LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Heating of the ITM by the Compensation Plate in Advanced LIGO

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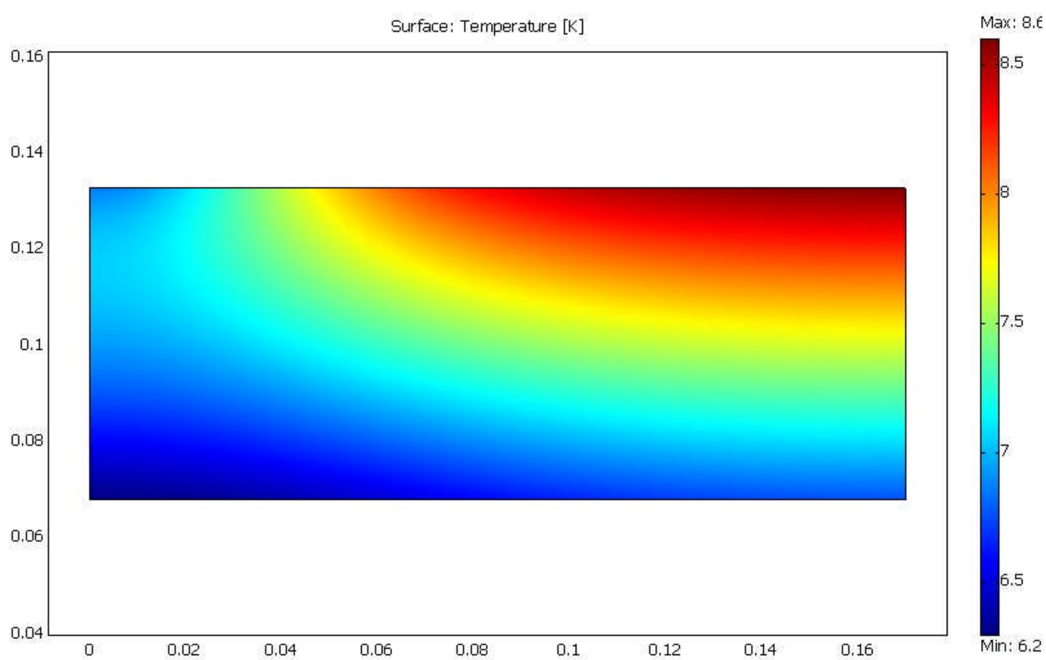
1 Introduction

Previous analyses of the thermal compensation in Advanced LIGO have assumed no thermal interaction between the compensation plate and input test mass. This technical note analyzes this interaction and shows that it contributes a significant thermal aberration into the ITM. It then discusses various strategies for minimizing and adapted to this effect.

2 Radiant Heating of the ITM by the CP

Figure 1 shows the thermal distribution in (r,z) for the compensation plate with the optimized annular heating pattern for 0.5 W power absorbed in the ITM. The overall heating is 7.5 W. Note that the bottom face of the CP has a nearly uniform temperature of 6.6K above ambient. This face will radiate as a blackbody to the AR face of the ITM.

Figure 1: temperature profile of CP with annular



compensation.

We now derive the irradiance upon the ITM AR face due to this temperature distribution. Referring to Figure 2, the CP and ITM faces form two disks of radius 0.17m, centered along a shared axis, separated by distance z. The irradiance onto a plane from a surface emitter facing it from distance z is given by

$$I = \frac{J_0 \cos^4(\varphi)}{z^2}$$

where J_0 is the surface emitter's radiance and φ is the angle from normal of the line connecting the surface emitter to the point on the plane.[1]

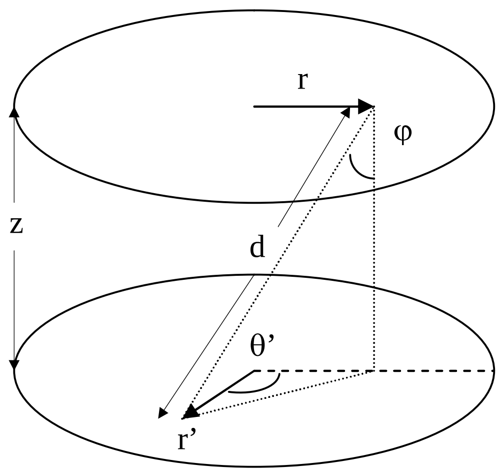


Figure 2: Diagram for derivation of heating pattern on ITM AR face.

