Finite element analysis of the stay design for the beam splitter structure Tim Hayler RAL 2007 LIGO-T070161-00-K

1. Beam splitter assembly and space envelope



Fig 1. Beam splitter assembly and space envelope

2. Box section frame with varying cross section of members.

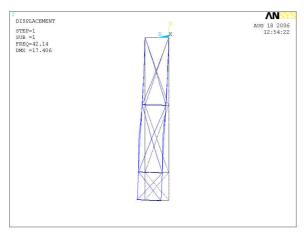


Fig 2. 20mm Square box section, 2mm wall thickness, first frequency of 42Hz.

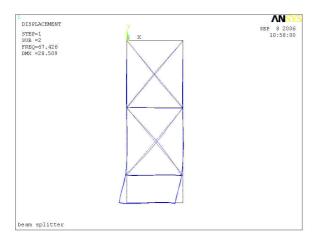


Fig 3. 20mm Square box section, 2mm wall thickness, second frequency 67Hz.

Table 1. Frequency versus increasing cross section.

Size of section [mm]	1 st and 2 nd Frequency [Hz]
20 x 20 x 2	42, 67
30 x 30 x 2	43, 83
40 x 40 x 2	44, 89
50 x 50 x 2	44, 91
50 x 50 x 4	44, 91

In the first mode the structure moves in the longitudinal direction, by increasing the section size of the cross bracing the frequency does not improve.

In the second mode the structure moves in the traverse direction, by increasing the section size of the cross bracing you can increase the frequency.

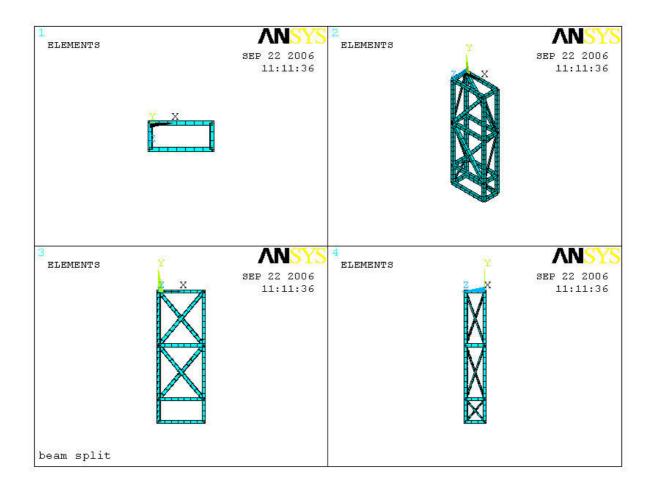


Figure 4. A structure with non uniform size of members. General member size 50 x 50mm, cross bracing members affecting the first mode 20 x 20mm. The first two frequencies are 41Hz and 87Hz.

Table 2. Non uniform sections

Size of section [mm]	1 st and 2 nd Frequency [Hz]
Uniform 50 x 50 x 2	44, 91
Non uniform, majority 50 x 50 x 2, cross bracing members affecting the first mode 20 x 20 x 2.	41, 87

There may be some merit in optimizing the size of individual members.

3. Contribution made by stays

Uniform structure 30 x 30 x 2 mm box section

Table 3. Contribution made by four stays at varying angles to the vertical.

Angle of stays from vertical [degrees]	1 st Frequency [Hz]	Mode shape 8 previous stay modes
10	79	Combination of longitudinal and traverse
15	83	Predominantly traverse
20	84	traverse
25	84	traverse
30	84	traverse
35	83	traverse

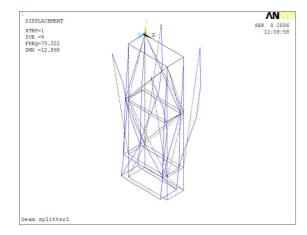


Fig 5. Isometric view of structure with four stays.

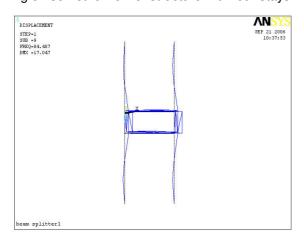


Fig 6. Plan view of structure with four stays.

