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Torsional Modes in Suspension Fibres/Ribbons

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1 Introduction

1.1 Purpose and Scope

With the move to flat ribbons rather than round fibres or wires in the final stage of the suspension, torsional modes are likely to be at rather lower frequencies because of the lower torsional elasticity and the greater moment of inertia for the same cross section, and the question arises whether they are likely to be an issue. This document presents the relevant theory and applies it to the AdvLIGO monolithic stage.

1.2 Version history

9/17/07: Pre-rev-00 draft #1.

9/19/07: Pre-rev-00 draft #2. Coupling to GW analysis added. Conclusions revised. Yaw thermal noise plot added.

9/20/07: Rev. 00.

1.3 References

L.D. Landau and E.M. Lifschitz, Theory of Elasticity (Course of Theoretical Physics, Volume 7), 3rd Edition, 1986.

R.J. Roark, Formulas for Stress and Strain, 5th Edition, 1975.

P. Willems et al., Search for stress dependence in the internal friction of fused silica, Physics Letters A 319 (2003) 8–12.

2 Theory

2.1 Intrinsic rigidity

For the purposes of torsional modes, a ribbon (or a round wire/fibre) can be treated as a beam. There are two components to the torsional rigidity, one intrinsic due to the elastic modulus of the material, and one induced by longitudinal tension. We consider the intrinsic component first. The intrinsic torsional rigidity of a beam is given by

$$R_{intrinsic} = \frac{GJ_e}{l}$$

where l is the length,

$$G = \frac{E}{2(1+\sigma)}$$

is the shear modulus (E and σ are the Young's modulus and Poisson's ratio), and J_e is the effective polar moment of area. For a round wire/fibre of radius r, J_e is equal to the actual polar moment of area,

$$J = \int r^2 \cdot dA$$

that is,

$$J_{e(fibre)} = J_{fibre} = \frac{1}{2}\pi r^4$$

and for other cross sections, it is somewhat smaller, relative to either the polar moment itself or the cross-sectional area. In general the computation of J_e is quite involved (see e.g., Landau and Lifschitz, §16, p.59ff) and a simple formula for J_e can only be given in a handful of simple geometries. The formula for a rectangular beam of width w and thickness t comes out in terms of an infinite series but can be approximated (Roark, p. 290) as

$$J_{e(ribbon)} = wt^3\left(\frac{1}{3} - \frac{3.36}{16}\frac{t}{w}\left[1 - \frac{t^4}{12w^4}\right]\right)$$

2.2 Stress-induced rigidity

If the beam is under tension $T = \sigma_T A$, where σ_T is tensional stress and A is area, then (Roark, p.317) there is an additional component of torsional rigidity

$$R_{stress} = \frac{\sigma_T J}{l}$$

(note that this term really does involve the polar moment of area J, not J_e, regardless of the type of cross-section). This is typically negligible for a round wire/fibre but can be significant for a thin ribbon. For a ribbon of rectangular cross-section,

$$J_{ribbon} = \frac{tw}{12}\left(t^2 + w^2\right)$$

2.3 Torsional waves

A beam as in the previous section supports torsional waves of velocity

$$v_T = \sqrt{\frac{Rl}{\rho J}} = \sqrt{\frac{J_e G + \sigma_T J}{\rho J}}$$

where ρ is mass density, ρJ is mass moment of inertia per unit length, and $1/Rl$ is torsional compliance per unit length. The boundary conditions are similar to those for violin modes, with a node at each end, and force the usual harmonic pattern of mode frequencies:

$$f_n = \frac{n v_T}{2l}$$

for n=1,2,3,… .

