Making use of Spring Compensation system for 2K

Eiichi Hirose[1], Keita Kawabe[2]
[1]Department of Physics, Syracuse University, Syracuse NY 13244
[2]LIGO Hanford Observatory, Richland WA 99354
May 25, 2007

Abstract

Radiation pressure effect on ASC was recently measured in both H1 and H2, and the results agreed to our simulation very well [1]. Since angular control system does not count radiation pressure effect, the system is getting ‘off-designed’ state when cavity laser power goes up. One way to prevent this problem is to feed torque due to radiation pressure back to each mirror in opposite direction and make radiation pressure invisible to ASC. There already exists front-end compensation system called SPRINGCOMP implemented by Sigg, Evans et al [2], which has not been in use so far. Using knowledge in [1], we calculate some values to make use of the system. This is internal technical document for LSC (LIGO Scientific Collaboration).

1. WFS sensing matrix

Driving each mirror at a certain frequency without control and reading each WFSs’ signal, we measured sensing matrix for both pitch and yaw. This can be done by using 9.7Hz notch filters [3]. Below are summary of the results. More complete results can be found at [4].

Table 1: Yaw sensing matrix measured on May 18, 2007 [ct per micro radian]

	ETMX	ETMY	ITMX	ITMY	RM	BS	MMT3
WFS1Q	186898.5	348784.7	55134.7	84034.5	12888	11167.5	4359
WFS2I	2661.1	6911.2	49630.4	82621	362955	219090.4	47892.6
WFS2Q	644.5	535.5	8743.4	13071.7	4322.1	36856.5	4821.4
WFS3I	1856.8	3090.2	4721	6837.7	53066.1	23158.3	63897.5
WFS4I	11303.5	21923	2210.1	3636	19504.5	5461.9	44572.2
QPDX	14505.5	146.9	7517.1	73.1	99.2	125.1	77.6
QPDY	79.2	29225.5	64.1	11122.8	77.8	68.6	292

Table 2: Pitch sensing matrix measured on May 18, 2007 [ct per micro radian]

	ETMX	ETMY	ITMX	ITMY	RM	BS	MMT3
WFS1Q	332479.9	606861.7	70524.8	88978.3	1991.7	3742.8	1704.7
WFS2I	4064	20105.5	53210.6	111818.2	242927.1	181518.8	65323.2
WFS2Q	515.8	3778.8	10172.9	18751.1	388.4	33673	4766.5
WFS3I	4147.1	5819.1	5047.4	9995.5	35754.6	18917.2	63874.7
WFS4I	30726.8	58493.8	2259.8	2652.9	29377.8	15374.5	55435.4
QPDX	32292.2	2269.6	11467.9	825.7	181.8	175.9	1559.1
QPDY	1809.4	71951.1	755	21949.5	149.5	63.9	199

Unit here is ct per micro radian. For instance, if we excite ETM mirror by one micro radian, WFS2Q yaw senses the excitation by 644.5 ct. The code calASC.m was used for this calculation [5], in which calibration of optical lever done by D. Cook in 2003 is being used.

Inverse of these matrices will give us information that how much each mirror tilt when each WFS senses some signals.

Table 3: Inverse of yaw sensing matrix [micro radian per ct]

	WFS1	WFS2A	WFS2B	WFS3	WFS4	QPDX	QPDY
ETMX	1.33E-03	-8.53E-04	-1.73E-05	1.10E-02	-1.50E-02	-6.56E-03	-5.51E-03
ETMY	-8.13E-04	5.27E-04	1.01E-05	-6.83E-03	9.27E-03	4.00E-03	3.35E-03
ITMX	-2.56E-03	1.65E-03	3.31E-05	-2.13E-02	2.90E-02	1.28E-02	1.06E-02
ITMY	2.14E-03	-1.39E-03	-2.68E-05	1.80E-02	-2.44E-02	-1.05E-02	-8.74E-03
RM	-3.84E-05	2.80E-05	-1.74E-05	-3.23E-04	4.38E-04	1.90E-04	1.57E-04
BS	-1.65E-04	1.10E-04	3.05E-05	-1.43E-03	1.94E-03	7.71E-04	6.33E-04
MMT3	5.28E-05	-3.71E-05	3.84E-06	4.65E-04	-6.08E-04	-2.58E-04	-2.13E-04

Table 4: Inverse of pitch sensing matrix [micro radian per ct]

