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Nonlinear Pitch Effects and Potential Capsize in the Quad Suspension Noise Prototype

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1 Introduction

1.1 Purpose and Scope

Experience with the AdvLIGO quad suspension prototype reveals that there is a significant non-linearity in the pitch restoring torque. Outside a fairly narrow stable zone, the pendulum will capsize in pitch. This document describes work done to model the effect using the Mathematica pendulum modeling toolkit.

1.2 Version history

3/26/07: -00.

2 Pendulum

The quad pendulum intended for the test mass suspension in AdvLIGO is described in general terms in T010103-05, "Advanced LIGO Suspension System Conceptual Design". At the time of writing, the noise prototype had just been suspended at Rutherford Appleton Labs (RAL). The parameter set `20060515noise` in Section 5.1 of this document is a good approximation to the as-built system, except that the current build uses metal wires instead of fused silica ribbons, and for simplicity, the off-diagonal components of the moments of inertia of the upper masses have been neglected. (The parameter set is for use with the `mbquadlite2lateral` version of the quad model, and the names of the parameters are as in the diagram in Section 2.3 of Appendix C of T010103-05.)

Upon assembly it was noticed that the behaviour in pitch was gratifyingly close to that predicted by the model, provided that the payload was exactly as designed. Even a slight deficiency in mass such as leaving off the eddy current damping units would cause the system to be unstable in pitch. From experience with the controls prototype, this is known to be due to the changed position of the spring tips of the blades in the upper and upper-intermediate mass, which move up when the payload is decreased. This decreases the parameters `dn` and `d1`, which are the vertical distances from the centres of mass to the wire attachment points. The pitch restoring force is a sum over these and the other `d` parameters, weighted by the masses, so decreasing any of them makes the pendulum more unstable.

More interestingly, even for the full payload, the pendulum was only stable in pitch for a relatively modest range of ±0.11 rad. A similar effect had been observed in earlier prototypes, but it hadn't been carefully investigated because the design had had a series of ad hoc modifications to take into account newly discovered effects in the blades, which had left the exact state a little uncertain, and the values of some of the relevant parameters were difficult to measure.

With the new prototype conforming well to the model in other respects, it was a good opportunity to revisit the conditional stability effect. The suspicion was that as the upper masses pitched, the vertical load from the wires was presented to the blades more edge-on, with the component in the working direction correspondingly reduced. This would cause the blades to curl up towards their unstressed positions, decreasing `dn` and `d1` and the pitch stability.

3 Model

The emphasis in prior modeling of the suspension has been on the normal mode formulation. However this involves linearization about the equilibrium position and thus throws away information about the suspected non-linear effect. To investigate possible capsize events requires backing up to the point in the calculation where the equilibrium point is found. This is done by a routine in the pendulum modeling toolkit (T020205-01) by constructing a symbolic formulation of the potential function and minimizing it using the Mathematica `FindMinimum[]` function.

If the system is not unconditionally stable there will be additional local equilibrium points with some of the masses upside down. Which endpoint the potential minimization routine finds as a function of starting state will be a reasonable proxy for what the real system does. To implement this, the minimization code was run multiple times with different starting states. The results are in the Mathematica notebook `ASUS4XLLateralModelCalc20060515NoiseTiltTest.nb` in the directory `20060515noisetilttest` in the T070075-00.zip archive accompanying this document.

A typical run was done in two stages as follows. In Step 1, the pitch of the top mass (`pitch0`) was frozen at a non-zero value and the minimum with respect to all the other coordinates was found. This simulates deliberately tilting the top mass under control. For large values of `pitch0`, the bottom three masses capsized immediately, in which case there was no point continuing to the second step. If they stayed up then in Step 2, the top mass was released and the minimum with respect to all the coordinates was found.

By trial and error the approximate endpoints of three zones were established:

Zone 1: pendulum is stable in the upright position

Zone 2: bottom three masses are stable alone, but capsize with top mass

Zone 3: even the bottom three masses capsize

For the default parameters, the results were

Zone 1: <0.17 rad

Zone 2: 0.18-0.22 rad

Zone 3: >0.23 rad

This is reasonable agreement with what was measured given the difficulty of measuring `dn` and `d1`. It tells us that the top mass is the weak link, but that the upper intermediate mass is not far behind. At also confirms the blade theory. At 0.17 rad in Step 1, the intermediate blades (on the top mass) had risen by 3.9 mm and the lower blades (on the upper intermediate mass) had risen by 1.3 mm. At 0.22 rad in Step 2, the lower blades had risen by 3.0 mm.

