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Beam Jitter Noise Contribution from Suspensions in IO

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1 Introduction

1.1 Purpose

This document describes the beam jitter contribution from various optical components in the IOO chain. The main source beam jitter is the seismic vibrations that are filtered through the suspensions. This document analyzes marginally stable cavity design where the mod ematching telescope mirrors are mounted on SOSs.

1.2 Scope

This document is prepared for the purpose of analyzing if the motion sensitivity to beam jitter in optical components in IO requires triple suspensions. Typical readers of this include people involved in designing input optics, alignment sensing, and control system.

1.3 Definitions

1.4 Acronyms

1.4.1 LIGO Documents

1. P. Fritchel, “HAM Seismic Isolation Requirements,” LIGO Technical Note, T060075-00-D.

1.4.2 Non-LIGO Documents

2. G. Mueller, "Beam jitter coupling in advanced LIGO," Opt. Express **13**, 7118-7132 (2005) http://www.opticsinfobase.org/abstract.cfm?URI=oe-13-18-7118

2 General description

The requirement on the beam jitter $\Delta\phi$ of the beam after the mode cleaner at a given frequency in terms of the amplitude of TEM_{10} mode is given by $|a_{10}| \le 7\cdot 10^{-9}\sqrt{1+\left(\frac{230Hz}{f}\right)^4}\ /\sqrt{\mathrm{Hz}}$ assuming a misalignment of $10^{-9}\,\mathrm{rad}$ for the ITM.[2] At 10 Hz this leads to $|a_{10}| \le 3.7\times10^{-6}/\sqrt{\mathrm{Hz}}$. Thus the total angular beam jitter contribution from the input optics should be less than this value.

The input section has one mode cleaner cavity that suppresses the beam jitter from the PSL by a factor 0f 250 (i.e., by 0.004). All the steering mirrors and the MMT mirrors contribute to the beam jitter also. For the case of steering mirrors and MMT mirrors, the contribution to the noise is suppressed by the transfer function of their respective suspension systems. The contribution of each component to angular beam jitter can be calculated by using ABCD matrices from the component to the final point in the chain. We have selected AR side of PRM as the point of evaluation. Using ABCD law, the contribution to the beam jitter is given by:

$$\Delta\tilde{\phi}_x = \sqrt{|A\times\tilde{x}|^2 + |B\times\tilde{\alpha}|^2} \quad \text{and} \qquad \textbf{(1)}$$

$$\Delta\tilde{\phi}_\alpha = \sqrt{|C\times\tilde{x}|^2 + |D\times\tilde{\alpha}|^2}$$

Here $\tilde{x}$ is the frequency dependent displacement due to HAM chamber and $\tilde{\alpha}$ is the pitch or yaw of the optical component due to HAM motion. $\Delta\tilde{\phi}_x$ is the contribution to beam jitter due to displacement fluctuations and $\Delta\tilde{\phi}_\alpha$ is the contribution due to angular fluctuations.

Taking into account the suppression due to the optical mount, this can be written as:

$$\Delta\tilde{\phi}_x = \sqrt{|A\cdot\tilde{x}\cdot\tilde{\gamma}_x|^2 + |B\cdot\tilde{\alpha}\cdot\tilde{\gamma}_\alpha|^2 + |B\cdot\tilde{x}\cdot\tilde{\gamma}_{x\alpha}|^2} \quad \text{and} \qquad \textbf{(2)}$$

$$\Delta\tilde{\phi}_\alpha = \sqrt{|C\cdot\tilde{x}\cdot\tilde{\gamma}_x|^2 + |D\cdot\tilde{\alpha}\cdot\tilde{\gamma}_\alpha|^2 + |D\cdot\tilde{x}\cdot\tilde{\gamma}_{x\alpha}|^2}$$

where $\tilde{\gamma}_x$ is the damping to displacement (i.e., *x* to *x* coupling mainly), $\tilde{\gamma}_\alpha$ is damping to angular motion (i.e., yaw to yaw mainly), and $\tilde{\gamma}_{x\alpha}$ is the damping to displacement dependent angular motion (i.e., *x* to pitch mainly). Note that here we have only considered the dominant coupling mechanisms. The transfer functions for suspensions are taken from private communication with Mark Barton at University of Glasgow. For the case of fixed mounts, we can take these two damping factors as 1. All other degrees of freedom have much lower damping factors. Note that the resultant contribution to the beam jitter by these mechanisms is being added incoherently. Note that the amplitude of the TEM_{10} mode is related to these two contributions according to:

$$\left|a_{10}\right|_{\Delta\phi} = \sqrt{\left|\Delta\tilde{\phi}_x \cdot \frac{1}{w}\right|^2 + \left|\Delta\tilde{\phi}_\alpha \cdot \frac{\pi w}{\lambda}\right|^2} \qquad \textbf{(3)}$$

where w is the beam radius.

