Preliminary finite element analysis of beam splitter design

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RAL 2007

T070160-02-K

1. Introduction

This note takes a simple approach to understanding the contribution made by elements of the beam splitter design, in particular the addition of stays. A representation of the beam splitter structure is made using an ANSYS finite element model made up of beam and shell elements, later the full CAD model is used with ANSYS workbench.

2. Initial simple model

The model is made up of beam elements representing the simple truss frame as shown in figure one. All members in the frame are 50mm by 50mm box section with a 5mm wall thickness.

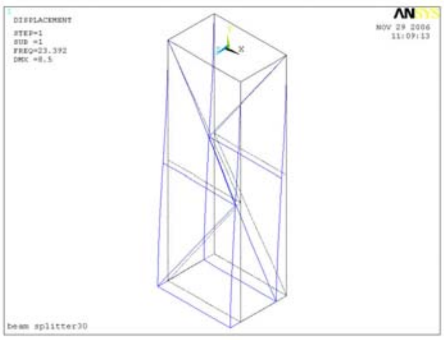

Fig 1. Simple truss structure with a first modal frequency of 23.4Hz.

Table 1. Simple structure showing first five modal frequencies

Mode	Frequency	Mode shape
1	23.3	longitudinal
2	79.1	torsional
3	90.1	panting
4	113.43	panting
5	126.26	traverse

3. Adding stays to the simple structure

In this section the model from section two now includes the addition of stays; the stays are made from the same sections as the main structure 50mm x 50mm box section with a 5mm wall thickness.

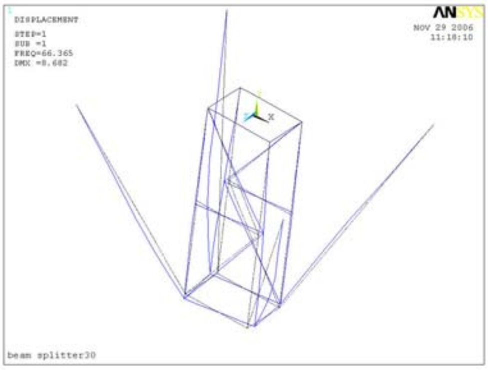

Fig 2. Simple structure with the addition of stays, first modal frequency of 66.4Hz.

Table 2. Simple structure with the addition of stays, first five modal frequencies.

Mode	Frequency 30 Deg	Mode shape
1	64.2	torsional
2	75.3	longitudinal
3	84.2	
4	92.1	
5	92.7	

Table 3. Simple structure with the addition of stays, first five modal frequencies versus various included angles.

Mode	Frequency 10 Deg	Frequency 15 Deg	Frequency 20 Deg	Frequency 25 Deg	Frequency 30 Deg	Frequency 35 Deg
1	47.1	58.8	67.4	66.4	66.2	60.8
2	68.5	68.1	67.6	72.8	75.3	74.5
3	88.8	88.5	87.8	86.6	84.2	79.0
4	110.3	109.8	106.3	100.4	92.1	82.9
5	110.9	110.4	107.3	100.5	92.7	83.6

4. Adding the shear plates to the simple structure

To improve the first fundamental frequency it is necessary to add shear plates to the side of the structure. Shear plates work in this design, firstly because the width of the structure is narrow, meaning a truss member would make little contribution, and secondly because they're easy to assemble. The shear plates are 5mm thick.

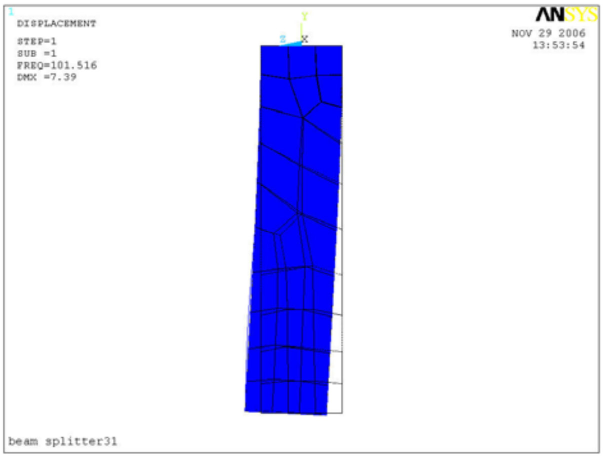

Fig 3. Simple structure with the addition of shear plates, first modal frequency of 101Hz.

Table 4. First five mode frequencies of the simple structure with shear plates.

Mode	Frequency	Mode shape
1	101.2	longitudinal
2	126.3	traverse
3	183.1	Plate mode
4	186.6	Plate mode
5	195.86	Plate mode

Table 5. Simple structure with shear plates, a comparison of the first five mode frequencies is made with and without an additional 12Kg of mass evenly distributed over the bottom four corners.