Table 4. Contribution made by eight stays at varying angles to the vertical.

Angle of stays from vertical [degrees]	1 st Frequency [Hz]	Mode shape 16 previous stay modes
10	84	Combination of longitudinal and traverse
15	94	Combination of longitudinal and traverse
20	104	Combination of longitudinal and traverse
25	111	Predominantly traverse
30	113	traverse
35	110	traverse

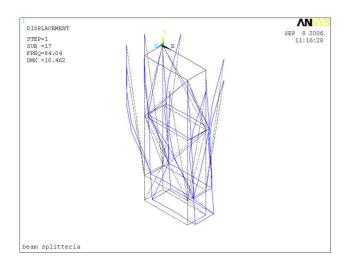


Fig 7. Isometric view of structure with eight stays.

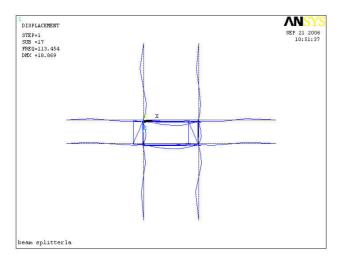


Fig 8. Plan view of structure with eight stays.

Table 5. Contribution made by four stays in star formation at varying angles to the vertical.

Angle of stays from vertical	1 st Frequency [Hz]	Mode shape
[degrees]		
10	81	rotational
15	92	rotational
20	100	rotational
25	106	rotational
30	100	rotational
35	89	rotational

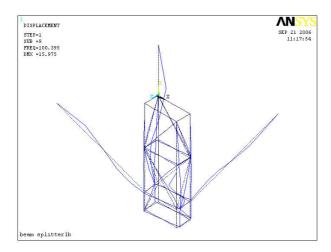


Fig 9. Isometric view of structure with four stays in star formation.

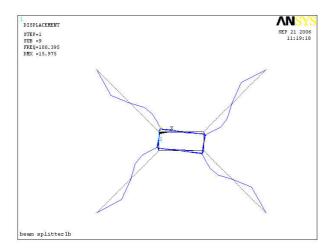


Fig 10. Plan view of structure with four stays in star formation.

Table 6. Results summary of contribution made by stays.

Configuration of stays, all members in structure 30 x 30 x 2 mm sections, stays at 30 ⁰ to the vertical.	Frequency Hz
No stays (table 1)	43
4 stays (figure 5)	87
8 stays (figure 7)	113
4 stays (45° star, figure 9)	100

4. Beam splitter following the space envelope

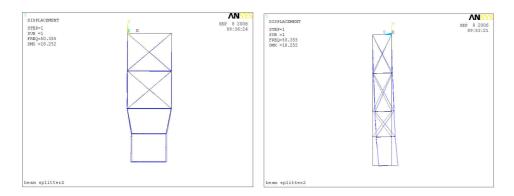


Fig 11. The first mode has the structure wagging in the longitudinal direction and the second mode has the structure wagging in the traverse direction, the frequencies at these modes are 50Hz and 58Hz respectively.

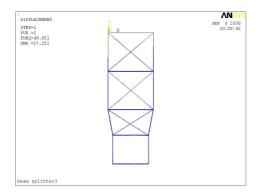


Fig 12. Additional cross bracing, the first mode has the structure wagging in the longitudinal direction and the second mode has the structure wagging in the traverse direction, the frequencies at these modes are 48Hz and 82Hz respectively. Additional cross bracing only adds mass in the first mode reducing the frequency but adds stiffness in the second mode increasing the frequency.

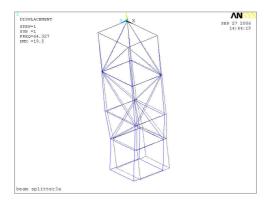


Fig 13. Increasing the narrow width from 200mm to 500mm, the first frequencies are 64Hz and 76Hz.