3 Application

3.1 AdvLIGO parameters

The following parameters were drawn from the Mathematica code of case `20061213TM` of the Quad Xtra-Lite Lateral model, which is the current reference model of the AdvLIGO Quad suspension.

```
vribbon = Recurse[{
   g -> 9.81,
   t -> 0.000115,
   w -> 0.00115,
   m -> 39.5696,
   A -> t w,
   J -> t w (t^2 + w^2)/12,
   Je -> w t^3 (1/3 - 3.36/16 t/w (1 - t^4/12/w^4)),
   sigmaT -> m g/4/A,
   Gsilica -> Ysilica/2/(1 + sigmasilica),
   Ysilica -> 7.27*^10,
   sigmasilica -> 0.17,
   rhosilica -> 2202,
   l -> 0.6,
   v1 -> Sqrt[Je Gsilica/rhosilica/J],
   f1 -> v1/2/l,
   v2 -> Sqrt[sigmaT/rhosilica],
   f2 -> v2/2/l,
   vv -> Sqrt[v1^2 + v2^2],
   ff -> Sqrt[f1^2 + f2^2]
}]
```

Evaluating gives

```
{g → 9.81, t → 0.000115, w → 0.00115, m → 39.5696,
 A → 1.3225 × 10^-7, J → 1.47208 × 10^-14, Je → 5.46273 × 10^-16,
 sigmaT → 7.33795 × 10^8, Gsilica → 3.10684 × 10^10, Ysilica → 7.27 × 10^10,
 sigmasilica → 0.17, rhosilica → 2202, l → 0.6, v1 → 723.585,
 f1 → 602.988, v2 → 577.27, f2 → 481.058, vv → 925.644, ff → 771.37}
```

That is, the two elasticity terms separately would give fundamental torsional mode frequencies of `f1`=602 Hz (intrinsic) and `f2`=577 Hz (stress), for an RMS total of `ff`=771 Hz. A model of this system in the finite-element program ANSYS gave 770 Hz, which is good agreement. (The final frequencies may be slightly different because, among other slight variations in parameters, the shape of the real ribbons will inevitably be more of a round-cornered rectangle due to surface-tension effects during pulling.)

By contrast, with a fused silica fibre of the same cross-sectional area, the intrinsic component is much bigger (`f1`=3130 Hz) and the stress component is slightly smaller (`f2`=481 Hz) so as to be almost negligible in the RMS sum: `ff`=3167 Hz.

3.2 Effect on GW signal

The torsional modes should not appear to first order in the longitudinal displacement of the front face of the optic because they represent a torque noise signal, but not a linear force, so the optic will simply rotate about its COM. However they may be visible above other thermal noise in the yaw displacement, depending on the Q, and could conceivably couple in via some second-order effect.

There are three effects that need to be taken into account to estimate the Q: structural damping, thermoelastic damping and dissipation dilution. The measurements of Willems et al. (2003) give some clues as to the material loss. The low frequency torsion mode of two masses in their experiment had a Q of roughly half the vertical bounce mode, consistent with most of the structural loss being from surface effects.

Of the two rigidity terms discussed above, only the intrinsic one should contribute to damping and thermal noise because it is associated with first order strain changes of the elastic material. Conversely, the stress-induced rigidity will be dissipation diluted. By contrast with the violin mode where the tension contributes the vast majority of the rigidity, with torsion it is less than half, so dilution will not be very significant even for a ribbon (and negligible in the case of a round fibre, as in Willems et al.).

According to Phil Willems (email to Mark Barton, 18 September 2007), "for pure shear deformation there is no thermoelastic damping because there is no temperature change in the ribbon (this follows from the fact that thermal expansion in glass is purely volumetric)". Since the torsion mode is purely shear, this condition is satisfied. And while there may be slight departures from this in the neck regions, because there is little dilution and other dissipation is uniformly distributed, end effects are likely to be small.

Because the lack of thermoelastic damping tends to cancel the lack of dissipation dilution, the Q's of the violin and torsional modes are likely to be not so dissimilar. The case `20061213TM` assumes 1.18×10^{-7} for the fibre longitudinal damping. Assuming half this for the intrinsic torsion, the torsional modes should be just visible in the yaw thermal noise, as seen in Figure 1. (In generating this figure, use was made of an add-on package to Mark Barton's pendulum modeling toolkit, ViolinModes.nb, contributed by Ben Lee of the University of Western Australia.)