Mode	Frequency No Extra Mass (results taken from table 4)	Frequency Extra 12Kg
1	101.2	78.3
2	126.3	100.59
3	183.1	137.4
4	186.6	172.82
5	195.86	186.62

5. Adding stays and shear plates to the simple structure

Stays are now re-introduced to the simple structure and shear plates at an included angle of 20 degrees.

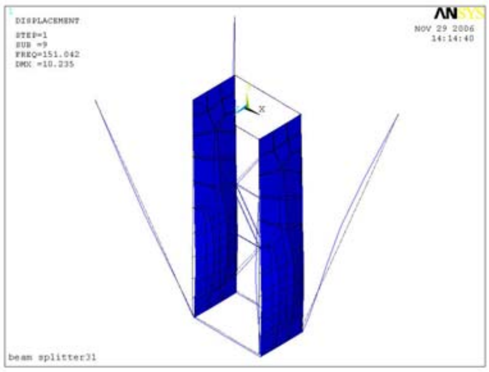

Fig 4. Simple structure comprised of truss frame, stays and side plates

Table 6. Simple structure with stays and side plates.

Mode	Frequency	Mode shape
1	105.0	
2	105.4	
3	107.7	
4	109.8	
5	110.9	
6	112.1	
7	114.0	
8	114.4	
9	147.8	traverse
10	176.3	longitudinal

Table 7. Simple structure with stays and shear plates, a comparison of the mode frequencies is made with and without an additional 12Kg of mass evenly distributed over the bottom four corners of the main structure.

Mode	Frequency (results taken from table 5)	Frequency Extra 12Kg
1	105.0	102.3
2	105.4	103.2
3	107.7	107.5
4	109.8	109.1
5	110.9	110.2
6	112.1	111.8
7	114.0	114.0
8	114.4	114.4
9	147.8	133.8
10	176.3	149.8

6. Exploring the nature of support at either end of the stay, built in or pin jointed.

6.1 Single beam built in at one end and pin jointed at the other

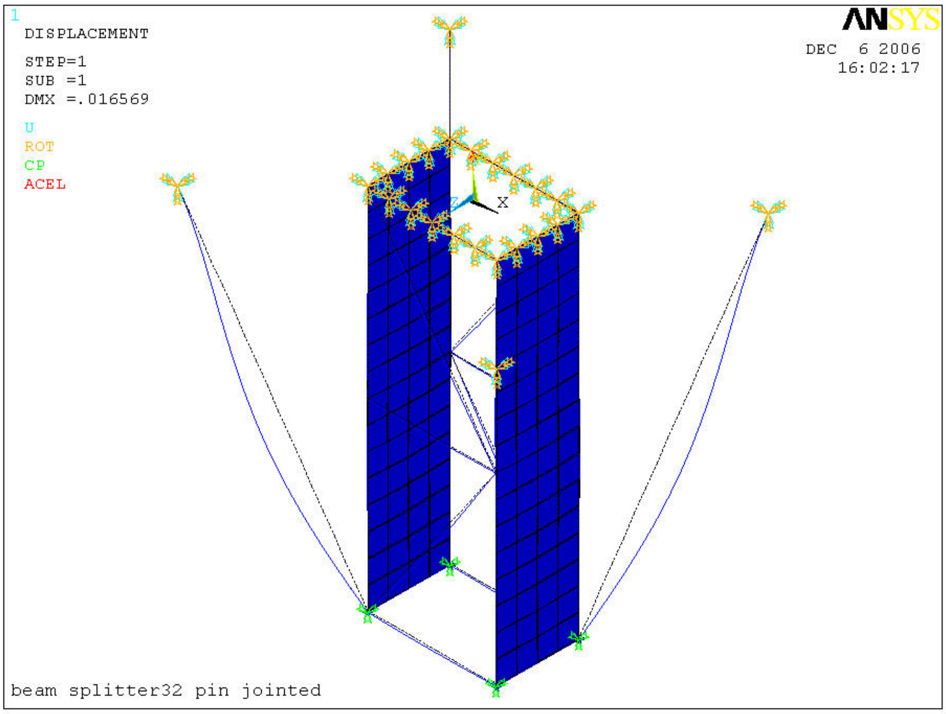

Fig 5. Simple structure comprised of truss frame, stays and side plates. Stays are built in at the free end and pin jointed at the attachment to the main body of the structure.

6.1.1 For a single beam built in at one end and pin jointed at the other

$f = (1/ 2\pi)\ 15.42 \sqrt{(EI/ml^4)}$

Where;

m = mass/length
mass = 0.407e-2 (metric tonne)
l = 1676mm
$E = 70e3\ N/mm^2$
$I = 307500\ mm^4$

Then f = 82.74Hz

Table 8. Simple structure and shear plates with stays pin jointed at the stay to main structure interface and built in at the stay to seismic table interface. A comparison of the mode frequencies is made with and without an additional 12Kg of mass evenly distributed over the bottom four corners.