	WFS1	WFS2A	WFS2B	WFS3	WFS4	QPDX	QPDY
ETMX	-1.15E-04	7.19E-06	-9.77E-07	-4.81E-04	5.26E-04	7.49E-04	5.60E-04
ETMY	5.23E-05	-3.14E-06	3.24E-07	2.05E-04	-2.24E-04	-3.37E-04	-2.50E-04
ITMX	3.27E-04	-2.01E-05	2.10E-06	1.36E-03	-1.49E-03	-2.03E-03	-1.59E-03
ITMY	-1.73E-04	1.04E-05	-1.02E-06	-6.80E-04	7.43E-04	1.11E-03	8.74E-04
RM	1.06E-05	3.95E-06	-2.30E-05	3.89E-05	-4.58E-05	-6.78E-05	-5.31E-05
BS	-6.64E-06	8.64E-07	2.93E-05	-5.09E-05	5.46E-05	2.16E-05	1.33E-05
MMT3	8.12E-09	-2.69E-06	4.24E-06	2.05E-05	-2.76E-06	1.22E-07	1.04E-06

For example, suppose each WFS senses a signal $s_1, s_{2A}, s_{2B}, s_3, s_4, s_{QX}, s_{QY}$ in ct, we can, in principle, deduce pitch angle of ETMX by the following way.

$$\begin{aligned}\theta_{ETMX} &= -1.15\times10^{-4} s_1 + 7.19\times10^{-6} s_{2A} - 9.77\times10^{-7} s_{2B} \\ &\quad - 4.81\times10^{-4} s_3 + 5.26\times10^{-4} s_4 + 7.49\times10^{-4} s_{QX} + 5.60\times10^{-4} s_{QY} \quad [\mu rad]\end{aligned} \tag{1}$$

Red values in the tables will be used in SPRINGCOMP system.

2. Torque due to radiation pressure

In a cavity, two mirrors are not independent, but instead they are coupled. And, there is a relation between mirror's tilt angle and point on which laser is acting [6].

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{L}{1-g_1 g_2} \begin{pmatrix} g_2 & 1 \\ 1 & g_1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \tag{2}$$

, where $g_1 = 1 - L/R_1$, $g_2 = 1 - L/R_2$, L and R are cavity length and radius of curvature of each mirror respectively.

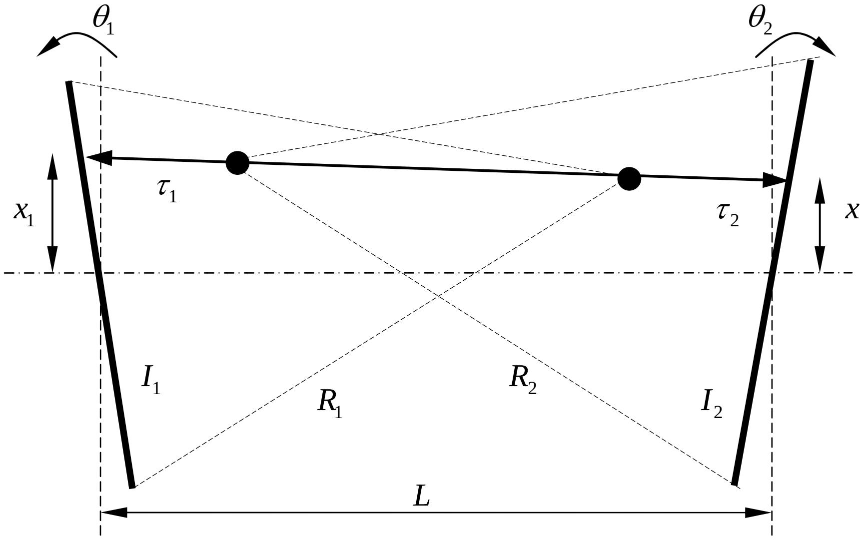

Figure 1: Schematic of a cavity. θ here can be either yaw or pitch.

Torque due to laser power stored in the cavity is simply given by

$$\tau_i = \frac{2Px_i}{c}, \tag{3}$$

where P is laser power in the cavity and x_i is distance from the center and c is sped of light. In order to estimate laser power stored in cavity, we use result of [1], where we measured yaw resonant frequency shift due to radiation pressure by changing input PSL power (figure 2). PSL powers in x axis are measurement points, while y axis shows estimated power stored in cavity by our Simulink® model. (We fit resonant frequency by changing laser power in the model) Equations in the plot are linear trend that we forced passing the origin. For 2K, relation between PSL power and cavity power P is then,

$$P = 3.7931 \times 10^3 PSL \quad [W] \tag{4}$$

Using (1), (2), (3), and (4), we can associate WFS output signal with torque due to radiation pressure. Real expression for (2) will become the following.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3912.245 & 5361.224 \\ 5361.224 & 4606.122 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \tag{5}$$