5. Beam splitter with stays on one side only

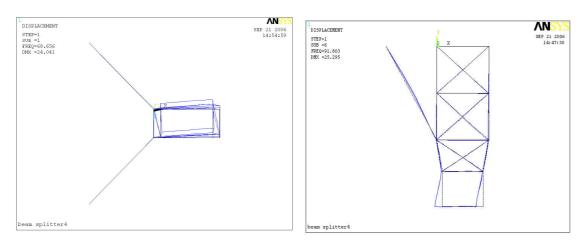


Fig 14. Bracing one side results in more movement on the free side, braces are at 30 degrees to vertical, the first two frequencies are 69Hz and 92Hz.

Table 7 comparison for number of stays

Number of stays	Frequency Hz
2 stays	69, (longitudinal), 92 (traverse)
4 stays	90, (longitudinal), 96 (traverse)

6. Space envelope for stays

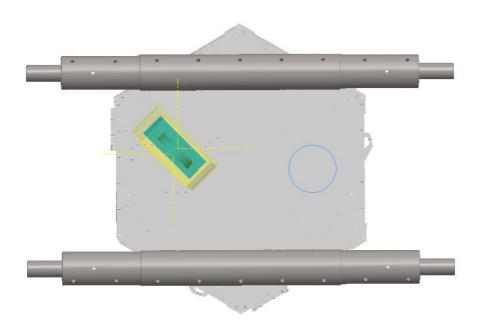


Fig 15. Plan view of beam splitter on the seismic table.

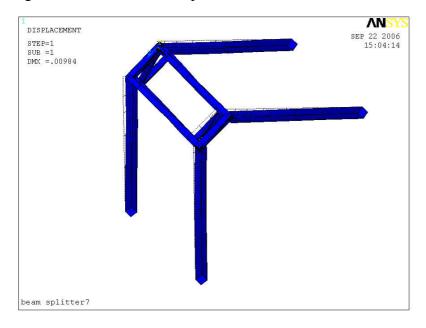


Fig 16. Plan view of the beam splitter with most likely configuration of stays.

7. Nature of the stays

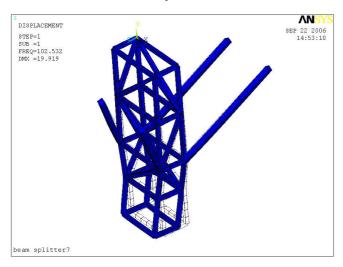


Fig 17. Uniform members in the structure 40 x 40 with larger stays 60 x 60

Table 8. Uniform members in the structure 40 x 40 with increasing stay cross section

Section of stays	Frequency Hz	Mode shape respectively
30 x 30	95, 110 (8 previous stay modes 80 – 85Hz)	Longitudinal, traverse (8 previous stay modes 80 – 85Hz)
40 x 40	96, 107	Longitudinal, traverse
50 x 50	100, 112	Longitudinal, traverse
60 x 60	102,114	Longitudinal, traverse

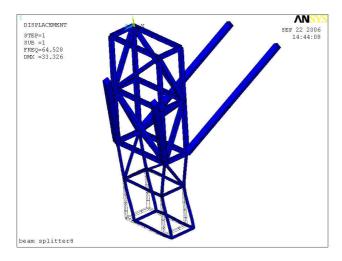


Fig 18. Reduced section of members in lower part of the structure. Stays 60 x 60, main structure 40 x 40, lower portion 20 x 20, frequencies 65Hz, 67Hz.

Following cases have main structural members 40 x 40 x 2 mm and the cross section of stays 60 x 60 x 2mm.

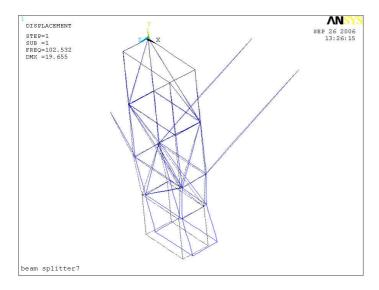


Fig 19. First frequency 102Hz (longitudinal) 114 (traverse)

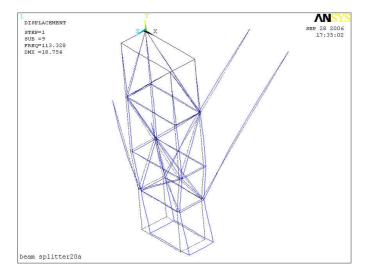


Fig 20. First frequency 113Hz (traverse) 121Hz (longitudinal) (8 previous stay modes 84 – 87Hz). This model has coupled nodes (x, y, z and not rotations) at the stay and structure interface.