Mode	Frequency	Frequency with Extra 12Kg
1	80.7	80.4
2	80.8	80.6
3	81.0	80.9
4	81.5	81.5
5	81.8	81.8
6	81.8	81.8
7	81.8	81.8
8	81.8	81.8
9	141.8	125.1
10	175.2	146.8

The results in table 8 indicate that the first eight modes are stay modes as they agree with the theory result of ≈ 80Hz and are unchanged by the addition of extra mass to the main structure. Modes nine and ten reduce with the addition of extra mass to the main structure indicating the structure is moving as one.

6. 2 For a single beam pin jointed at both ends.

$f = (1/2\pi)\, \pi^2 \sqrt{(EI/ml^4)}$

Then f = 52.9Hz

6.3 For a single beam built in at both ends.

$f = (1/2\pi)\, 22.37 \sqrt{(EI/ml^4)}$

Then f = 120Hz

7. Moving the stay contact point half way up the structure to eliminate stay modes

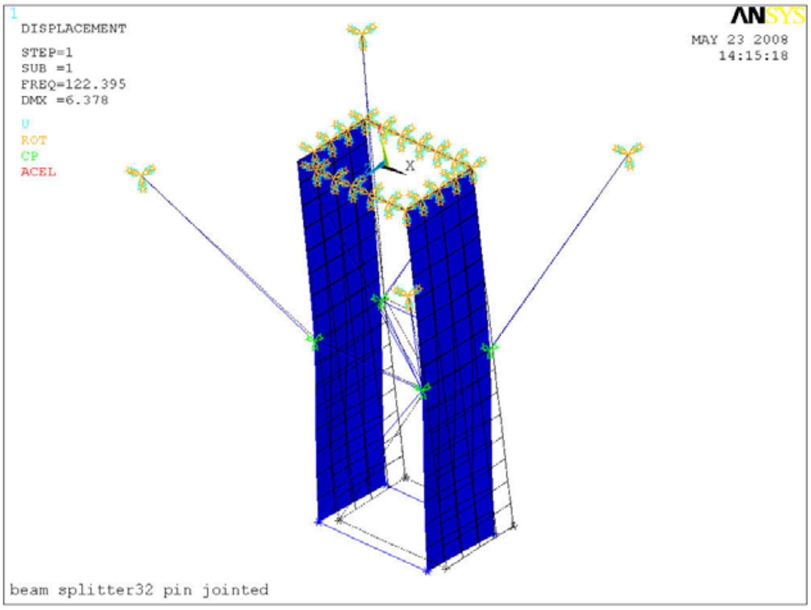

Fig 6. Simple structure comprised of truss frame, stays and side plates. Stays are built in at the free end and pin jointed at the attachment to the main body of the structure. In this case the stays contact the main structure at the half way point

Table 9. Simple structure and shear plates with stays pin jointed at the stay to main structure interface and built in at the stay to seismic table interface. In this case the stays contact the main structure at the half way point. A comparison of the mode frequencies is made with and without an additional 12Kg of mass evenly distributed over the bottom four corners

Mode	Frequency	Frequency with Extra 12Kg
1	153	122
2	167	132
3	177	139
4	186	186
5	195	194
6	197	197
7	229	202
8	234	209
9	235	235
10	237	235