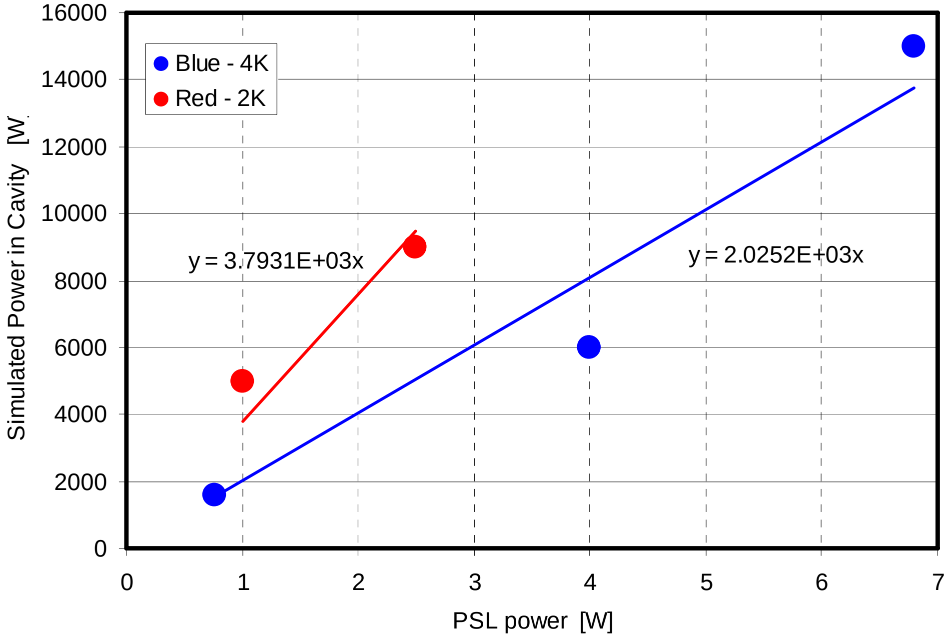

Figure 2: Cavity power estimation from [1]

Here, values of arm length, radius of curvature, and g parameters are 2000m, 14200m(ITM), 7400m(ETM), 0.8592(g1), and 0.7297(g2) respectively.

3. Coil count for producing a torque

Finally, we need to know how many counts we need to send to each coil to cancel the above torque produced by radiation pressure. For this purpose, we use DC calibration values done by V3 length calibration.

$$\frac{dx}{dETMX_{EXC}} = \frac{(-0.860nm/ct)}{1-(f/0.749Hz)^2} \tag{6}$$

$$\frac{dy}{dETMY_{EXC}} = \frac{(-0.896nm/ct)}{1-(f/0.764Hz)^2} \tag{7}$$

$$\frac{dy}{dITMX_{EXC}} = \tag{8}$$

$$\frac{dy}{dITMY_{EXC}} = \tag{9}$$

Therefore, force driven by each coil per count can be calculated as follows.

$$
\begin{aligned}
F_{coil} &= 0.25 \times m\omega^2 x \\
&= 0.25 \times 10[kg] \times (2\pi f)^2 \times DC\,[nm/ct] \qquad (10) \\
&= 157.9 \times f^{\,2} \times DC\,[\text{nN/ct}]
\end{aligned}
$$

Distance between coils attached on each mirror are $l_1 = 161.65m$, $l_2 = 161.39m$ respectively.

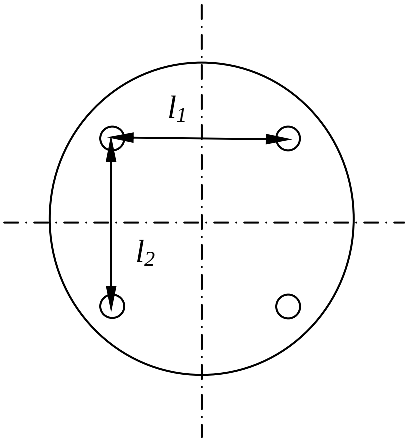

Figure 3: Distance between coils

Therefore, if we use all four coils (two coils in each direction), immediately we can calculate torque per count in pitch and yaw.

$$\tau_{pitch} = 50.97 \times f^{\,2} \times DC\,[nNm/ct] \qquad (11)$$

$$\tau_{yaw} = 51.05 \times f^{\,2} \times DC\,[nNm/ct] \qquad (12)$$

Thus, using (3), (11), and (12), we will get how many counts to be sent to each coil to cancel radiation pressure.

References

[1] E. Hirose, K Kawabe, G070118-00, http://www.ligo.caltech.edu/docs/G/G070118-00.pdf
[2] D. Sigg, M. Evans, unpublished?
[3] ~cvs/lho/scripts/h2/ASC/sense3
[4] ~cvs/lho/scripts/h2/ASC/yawH2_070518_1624.txt | pitchH2_070518_1624.txt
[5] ~cvs/lho/scripts/h2/ASC/calASC.m
[6] Siegman, Lasers, University Science Books