From Figure 2, it is clear that

$$\cos(\varphi) = \frac{z}{d} = \frac{z}{\sqrt{(r'\cos(\theta) - r)^2 + (r\sin(\theta))^2 + z^2}}$$

The blackbody radiance of the emitting surface is given by $J_0 = \varepsilon\sigma T^4(r')/\pi$, where ε is its emissivity, $T(r')$ its temperature, and σ is the Stefan-Boltzmann constant. Note that since we are describing a radiance (W/m^2 steradian) and not an intensity (W/m^2), the formula includes a factor of π. Finally, the net irradiance at point r on the ITM from the whole CP requires us to integrate over r' and θ':

$$I(r) = \int_0^{.17m} r'dr' \int_0^{2\pi} d\theta' \frac{J_0 \cos^4(\varphi)}{z^2} = \frac{\varepsilon\sigma z^2}{\pi} \int_0^{.17m} r'dr' T^4(r') \int_0^{2\pi} d\theta' \frac{1}{\left[(r'\cos(\theta) - r)^2 + (r'\sin(\theta))^2 + z^2\right]^2}$$

1

[1] Two powers of the cosine come from the distance between the surface emitter and point on the plane, one power comes from the Lambertian radiance pattern of an incandescent surface emitter, and one power comes from the foreshortening of the plane as seen from the surface emitter. See p. 209, Modern Optical Engineering, 2nd ed., Warren Smith for details.

We can simplify this integral by noting that $T(r')$ is nearly constant over the CP face, so we can move it out of the integrand. Further, the integration over θ will be simple if we re-express the remaining integrand:

$$I(r) = \frac{\varepsilon\sigma z^2 T^4}{\pi} \int_0^{.17m} r'dr' \int_0^{2\pi} d\theta' \frac{1}{\left[\left(r'^2 + r^2 + z^2\right)^2 - 2rr'\cos(\theta)\right]^2}$$

Integrals of this form have the solution

$$\int_0^{2\pi} dx \frac{1}{(a + b\cos x)^2} = \frac{2\pi a}{\left(a^2 - b^2\right)^{3/2}}$$

so long as $a > |b|$. In our case, $a = r'^2 + r^2 + z^2$, $b = -2rr'$, and we can use the law of cosines to verify that $a > |b|$. This reduces the integral to

$$I(r) = \frac{\varepsilon\sigma z^2 T^4}{\pi} \int_0^{.17m} dr' \frac{2\pi r'\left(r'^2 + r^2 + z^2\right)}{\left[\left(r'^2 + r^2 + z^2\right)^2 - 4r^2 r'^2\right]^{3/2}}$$

This integral may be solvable by general techniques, but instead we solved it numerically using Mathematica. The solution for the heating of the ITM surface is shown in Figure 3.

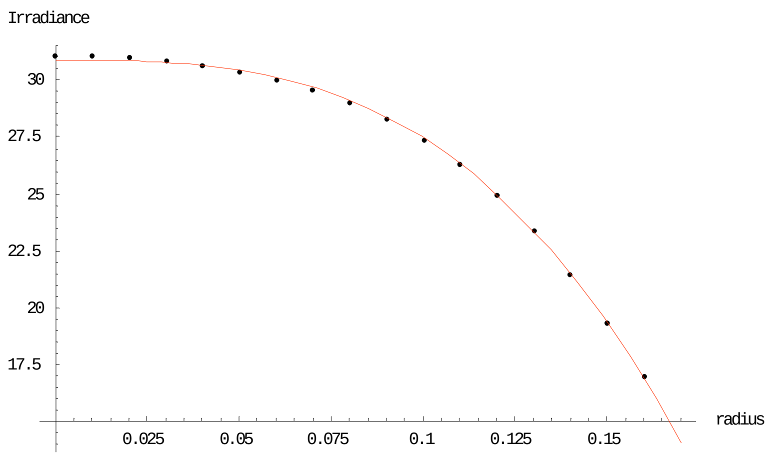
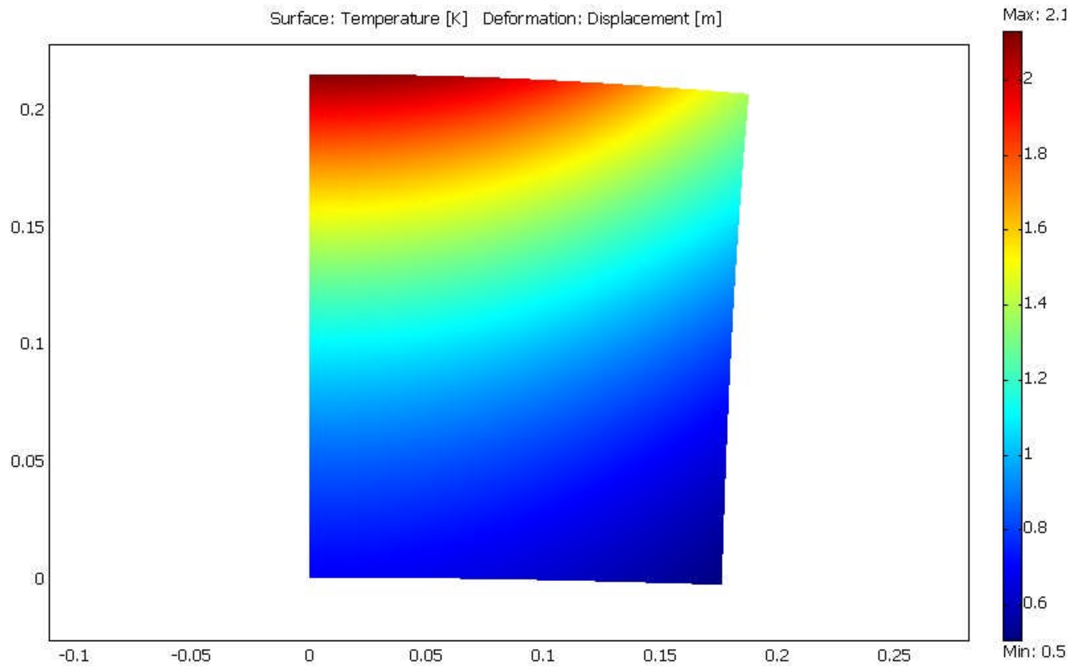