From 6.1.1 the stay modes begin at 244Hz

8. Beam splitter structure with stays using Pro Engineer CAD model and ANSYS workbench

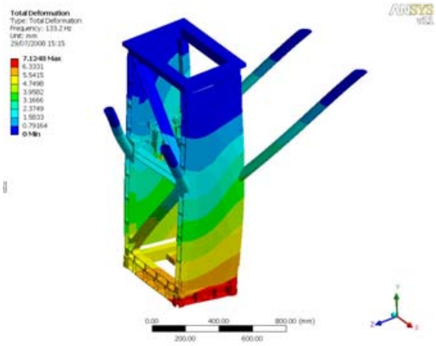

Fig 6. Beam splitter with no lower structure, first frequency 133Hz

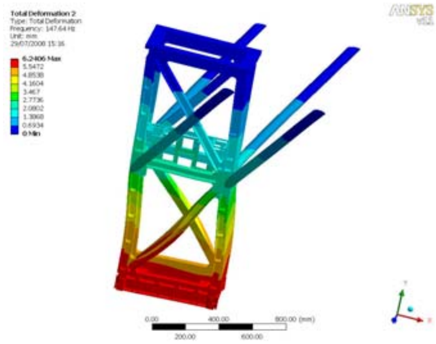

Fig 7. Beam splitter with no lower structure, second frequency 148Hz

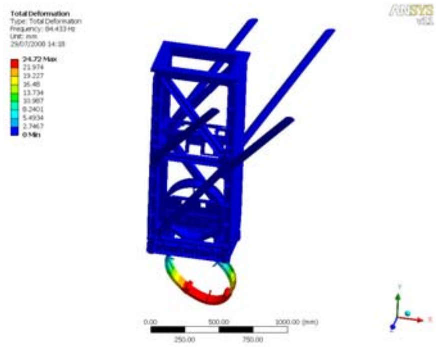

Fig 8. Beam splitter with lower structure, first frequency 85Hz

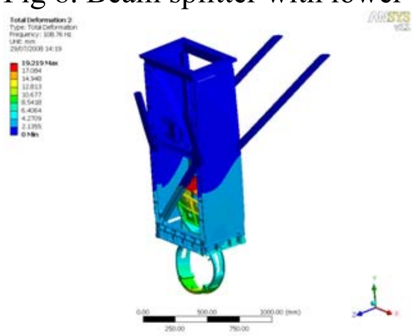

Fig 9. Beam splitter with lower structure, second frequency 109Hz

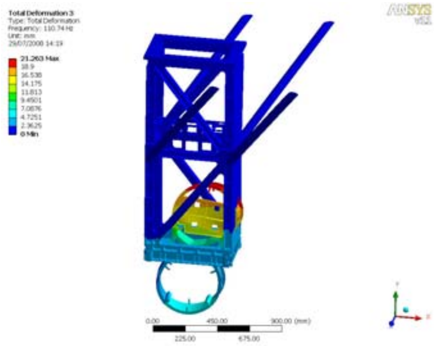

Fig 10. Beam splitter with lower structure, third frequency 111Hz

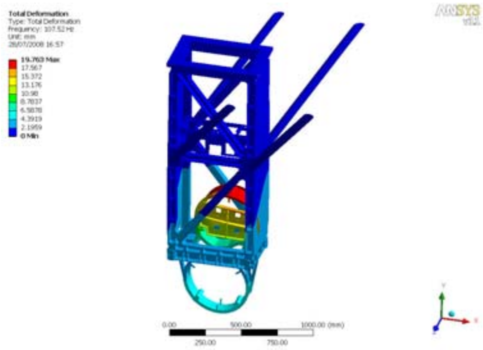

Fig 11. Beam splitter with lower structure and lower structure stays, first frequency 108Hz

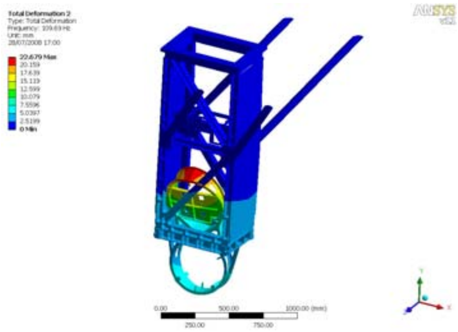

Fig 12. Beam splitter with lower structure and lower structure stays, second frequency 110Hz

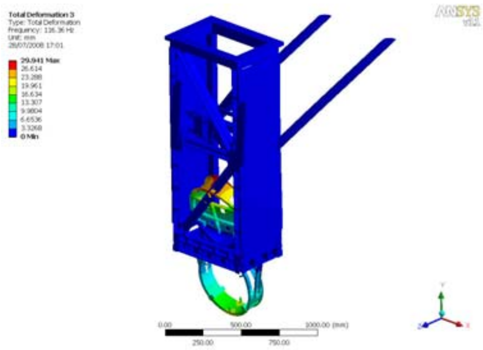

Fig 13. Beam splitter with lower structure and lower structure stays, third frequency 116Hz

Table 10. Comparison of different configurations for the beam splitter structure.

	Configuration of Beam splitter structure		
Modal frequency	**No lower structure (figures 6,7) [Hz]**	**With lower structure (figures 8,9,10) [Hz]**	**With lower structure and lower structure stays (figures 11,12,13) [Hz]**
1st frequency	**133**	**85**	**108**
2nd frequency	**148**	**109**	**110**
3rd frequency		**111**	**116**

Conclusion

A structure with shear plates but without stays and an additional 12Kg evenly distributed over the bottom four corners gives a first frequency of 78.3Hz.

A structure with shear plates and stays built in at the free end and pin jointed at the attachment to the bottom of the main body of the structure, and with an additional 12Kg distributed over the bottom four corners, gives a first frequency of 80Hz. The first 8 modes of this structure are all stay modes at around 80Hz, in the ninth mode the structure moves as one in the traverse direction at a frequency of 125Hz.

A structure with shear plates, stays built in at the free end and pin jointed at the attachment to the main body of the structure at the half way point, and an additional 12Kg, gives a first frequency of 122Hz. In the predominantly longitudinal direction, with no stay modes until 244Hz.

A structure with stays is less sensitive to the addition of mass.

Looking at the effective mass in appendix B for a simple structure, as described in table 5, the first mode frequency is 78.3Hz, predominantly in the longitudinal (Z) direction, with an effective mass in the longitudinal direction of 0.34e-1. By adding stays to the simple structure, as described in table 7, this longitudinal mode is now seen again by mode 10 with an effective mass of 0.22e-1. The only other modes with a similar effective mass are when the stays are all panting uniformly in the longitudinal (mode 4 in z) or traverse (mode 1 in x) direction.