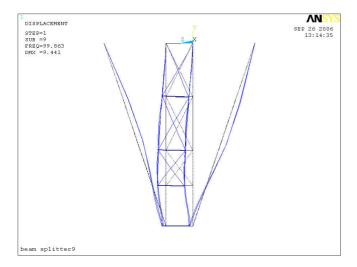


Fig 21. First two frequencies 99Hz (longitudinal) and 131Hz (traverse) (8 previous stay modes 64-75Hz)

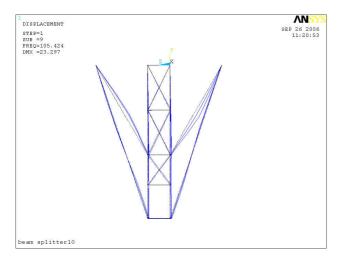


Fig 22. 128Hz (longitudinal), 134Hz (traverse) (8 previous stay modes 67-75Hz)

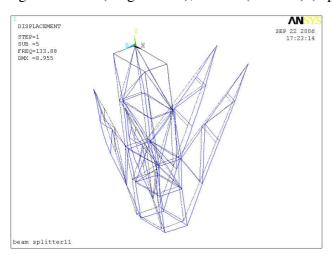


Fig 23. 134Hz (traverse), 143 (longitudinal) (4 previous stay modes 82- 102Hz)

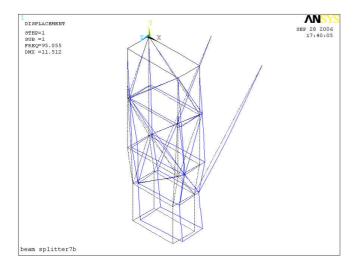


Fig 24. Decreasing the angle of the stays to the vertical from 30^0 to 20^0 , first two frequencies 95, 108Hz (no previous stay modes)

Table 9. Location of stays on the structure

Stay attachment position on the structure	Frequency [Hz]
Structure split (figure 19)	102, 114
Above test mass (figure 20)	113, 121
Bottom of the structure (figure 21)	99, 131
Two stays, Split and bottom (figure 22)	128, 134
Two stays, Space frame from split and bottom (figure 23)	134, 143

Conclusion

The numbers for frequency in this note can not be assumed to be those of the final design, all the models here are idealized structures made entirely from box section perfectly connected together at their neutral axis.

The frequency for the beam splitter structure can be doubled with the use of stays providing you accept a series of stay modes. You can eliminate stay modes by changing the stiffness to length ratio e.g. by attaching the stays higher up the structure as in the case of figure 19, by decreasing the angle of the stays to the vertical from 30^0 to 20^0 as in the case of figure 24, or by making the stays more elaborate as in the case of figure 23. All these measures will reduce the structures fundamental frequency.

In terms of fitting everything in, you can change the position of each stay subtly in any direction without making a big difference to the result.

In figure 18 the model has coupled nodes at the interface between the stays and the main structure, the nodes are coupled in 3 degrees of freedom x, y, z and not rotational degrees of freedom (rotx, roty, rotz) this isolates the stay modes from the structure. As a result all the stay frequencies are very close together. Is there any advantage in having the stays attached to the structure by a ball joint?

The simplest solution could well be four stays coming off the structure at 45^{0} as shown in figure 16, attached to the structure just above the test mass as shown in figure 17, at 30^{0} to the vertical. The final solution depends on our attitude to individual stay modes, do they have sufficiently low mass. If we want a solution without stay modes, do the stays still adequately raise the frequency?

