LIGO Laboratory / LIGO Scientific Collaboration

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Complex Modulation

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1. Introduction

RF modulation is needed to sense and control the interferometer length and alignment DOFs. Three modulation frequencies are needed, one to lock the mode cleaner and two other frequencies to extract control signals of the main interferometer (including Michelson, power recycling cavity, signal recycling cavity and arm cavities). One of the main problems[1] with using multiple phase modulation frequencies is that, if the modulations are applied in series, each modulation modulates all previously present frequency components, .i.e. Sidebands on Sidebands are generated. Those Sidebands on Sidebands can obfuscate the error signals.

1.1 Avoiding Sidebands on Sidebands

Currently, three techniques are known to circumvent the sidebands on sidebands problem. The baseline design for Advanced LIGO solves this problem by using a Mach-Zehnder interferometer and applying the modulations in different arms, see here[2]. An alternate solution to this problem is the possibility of the use of *complex modulation* to synthesize arbitrary modulation configurations. This is investigated in the rest of this document and was previously presented here[3]. In addition, it is also possible to use two-phase locked lasers to generation a frequency-shifted sub-carrier, however this technique is only mentioned for completeness and not further pursuit in this document.

1.2 Complex modulation

This approach uses the simultaneous application of phase and amplitude modulation to generate sidebands without mixing terms. This was called complex modulation, and we briefly discuss the underlying principle:

An amplitude- and phase modulated light field in general can be described as:

$$E(t) = \frac{E_0}{2}\exp(i\omega t + f(t)) + c.c. \quad \textbf{(1.1)}$$

where the modulation function can be divided in

$$f(t) = \phi(t) + i\alpha(t) \quad \textbf{(1.2)}$$

To create an arbitrarily chosen field E_{new} the following equation has to be solved:

$$E_{new}(t) = E_{old}(t)\cdot\exp(f(t)) + c.c. \quad \textbf{(1.3)}$$

leading to the following coefficient for the amplitude and phase modulations:

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[1] LIGO–T040119–00–R, “Mach-Zehnder interferometer for Advanced-LIGO optical configurations to eliminate sidebands of sidebands”, O. Miyakawa, et. al.

[2] LIGO-T07XXXX-01-D, “Input Optics Subsystem Preliminary Design Document”, UF-LIGO group

[3] LIGO-G060452-00-Z, “Complex Optical Modulation”, V. Quetschke, et. al.

$$\alpha(t) = \ln\left|\frac{E_{new}(t)}{E_{old}(t)}\right| \qquad \phi(t) = \arg\left(\frac{E_{new}(t)}{E_{old}(t)}\right) \tag{1.4}$$

To follow this approach and to investigate complex modulation it is necessary to impose arbitrary phase and amplitude modulations on the carrier laser light.

1.3 Experimental realization

A proof-of-principle experiment has been set-up to investigate the usability of this approach. Figure 1.1 shows the schematic setup. The amplitude modulation was generated with an EOM between 45° polarizers. The unwanted extraneous phase modulation was compensated with the second EOM that is mounted in the standard phase modulation orientation.

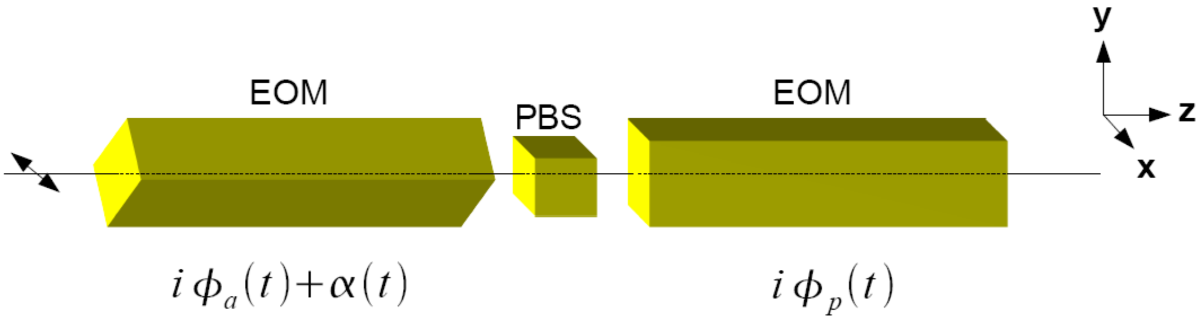

Figure 1.1: Complex modulation

In this configuration, the modulation function can be written as:

$$f(t) = \phi_a(t) + \phi_p(t) + i\alpha(t) \tag{1.5}$$

Figure 1.2 shows the schematic layout of the experiment. The modulation function was calculated by a PC running Matlab that was equipped to upload the modulation function into a Tektronix AFG3252 arbitrary function generator.