Looking at effective mass for a simple structure with pin jointed stays, as described in table 8, there are eight stay modes at ≅ 80Hz before a predominately traverse mode at 125Hz, followed by a predominately longitudinal mode at 146Hz. The traverse ninth mode has an effective mass of 0.3e-1, a similar effective mass is only seen amongst the stay modes by mode one, in the x direction, when all the stays are panting uniformly in the same direction.

Appendix A

Comparison of beam model to CAD/workbench model

The beam model has been changed slightly to suit existing CAD geometry for the purpose of a workbench comparison.
The shear plates on the beam model only connect where there are key points i.e. at the top and bottom corners and where the cross members meet half way down the structure on either side, each shear plate has six contact points in total.

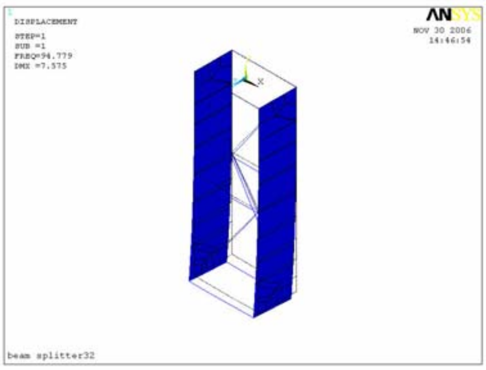

Fig 6. Simple structure with shear plates in beams and shells.

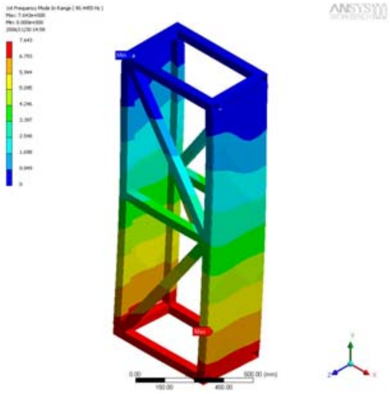

Fig 7. Simple structure with shear plates taken from CAD model into ANSYS workbench.

Table 8. Models as in figures 6 and 7 with side plates.

Frequency	Beam model	Workbench model
1st mode longitudinal	94.1	90.5
2nd mode traverse	120.6	117.7
3rd mode torsion	169.6	171.0

Appendix B

Effective mass for the simple structure with the addition of shear plates and 12Kg of mass evenly distributed over the bottom four corners as described in table 5.

***** PARTICIPATION FACTOR CALCULATION ***** X DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	78.3020	0.12771E-01	0.89018E-01	0.468555	0.792426E-02	0.180021
2	100.590	0.99414E-02	0.18998	1.000000	0.360943E-01	1.00000
3	137.403	0.72779E-02	-0.31554E-05	0.000017	0.995628E-11	1.00000
4	172.820	0.57864E-02	-0.29051E-05	0.000015	0.843969E-11	1.00000
5	186.617	0.53586E-02	0.59131E-05	0.000031	0.349645E-10	1.00000
				SUM OF EFFECTIVE MASSES=	0.440186E-01	

***** PARTICIPATION FACTOR CALCULATION ***** Y DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	78.3020	0.12771E-01	0.33143E-06	0.000086	0.109843E-12	0.462877E-08
2	100.590	0.99414E-02	0.10167E-05	0.000264	0.103363E-11	0.481861E-07
3	137.403	0.72779E-02	0.29496E-02	0.767345	0.869992E-05	0.366615
4	172.820	0.57864E-02	-0.38439E-02	1.000000	0.147752E-04	0.989242
5	186.617	0.53586E-02	-0.50526E-03	0.131446	0.255285E-06	1.00000
				SUM OF EFFECTIVE MASSES=	0.237304E-04	

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	78.3020	0.12771E-01	0.18384	1.000000	0.337972E-01	0.847821
2	100.590	0.99414E-02	-0.77887E-01	0.423667	0.606638E-02	1.00000
3	137.403	0.72779E-02	-0.14453E-06	0.000001	0.208897E-13	1.00000
4	172.820	0.57864E-02	0.24984E-05	0.000014	0.624195E-11	1.00000
5	186.617	0.53586E-02	-0.10783E-05	0.000006	0.116281E-11	1.00000
				SUM OF EFFECTIVE MASSES=	0.398635E-01	

***** PARTICIPATION FACTOR CALCULATION *****ROTX DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	78.3020	0.12771E-01	-251.03	1.000000	63017.1	0.844164
2	100.590	0.99414E-02	107.86	0.429656	11633.3	1.00000
3	137.403	0.72779E-02	-0.21244E-04	0.000000	0.451311E-09	1.00000
4	172.820	0.57864E-02	-0.19693E-02	0.000008	0.387798E-05	1.00000
5	186.617	0.53586E-02	0.19835E-02	0.000008	0.393412E-05	1.00000
			SUM OF EFFECTIVE MASSES=		74650.4	