Design for Beam splitter

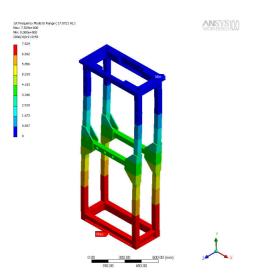


Fig 25. Basic frame, first three frequencies, 17Hz, 35Hz, 38Hz.

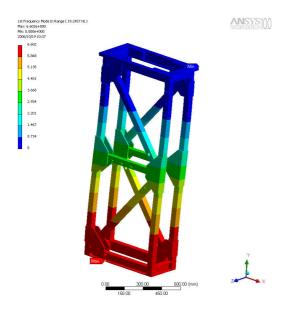


Fig 26. Basic frame with cross bracing, first three frequencies, 19Hz, 61HZ, 98Hz

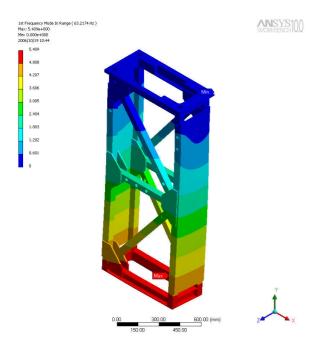


Fig 27. Basic frame with cross bracing and shear plates, first three frequencies, 63Hz, 98Hz, 112Hz

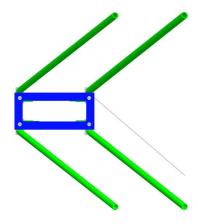


Fig 28. Configuration of stays.

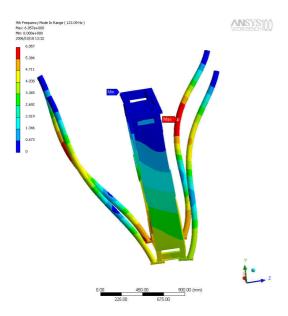


Fig 29. Longitudinal mode shape frequency 123Hz, eight previous stay modes.

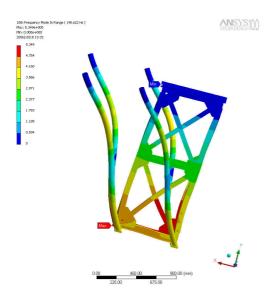


Fig 30. Traverse mode shape frequency 149Hz, eight previous stay modes.

Table 10. Comparison between workbench solution and beam models for figure 28 configuration.

Finite element model	Longitudinal mode shape frequency Hz	Traverse mode shape frequency Hz
Work bench solution	123	149
Beam model solution	131	153

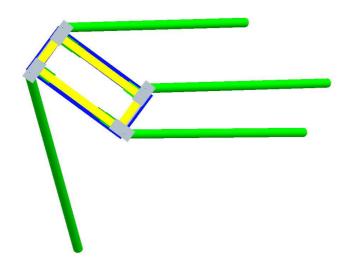


Fig 31. Alternative arrangement for stays.

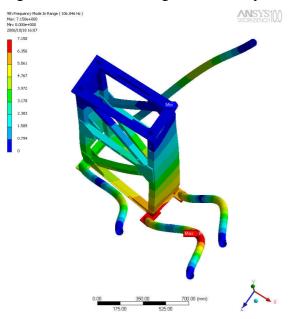


Fig 32. Longitudinal and traverse mode shape frequency 107Hz and 141Hz respectively.

Table 11. Comparison between workbench and beam models for figure 31 configuration.

Finite element model	Longitudinal mode shape frequency Hz	Traverse mode shape frequency Hz
Work bench solution	107	141
Beam model solution	121	148

Table 12. Beam model solution for the configuration in figure 31. The model has an additional 12Kg of mass spread over the bottom four corners of the structure.

Extra 12Kg mass	First frequency Hz	Second frequency Hz
Beam model solution	88	131
60 diameter stays	8 previous stay modes	
Beam model solution	78	115
40 diameter stays	8 previous stay modes	
Beam model solution	72	90
20 diameter stays	16 previous stay modes	

Try stays with configuration as shown in figure 28, but 15 degrees to the vertical.