To check the sensitivity on these parameters, `dn` and `d1` were increased by 2 mm each:

Zone 1: <0.22 rad

Zone 2: 0.23-0.27 rad

Zone 3: >0.28 rad

As might be hoped, this provides a modest but potentially useful increment in stability.

4 Conclusions

The lessons in this study are:

(i) The stable zone is not huge but should be adequate. In the first instance, we should simply keep the stops tight enough to prevent the pendulum getting outside the stable zone.

(ii) Payloads need to be calculated precisely and if necessary should err on the heavy side.

(iii) In an emergency, stability can be increased a useful amount by adding trim mass or otherwise increasing `dn` and `d1`. This would tend to increase the frequency of the first two pitch modes (#1 and #9 in the table in Section 5.2), but this is relatively innocuous provided they don't land exactly on the frequency of some other mode.

5 Appendix

5.1 Parameters

```
overrides0 = {        (* AFCP = as for controls prototype *)
ribbons -> True,
     nx -> 0.1300, (* AFCP, T040214-01 *)
     ny -> 0.5000, (* AFCP, T040214-01 *)
     nz -> 0.0840, (* AFCP, 040214-01 *)
   denn -> 4000, (* AFCP, T040214-01 *)
     mn -> 22.285, (* AFCP, measured, T040229-12 *)
    Inx -> 0.4557, (* AFCP, MPL, 9/1/05 *)
    Iny -> 0.0712, (* AFCP, MPL, 9/1/05 *)
    Inz -> 0.4546, (* AFCP, MPL, 9/1/05 *)
   Inxy -> 0, (* try to design symmetrically *)
   Inyz -> 0, (* try to design symmetrically *)
   Inzx -> 0, (* try to design symmetrically *)
   Inxz -> Inzx,
   Inzy -> Inyz,
   Inyx -> Inxy,
     ux -> 0.1300, (* AFCP, T040214-01 *)
     uy -> 0.5000, (* AFCP, T040214-01 *)
     uz -> 0.0840, (* AFCP, T040214-01 *)
   den1 -> 4000, (* AFCP, T040214-01 *)
     m1 -> 21.8000, (* AFCP, measured, T040229-12 *)
    I1x -> 0.5106, (* AFCP, MPL, 9/1/05 *)
    I1y -> 0.0598, (* AFCP, MPL, 9/1/05 *)
    I1z -> 0.5136, (* AFCP, MPL, 9/1/05 *)
     ix -> 0.2000, (* M050397-02 *)
     ir -> 0.1700, (* M050397-02 *)
     if -> 0.0950, (* M050397-02 *)
   den2 -> 2200, (* Bench *)
     m2 -> den2 int[1,ir,ix,if],
    I2x -> den2 int[MOIintegrands[[1,1]],ir,ix,if],
    I2y -> den2 int[MOIintegrands[[2,2]],ir,ix,if],
    I2z -> den2 int[MOIintegrands[[3,3]],ir,ix,if],
     tx -> 0.2000, (* M050397-02 *)
     tr -> 0.1700, (* M050397-02 *)
     tf -> 0.0950, (* M050397-02 *)
   den3 -> 2200, (* Bench *)
     m3 -> den3 int[1,tr,tx,tf],
    I3x -> den3 int[MOIintegrands[[1,1]],tr,tx,tf],
    I3y -> den3 int[MOIintegrands[[2,2]],tr,tx,tf],
    I3z -> den3 int[MOIintegrands[[3,3]],tr,tx,tf],
tlnspec -> 0.416, (* NR 4/3/06 *)
tl1spec -> 0.277, (* NR 4/3/06 *)
```

```
tl2spec -> 0.341, (* NR 4/3/06 *)
tl3spec -> 0.602, (* NR 4/3/06 *)
     ln -> 0.44094, (* derived *)
     l1 -> 0.309029, (* derived *)
     l2 -> 0.340707, (* derived *)
     l3 -> 0.6, (* derived *)
     rn -> 5.200 10^-04, (* AFCP, measured, MB, 9/29/05 *)