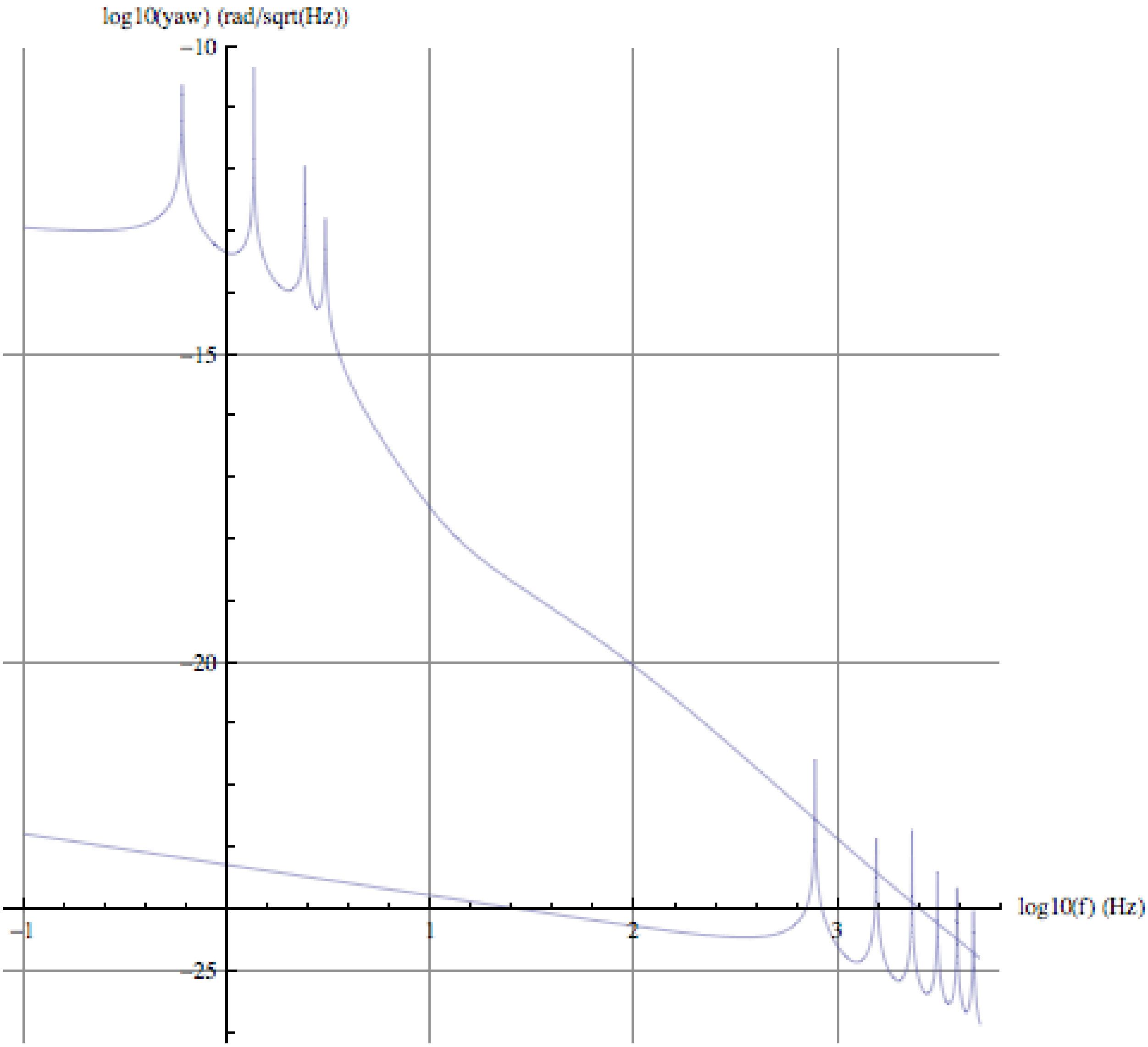

Figure 1: Yaw thermal noise – from torsional modes, other, sum