***** PARTICIPATION FACTOR CALCULATION *****ROTY DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	78.3020	0.12771E-01	0.40197E-03	0.000007	0.161576E-06	0.412036E-10
2	100.590	0.99414E-02	0.12416E-02	0.000021	0.154152E-05	0.434309E-09
3	137.403	0.72779E-02	59.897	1.000000	3587.59	0.914875
4	172.820	0.57864E-02	-18.239	0.304500	332.643	0.999702
5	186.617	0.53586E-02	1.0808	0.018044	1.16804	1.00000
			SUM OF EFFECTIVE MASSES=		3921.40	

***** PARTICIPATION FACTOR CALCULATION *****ROTZ DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	78.3020	0.12771E-01	109.61	0.432943	12015.4	0.157852
2	100.590	0.99414E-02	253.19	1.000000	64103.1	1.00000
3	137.403	0.72779E-02	-0.43944E-02	0.000017	0.193104E-04	1.00000
4	172.820	0.57864E-02	-0.25391E-02	0.000010	0.644690E-05	1.00000
5	186.617	0.53586E-02	0.81265E-02	0.000032	0.660406E-04	1.00000
			SUM OF EFFECTIVE MASSES=		76118.6	

Effective mass for the simple structure with the addition stays, shear plates and 12Kg of mass evenly distributed over the bottom four corners as described in table 7.

***** PARTICIPATION FACTOR CALCULATION ***** X DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	102.341	0.97712E-02	0.18878	1.000000	0.356370E-01	0.617922
2	103.242	0.96860E-02	0.34480E-04	0.000183	0.118886E-08	0.617922
3	107.583	0.92952E-02	0.17770E-06	0.000001	0.315774E-13	0.617922
4	109.046	0.91705E-02	-0.34449E-01	0.182485	0.118674E-02	0.638499
5	110.185	0.90756E-02	0.32938E-01	0.174483	0.108494E-02	0.657311
6	111.841	0.89413E-02	-0.35379E-05	0.000019	0.125165E-10	0.657311
7	113.971	0.87741E-02	-0.26422E-05	0.000014	0.698108E-11	0.657311
8	114.430	0.87389E-02	0.15371E-02	0.008143	0.236281E-05	0.657352
9	133.765	0.74758E-02	-0.11475	0.607838	0.131667E-01	0.885654
10	149.819	0.66747E-02	0.81207E-01	0.430174	0.659462E-02	1.00000
				SUM OF EFFECTIVE MASSES=	0.576724E-01	

***** PARTICIPATION FACTOR CALCULATION ***** Y DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	102.341	0.97712E-02	-0.12242E-06	0.000005	0.149857E-13	0.119848E-10
2	103.242	0.96860E-02	0.21325E-02	0.078727	0.454763E-05	0.363698E-02
3	107.583	0.92952E-02	-0.22599E-01	0.834298	0.510714E-03	0.412082
4	109.046	0.91705E-02	0.12193E-06	0.000005	0.148676E-13	0.412082
5	110.185	0.90756E-02	-0.46935E-07	0.000002	0.220289E-14	0.412082
6	111.841	0.89413E-02	-0.11811E-02	0.043603	0.139497E-05	0.413198
7	113.971	0.87741E-02	-0.27087E-01	1.000000	0.733729E-03	1.00000
8	114.430	0.87389E-02	-0.12751E-06	0.000005	0.162578E-13	1.00000
9	133.765	0.74758E-02	-0.11274E-05	0.000042	0.127113E-11	1.00000
10	149.819	0.66747E-02	-0.63574E-05	0.000235	0.404162E-10	1.00000
				SUM OF EFFECTIVE MASSES=	0.125039E-02	

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	102.341	0.97712E-02	0.70886E-01	0.476587	0.502479E-02	0.888423E-01
2	103.242	0.96860E-02	0.14142E-04	0.000095	0.199994E-09	0.888423E-01
3	107.583	0.92952E-02	0.38437E-06	0.000003	0.147737E-12	0.888423E-01
4	109.046	0.91705E-02	0.14506	0.975313	0.210437E-01	0.460912
5	110.185	0.90756E-02	-0.68763E-01	0.462312	0.472830E-02	0.544512
6	111.841	0.89413E-02	0.37610E-06	0.000003	0.141449E-12	0.544512
7	113.971	0.87741E-02	0.89322E-06	0.000006	0.797850E-12	0.544512
8	114.430	0.87389E-02	0.26122E-02	0.017563	0.682359E-05	0.544633
9	133.765	0.74758E-02	-0.60269E-01	0.405208	0.363237E-02	0.608856
10	149.819	0.66747E-02	-0.14874	1.000000	0.221225E-01	1.00000
				SUM OF EFFECTIVE MASSES=	0.565585E-01	