Figure 3: Heating profile on ITM AR face for annular CP heating. ITM-CP separation is 7cm.

A check on the accuracy of this calculation is to investigate the case when the ITM-CP separation is very small compared to their radius. This will be discussed in the next section. The red line in the figure is a fit of the function $I = a + br^3$ to the result; the goodness of this fit allowed us to use it as a boundary heating condition to a COMSOL model of the ITM. The result of that model is shown in Figure 4.

Figure 4: temperature profile and deformed shape of ITM heated by profile in Figure



3.

We see that there is both a flexure of the ITM due to the heating of the ITM face, and a radial thermal gradient inside the ITM. This latter dominates the thermal aberration, so we show in Figure 5 the thermorefractive profile through the ITM.

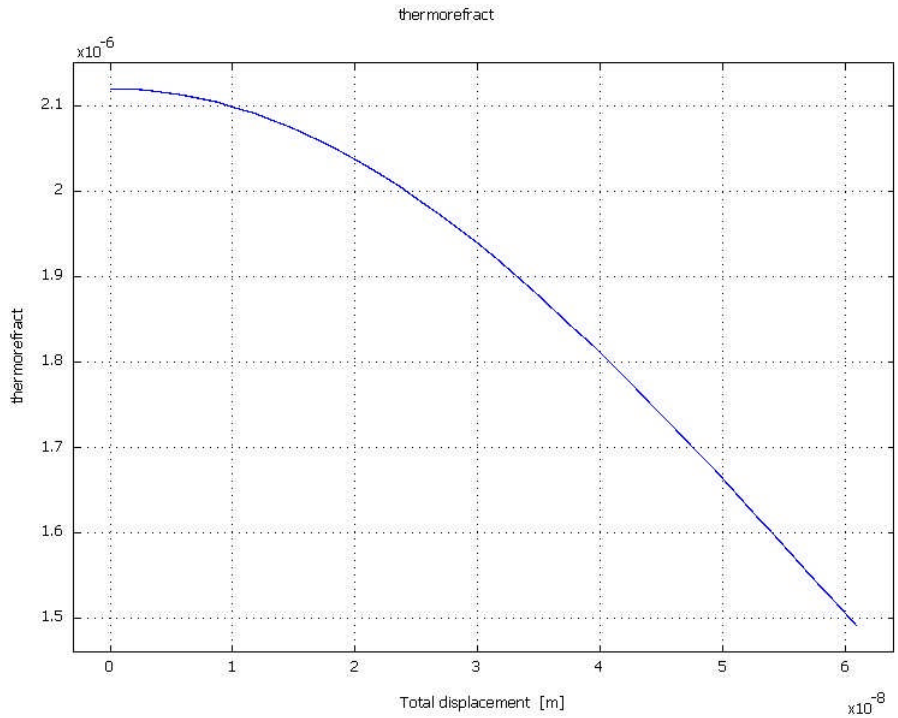


Figure 5: Thermorefractive profile through ITM axis.

