negligible. However, when we operate with a smaller SRM transmission for better high-frequency tuned performance, loss in the signal recycling cavity becomes more important. This favors low arm finesse, as shown in Fig. 4.

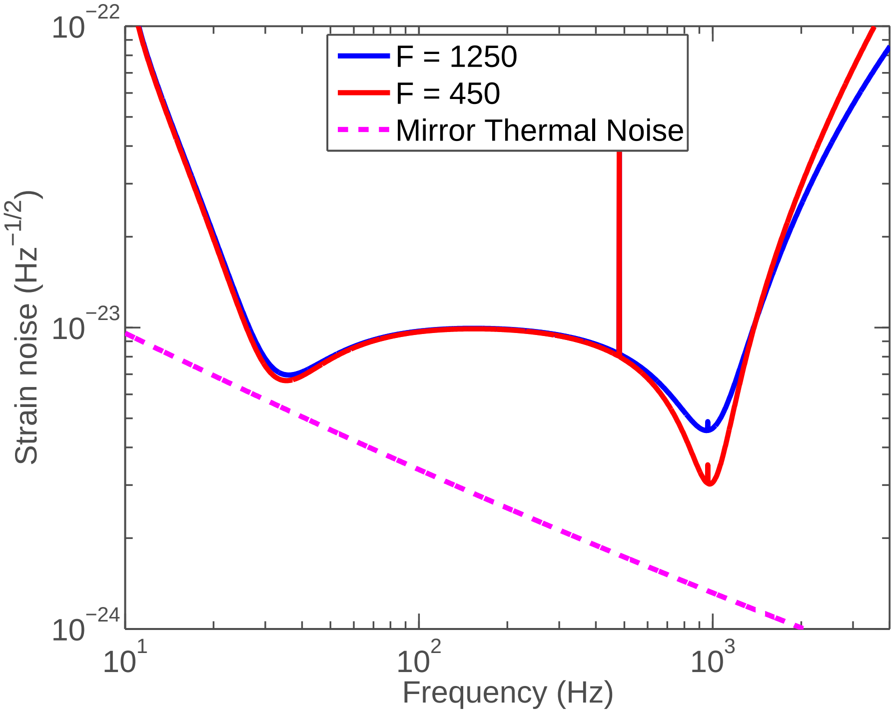

Figure 4: Strain noise for a high-frequency tuned configuration, for two values of the arm finesse. In each case, the SRM transmission is set equal to the ITM transmission. The lower arm finesse case has lower noise at the 1 kHz tuning frequency because of loss in the signal recycling cavity; the SRC loss in each case is taken to be 0.0085. The input power is 125 W in both cases. Because there is also loss in the power recycling cavity, the low finesse arm power (814 kW) is lower than the high finesse arm power (826 kW).

12 Conclusion

We are clearly motivated to reduce the arm finesse from the historical value of 1250. Based on the factors discussed in this note, we propose an arm cavity finesse of 450.