***** PARTICIPATION FACTOR CALCULATION *****ROTX DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	102.341	0.97712E-02	-77.104	0.351073	5945.00	0.666751E-01
2	103.242	0.96860E-02	-0.15177E-01	0.000069	0.230346E-03	0.666751E-01
3	107.583	0.92952E-02	-0.37429E-03	0.000002	0.140091E-06	0.666751E-01
4	109.046	0.91705E-02	-147.13	0.669918	21647.1	0.309454
5	110.185	0.90756E-02	57.690	0.262679	3328.18	0.346781
6	111.841	0.89413E-02	-0.31591E-03	0.000001	0.998008E-07	0.346781
7	113.971	0.87741E-02	-0.73037E-03	0.000003	0.533446E-06	0.346781
8	114.430	0.87389E-02	-2.7315	0.012437	7.46107	0.346864
9	133.765	0.74758E-02	100.01	0.455365	10001.7	0.459037
10	149.819	0.66747E-02	219.62	1.000000	48234.3	1.00000
				SUM OF EFFECTIVE MASSES=	89163.8	

***** PARTICIPATION FACTOR CALCULATION *****ROTY DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	102.341	0.97712E-02	0.15287E-01	0.000192	0.233705E-03	0.336409E-07
2	103.242	0.96860E-02	-79.815	1.000000	6370.50	0.917007
3	107.583	0.92952E-02	-11.556	0.144784	133.541	0.936230
4	109.046	0.91705E-02	0.23302E-03	0.000003	0.542985E-07	0.936230
5	110.185	0.90756E-02	-0.35284E-03	0.000004	0.124499E-06	0.936230
6	111.841	0.89413E-02	21.048	0.263705	443.008	0.999999
7	113.971	0.87741E-02	-0.91384E-01	0.001145	0.835110E-02	1.00000
8	114.430	0.87389E-02	0.90313E-03	0.000011	0.815651E-06	1.00000
9	133.765	0.74758E-02	-0.98479E-03	0.000012	0.969817E-06	1.00000
10	149.819	0.66747E-02	0.76792E-02	0.000096	0.589700E-04	1.00000
			SUM OF EFFECTIVE MASSES=		6947.06	

***** PARTICIPATION FACTOR CALCULATION *****ROTZ DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	102.341	0.97712E-02	193.61	1.000000	37485.1	0.452647
2	103.242	0.96860E-02	0.34731E-01	0.000179	0.120625E-02	0.452647
3	107.583	0.92952E-02	0.67930E-03	0.000004	0.461445E-06	0.452647
4	109.046	0.91705E-02	-28.097	0.145119	789.421	0.462179
5	110.185	0.90756E-02	31.534	0.162872	994.376	0.474187
6	111.841	0.89413E-02	0.73539E-03	0.000004	0.540801E-06	0.474187
7	113.971	0.87741E-02	0.64191E-03	0.000003	0.412052E-06	0.474187
8	114.430	0.87389E-02	13.317	0.068781	177.335	0.476328
9	133.765	0.74758E-02	-172.36	0.890238	29707.9	0.835061
10	149.819	0.66747E-02	116.87	0.603646	13659.2	1.00000
			SUM OF EFFECTIVE MASSES=		82813.3	

Effective mass for the simple structure with the addition of pin jointed stays at the stay to main structure interface, shear plates and 12Kg of mass evenly distributed over the bottom four corners as described in table 8.

***** PARTICIPATION FACTOR CALCULATION ***** X DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	80.4186	0.12435E-01	0.10836	0.612191	0.117420E-01	0.204627
2	80.6290	0.12402E-01	0.52580E-05	0.000030	0.276470E-10	0.204627
3	80.8915	0.12362E-01	-0.79118E-01	0.446983	0.625964E-02	0.313714
4	81.4710	0.12274E-01	0.47632E-06	0.000003	0.226876E-12	0.313714
5	81.8286	0.12221E-01	0.27000E-07	0.000000	0.729011E-15	0.313714
6	81.8311	0.12220E-01	0.26455E-02	0.014946	0.699843E-05	0.313836
7	81.8417	0.12219E-01	0.21374E-06	0.000001	0.456846E-13	0.313836
8	81.8420	0.12219E-01	-0.12063E-02	0.006815	0.145507E-05	0.313861
9	125.139	0.79911E-02	0.17700	1.000000	0.313306E-01	0.859857
10	146.792	0.68124E-02	-0.89676E-01	0.506631	0.804177E-02	1.00000
				SUM OF EFFECTIVE MASSES=	0.573825E-01	

***** PARTICIPATION FACTOR CALCULATION ***** Y DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	80.4186	0.12435E-01	0.78023E-07	0.000003	0.608763E-14	0.421066E-11
2	80.6290	0.12402E-01	0.36313E-04	0.001168	0.131867E-08	0.912092E-06
3	80.8915	0.12362E-01	0.39476E-07	0.000001	0.155834E-14	0.912093E-06
4	81.4710	0.12274E-01	0.78607E-04	0.002529	0.617910E-08	0.518602E-05
5	81.8286	0.12221E-01	-0.31085E-01	1.000000	0.966289E-03	0.668362
6	81.8311	0.12220E-01	0.30154E-06	0.000010	0.909250E-13	0.668362
7	81.8417	0.12219E-01	0.21896E-01	0.704398	0.479450E-03	0.999985
8	81.8420	0.12219E-01	-0.11340E-05	0.000036	0.128596E-11	0.999985
9	125.139	0.79911E-02	-0.59552E-06	0.000019	0.354640E-12	0.999985
10	146.792	0.68124E-02	-0.14484E-03	0.004660	0.209793E-07	1.00000
				SUM OF EFFECTIVE MASSES=	0.144577E-02	