Observe that the optical path through the ITM is now equivalent to a 12 km positive thermal lens, as opposed to the 5 km negative thermal lens in the compensation plate. Thus, the effectiveness of the compensation plate has been reduced about 40%. In addition, the optimal compensation profile has changed.

3 Implications for TCS

Figure 6 shows the temperature distribution for a CP heated by a 0.5 W Gaussian central spot, as would be used for a hot point design. In this case the power incident on the CP is 15x less than for the annular heating, and the average temperature of the back face is 10x lower, although with a stronger gradient, and the radiated power is only .22 W.

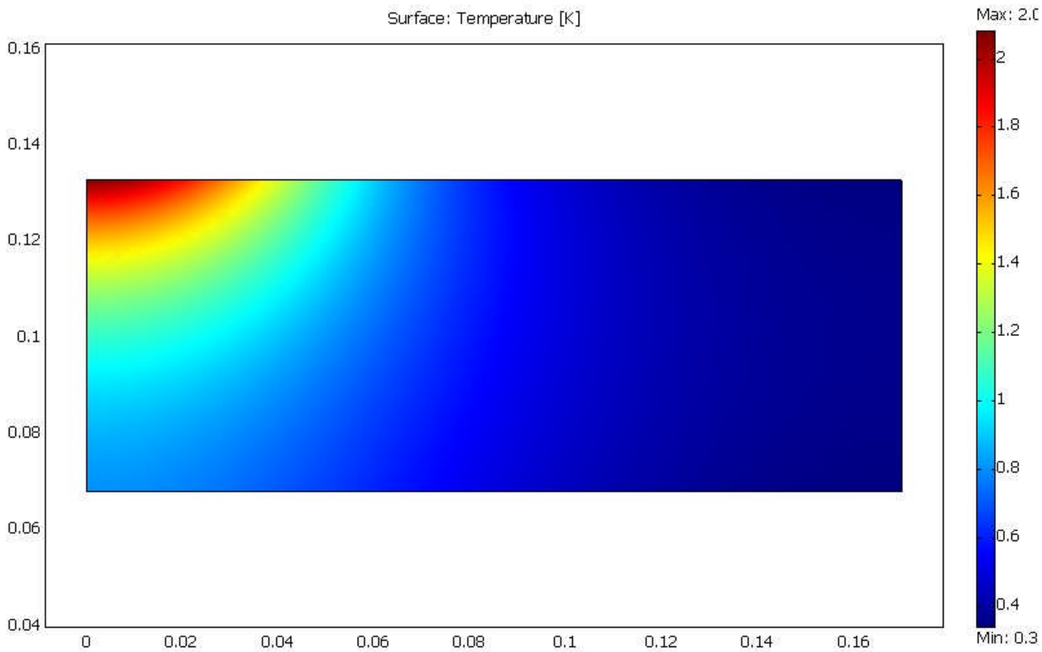


Figure 6: temperature profile for centrally heated CP.

The temperature distribution across the back face of the CP is no longer as uniform as it was for the annular compensation, although it would be much more uniform if the CP were thicker. To rough order, the temperature rise of the back face of the CP is 11x smaller, and so the effect on the ITM will also be 11x smaller- the induced positive thermal lens will have 132 km focal length. Therefore, one possible strategy for dealing with this issue is to use the hot point design.

There is a drawback to this approach. Normally, optics are made with flat or spherical surfaces, and can then be optimally coupled to each other in cavities. Since the self-heating of the interferometer creates aspherical aberrations, the baseline design to TCS uses aspherical compensation profiles, to restore net sphericity to the compensated optical wavefronts. In the hot point design, this is not possible with spherical optics. One solution to this problem would be to make the compensation plate with an aspherical surface, by means of a compensating polish or corrective coating. Corrective coatings with the requisite sag have already been made at SMA-Lyon for a mesa beam cavity experiment.

Another interesting possibility is to accept the heating of the ITM, but to try to make the temperature rise as uniform as possible. This can be done by insulating its barrel, and making the heating of the AR face uniform. Since the CP back face has nearly uniform temperature when it receives annular heating, simply moving it closer to the AR face of the ITM, so that each point on the CP heats only the opposite point on the ITM and no heat escapes out the edges, should make the heating very uniform. Making the CP thicker would both reduce the gap and make its back face temperature more even. As Figure 7 shows, the Mathematica model of the heating distribution in fact shows nearly uniform heating at the predicted blackbody level for z=5mm. Once again, the line is a simple functional fit for input to the COMSOL code.