The targeted displacement from HAM chamber $\tilde{x}$ is $4.0\times10^{-11} m/\sqrt{Hz}$ at 10 Hz while yaw angular motion $\tilde{\alpha}$ is $2.0\times10^{-12} rad/\sqrt{Hz}$ at 10 Hz.[2] The PSL beam jitter specifications are $374\times10^{-6}/\sqrt{Hz}$ for the amplitude of the TEM_{10} mode at 10 Hz. Allocating equal amount of contribution from angular and displacement beam jitter and multiplying with the conversion factors of w for displacement and $\lambda/(\pi w)$ for angle, $\tilde{x}_{PSL}$ is $5.3\times10^{-7} m/\sqrt{Hz}$ at 10 Hz angular motion $\tilde{\alpha}_{PSL}$ is $4.48\times10^{-8} rad/\sqrt{Hz}$ at 10 Hz. Mode cleaner provides a filtering of about 1/250. Using this formulation, contribution from various components is calculated as shown in the Table below.

Table 1. Contribution to Displacement Beam Jitter from IOO Mirrors

Optical Component	|TF| from ABCD Matrice		Mounting					TF x Damp x Expected Motio		Total
	Dis. to angle A (m/m)	Angle to angle B m/rad	Type	Dis. Damping (γx) m/m at 10 Hz	Dis.-to-Pitch (γxα) rad/m at 10 Hz	Angular Damping (gα) rad/rad at 10 Hz	No. of Elements	From Dis. m/?Hz at 10 H	Angular m/?Hz at 10 H	m/?Hz at 10 Hz
PSL/MC	16.3	308.5	MC Filterin	0.004	0.004	0.004	1	3.4E-08	7.8E-08	8.5E-08
From Steering Mirr	16.3	269.6	SOS	0.01	0.025	0.0063	3	6.5E-12	2.7E-10	8.1E-10
MMT1	16.3	260.3	SOS	0.01	0.025	0.0063	1	6.5E-12	2.6E-10	2.6E-10
MMT2	16.3	14.3	SOS	0.01	0.025	0.0063	1	6.5E-12	1.4E-11	1.6E-11
MMT3	0.1	15.4	LOS	0.06	0.01	0.002	1	1.6E-13	6.2E-12	6.2E-12
									Total Beam J	8.53E-08
									Total Coh. B.	8.53033E-08

Table 2. Contribution to Angular Beam Jitter from IOO Mirrors

Optical Component	|TF| from ABCD Matrices		Mounting					TF x Damp x Expected		Total
	Dis. to angle C (rad/m)	Angle to angle D rad/rad	Type	Dis. Damping m/m at 10 Hz	Dis.-to-Pitch (gxa) rad/m at 10 Hz	Angular Damping (ga) rad/rad at 10 Hz	No. of Elements	From Dis. rad/?Hz at 10 Hz	Angular rad/?Hz at 10 Hz	rad/?Hz at 10 Hz
PSL/MC	0.019	0.291	MC Filtering	0.004	0.004	0.004	1	2.78E-12	5.20E-12	5.90E-12
From Steering Mirror	0.019	0.247	SOS	0.01	0.025	0.0063	3	7.44E-15	2.47E-13	7.40E-13
MMT1	0.019	0.236	SOS	0.01	0.025	0.0063	1	7.44E-15	2.36E-13	2.36E-13
MMT2	0.019	0.078	SOS	0.01	0.025	0.0063	1	7.44E-15	7.77E-14	7.81E-14
MMT3	0.069	1	LOS	0.06	0.01	0.002	1	1.66E-13	4.00E-13	4.33E-13
									Total Beam Jitter	5.97E-12

The total contribution due to the mirrors in IOO can be calculated by using Eq. 3 where $\Delta\tilde{\phi}_x$ and $\Delta\tilde{\phi}_\alpha$ are the total beam jitter values in Table 1 and Table 2 respectively. The beam size to be used is 6.0 cm because we are calculating the amplitude of TEM_{10} mode at the AR side of the PRM. This provides a total beam jitter of $1.77\times10^{-6}/\sqrt{\text{Hz}}$ as compared to the $3.7\times10^{-6}/\sqrt{\text{Hz}}$ limit due to the ITM differential limit. The total contribution is still a factor of two lower than the proposed limit. Table 1 and Table 2 also show that the PSL noise is the main contributing noise source for the beam jitter and the SOSs used for the mode matching optics provide sufficient suppression. Table 3 shows the summary of the above results. Here column 2 of the table shows the total normalized contribution of the optical elements to the beam jitter noise. Column 3 shows the incoherent running sum of the noise contributions. Individually, PSL beam jitter is the main noise contributing factor that is scaled to one. The next significant source is the MMT_3 contribution that is about 23% of the beam jitter noise due to PSL. However as these noise factors add incoherently, the contribution is about 2.6%. Overall there the beam jitter noise contribution from the optical components in the IOO chain is about 4% of the PSL beam jitter contribution.

Table 3. Summary of the Beam Jitter Contribution from Noise Sources

Optical Component	Normalized Contribution	Individual Incoherent Contribution
PSL/MC	1	1
From three Steering Mirror	0.167	0.0139
MMT1	0.053	0.0014
MMT2	0.018	0.0141
MMT3	0.226	0.0263
	Total	**1.0403**

3 Conclusion

In conclusion, the beam jitter contribution from the IOO suspension is negligible as compared to the noise contributed from PSL beam jitter. The total beam contribution is still a factor of two lower than the beam jitter requirements in Advanced LIGO.