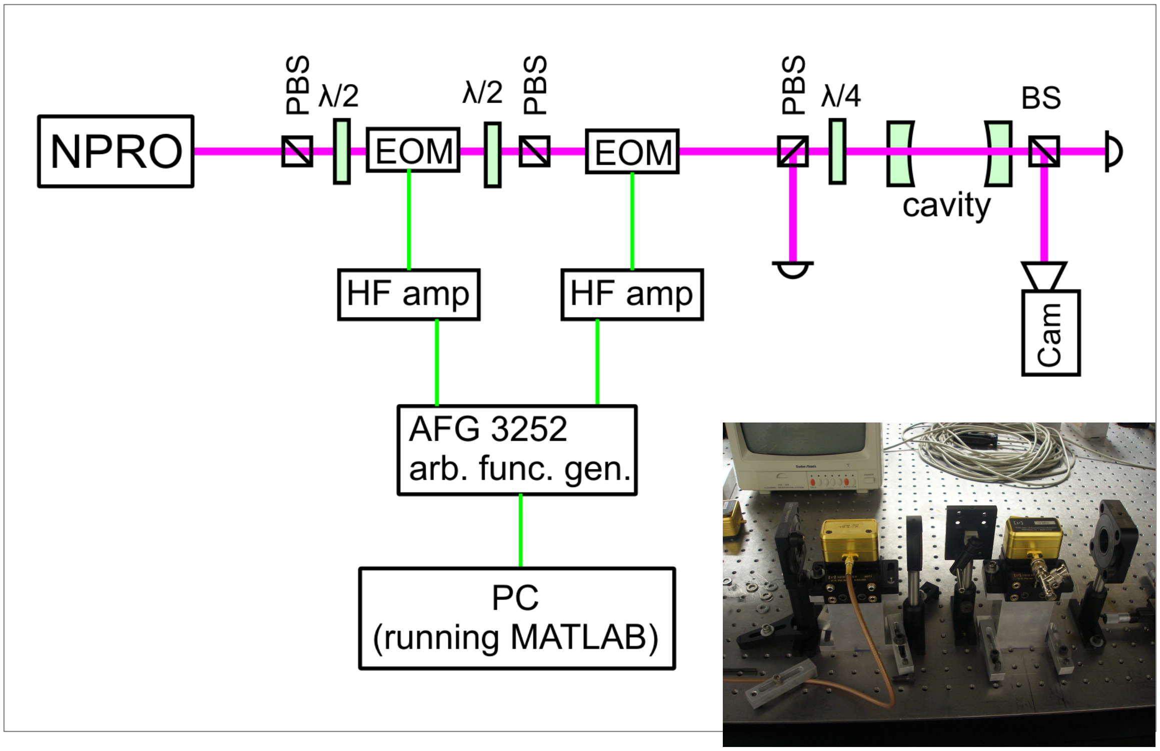

Figure 1.2: Schematic experimental setup. Inset in lower right corner shows the two EOMs.

To achieve a modulation depth of m = 0.7 the signals of the function generator were amplified by two RF amplifiers and fed into the modulators. We used an optical cavity to analyze the frequency components and a fast photodiode to measure the amplitude modulation.

To synthesize the desired modulation the transfer function of the modulators has to be flat, or at least to be stable so that it can be compensated by the calculated modulation function. During the measurements, we uncovered the following issues:

- Temperature-dependent AM
- Non flat transfer function probably caused by electrical and/or piezo resonances in the driver and the crystal prohibiting the compensation of mixing terms.
- The (currently used) HF amplifiers become nonlinear for large driving signals.

These problems are likely to be overcome by temperature stabilization of the crystal, the use of electronic and/or mechanical damping and superior amplifiers. Despite these problems the preliminary setup is already able to create and measure a single sideband modulation on the carrier.

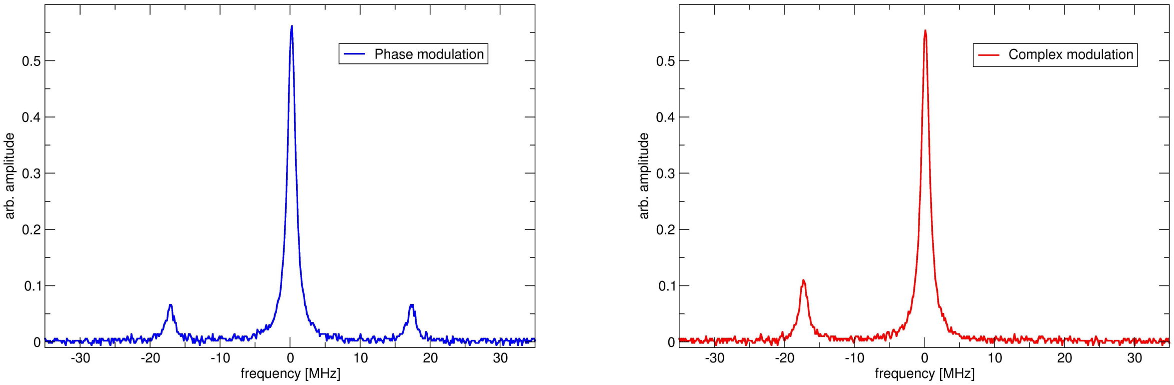

Figure 1.3: Left: Phase Modulation. Right: Single sideband from complex modulation.