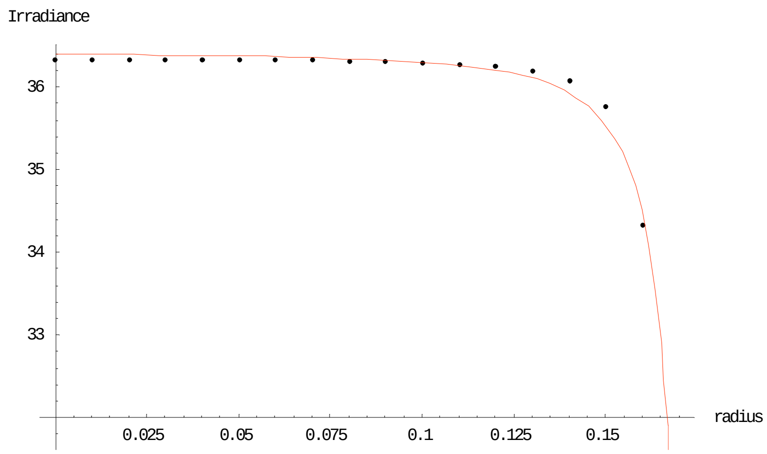


Figure 7: Heating profile on ITM AR face if the annularly heated CP of figure 1 is only 5mm away.

Figure 8 shows that the thermorefractive lens through the ITM is now 290 km, 24x larger than before, thanks to the insulation of the ITM barrel and the nearness of the CP. At this level, the flexure of the ITM becomes the dominant aberration. Since this heats the back face of the ITM, it provides ROC compensation in the proper direction to correct the arm cavity mode for its self-heating. However, the ROC change is only 5×10^{-6} diopters, far too small to make a real difference, and too small to allow for this technique to be effective without very powerful CO_2 lasers acting on the CP. Nevertheless, this indicates that the flexure is no real problem, either in the arm or in the recycling cavities. Elasto-optical effects need still be considered. Nevertheless, this indicates another strategy by which the CP-ITM heat transfer problem can be managed.

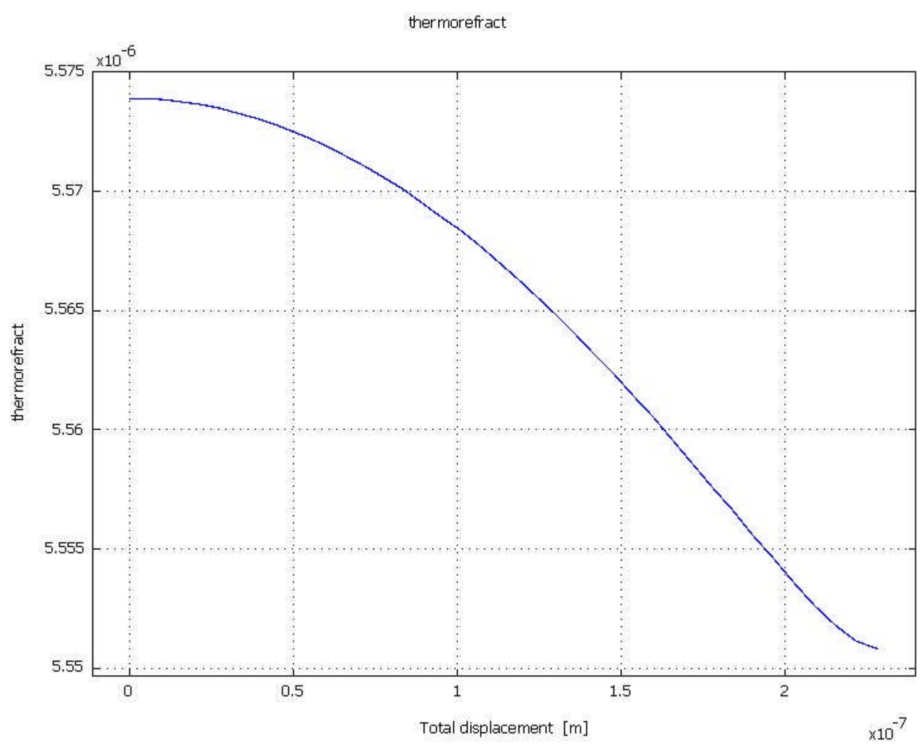


Figure 8: Thermorefractive profile through barrel-insulated ITM heated with profile of Figure 7.