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	80.4186	0.12435E-01	0.85115E-01	0.483372	0.724453E-02	0.130302
2	80.6290	0.12402E-01	0.10056E-04	0.000057	0.101118E-09	0.130302
3	80.8915	0.12362E-01	0.98510E-01	0.559441	0.970412E-02	0.304844
4	81.4710	0.12274E-01	-0.12093E-06	0.000001	0.146247E-13	0.304844
5	81.8286	0.12221E-01	0.14505E-07	0.000000	0.210384E-15	0.304844
6	81.8311	0.12220E-01	-0.39572E-02	0.022473	0.156593E-04	0.305125
7	81.8417	0.12219E-01	-0.15720E-06	0.000001	0.247107E-13	0.305125
8	81.8420	0.12219E-01	0.68784E-03	0.003906	0.473118E-06	0.305134
9	125.139	0.79911E-02	0.87332E-01	0.495964	0.762690E-02	0.442314
10	146.792	0.68124E-02	0.17609	1.000000	0.310061E-01	1.00000
				SUM OF EFFECTIVE MASSES=	0.555978E-01	

***** PARTICIPATION FACTOR CALCULATION *****ROTX DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	80.4186	0.12435E-01	-78.391	0.319943	6145.14	0.690842E-01
2	80.6290	0.12402E-01	-0.90685E-02	0.000037	0.822385E-04	0.690842E-01
3	80.8915	0.12362E-01	-87.658	0.357766	7683.96	0.155468
4	81.4710	0.12274E-01	0.12584E-03	0.000001	0.158369E-07	0.155468
5	81.8286	0.12221E-01	-0.11933E-03	0.000000	0.142392E-07	0.155468
6	81.8311	0.12220E-01	-6.5460	0.026717	42.8503	0.155950
7	81.8417	0.12219E-01	0.14061E-03	0.000001	0.197704E-07	0.155950
8	81.8420	0.12219E-01	0.29777E-01	0.000122	0.886691E-03	0.155950
9	125.139	0.79911E-02	-122.67	0.500643	15046.8	0.325107
10	146.792	0.68124E-02	-245.02	1.000000	60032.6	1.00000
				SUM OF EFFECTIVE MASSES=	88951.4	

***** PARTICIPATION FACTOR CALCULATION *****ROTY DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	80.4186	0.12435E-01	0.54494E-02	0.000078	0.296959E-04	0.611047E-08
2	80.6290	0.12402E-01	-69.648	1.000000	4850.91	0.998162
3	80.8915	0.12362E-01	0.25122E-02	0.000036	0.631137E-05	0.998162
4	81.4710	0.12274E-01	1.6084	0.023093	2.58703	0.998694
5	81.8286	0.12221E-01	-1.0320	0.014817	1.06505	0.998913
6	81.8311	0.12220E-01	-0.72663E-04	0.000001	0.527990E-08	0.998913
7	81.8417	0.12219E-01	-0.29538	0.004241	0.872498E-01	0.998931
8	81.8420	0.12219E-01	0.70142E-04	0.000001	0.491984E-08	0.998931
9	125.139	0.79911E-02	0.32528E-02	0.000047	0.105806E-04	0.998931
10	146.792	0.68124E-02	-2.2791	0.032724	5.19449	1.00000
				SUM OF EFFECTIVE MASSES=	4859.85	

***** PARTICIPATION FACTOR CALCULATION *****ROTZ DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	80.4186	0.12435E-01	97.376	0.419402	9482.12	0.114763
2	80.6290	0.12402E-01	0.45667E-02	0.000020	0.208548E-04	0.114763
3	80.8915	0.12362E-01	-68.592	0.295426	4704.82	0.171706
4	81.4710	0.12274E-01	0.61859E-03	0.000003	0.382658E-06	0.171706
5	81.8286	0.12221E-01	0.14646E-04	0.000000	0.214502E-09	0.171706
6	81.8311	0.12220E-01	2.8373	0.012220	8.05005	0.171803
7	81.8417	0.12219E-01	0.35188E-03	0.000002	0.123816E-06	0.171803
8	81.8420	0.12219E-01	11.564	0.049808	133.732	0.173422
9	125.139	0.79911E-02	232.18	1.000000	53906.9	0.825863
10	146.792	0.68124E-02	-119.95	0.516625	14387.8	1.00000
				SUM OF EFFECTIVE MASSES=	82623.5	