     r1 -> 3.5000 10^-04, (* AFCP, T040214-01 *)
     r2 -> 3.1000 10^-04, (* AFCP, T040214-01 *)
     t3 -> 0.000113, (* ribbon thickness - T010103-04 *)
     W3 -> 0.00113, (* ribbon width - T010103-04 *)
    M31 -> If[ribbons, W3 t3^3/12, r3^4/4],
    M32 -> If[ribbons, t3 W3^3/12, r3^4/4],
     A3 -> If[ribbons, W3 t3, Pi r3^2],
    kw3 -> Y3 A3/l3,
     Yn -> 2.2000 10^+11, (* T040214-01 *)
     Y1 -> 2.2000 10^+11, (* T040214-01 *)
     Y2 -> 2.2000 10^+11, (* T040214-01 *)
     Y3 -> Ysilica, (* measured, MB, 11/18/05, typo corrected *)
    ffn -> 0.807, (* from Ian's data, 11/30/05, linear fit version *)
    ff1 -> 0.641, (* from Ian's data, 11/30/05, linear fit version *)
    ff2 -> 0.608, (* from Ian's data, 11/30/05, linear fit version *)
   kffn -> 1 + ffn*Tan[Pi*thetan/180],
   kff1 -> 1 + ff1*Tan[Pi*theta1/180],
   kff2 -> 1 + ff2*Tan[Pi*theta2/180],
   kbuz -> 4*Pi^2*(59/60)^2*61*kffn, (* AFCP, measured, T040229-12 *)
   kbiz -> 4*Pi^2*(70/60)^2*50*kff1, (* AFCP, measured, T040229-12 *)
   kblz -> 4*Pi^2*(76/60)^2*39*kff2, (* AFCP, measured, T040229-12 *)
     dm -> 0.001-flexn,
     dn -> 0.001-flex1+(g*m13)/(2*kbix),
     d0 -> 0.001-flex1,
     d1 -> 0.001-flex2+(g*m23)/(2*kblx),
     d2 -> 0.001-flex2,
     d3 -> 0.001-flex3,
     d4 -> 0.001-flex3,
twistlength -> 0,                       (* T040214-01 *)
   d3tr -> 1.0000 10^-03,  (* T040214-01 *)
   d4tr -> 1.0000 10^-03,  (* T040214-01 *)
     sn -> 0,                           (* T040214-01 *)
     su -> 0.003,  (* T040214-01 *)
     si -> 0.003,  (* T040214-01 *)
     sl -> 0.015,  (* T040214-01 *)
    nn0 -> 0.250,  (* T040214-01 *)
    nn1 -> 0.090,  (* T040214-01 *)
     n0 -> 0.200,  (* T040214-01 *)
     n1 -> 0.060,  (* T040214-01 *)
     n2 -> 0.140,  (* T040214-01 *)
     n3 -> 0.164, (* CT, email to NR, 9/22/04 *)
     n4 -> 0.158, (* CT, email to NR, 9/22/04 *)
     n5 -> 0.158, (* CT, email to NR, 9/22/04 *)
    nwn -> 2,
    nw1 -> 4,
    nw2 -> 4,
    nw3 -> 4,
    mn3 -> mn+m13,
    m13 -> m1+m23,
    m23 -> m2+m3,
   kbux -> 1.0 10^5,  (* as for middle *)
   kbix -> 1.0 10^5,  (* Justin 11/29/05 *)
   kblx -> 0.8 10^5,  (* Ian 12/09/05 *)
  flexn -> Sqrt[nwn Mn1 Yn/(mn+m1+m2+m3)/g]*cn^(3/2),
  flex1 -> Sqrt[nw1 M11 Y1/(m1+m2+m3)/g]*c1^(3/2),
  flex2 -> Sqrt[nw2 M21 Y2/(m2+m3)/g]*c2^(3/2),
  flex3 -> Sqrt[nw3 M31 Y3/m3/g]*c3^(3/2),
 thetan -> 180 ArcSin[sin]/Pi,
```

```
theta1 -> 180 ArcSin[si1]/Pi,
theta2 -> 180 ArcSin[si2]/Pi,
theta3 -> 180 ArcSin[si3]/Pi,
```

[…damping stuff omitted…]

```
};
```

5.2 Frequencies

N	f	type			
1	0.324911	pitch3	pitch2		
2	0.440754	x3	pitch3	pitch2	
3	0.46409	y3	y2	roll3	
4	0.577646	z3	z2		
5	0.589397	yaw3	yaw2		
6	0.777094	roll3	roll2	roll1	
7	0.987463	pitch0	x2	pitch1	x1
8	1.04527	y2	y1	y3	
9	1.19542	pitch0	pitch1		
10	1.28021	yaw3	yaw1		
11	1.50846	pitch0			
12	1.9828	x0	x1		
13	2.10122	y0	y1	roll1	
14	2.30707	z0	z1		
15	2.3554	yaw0	yaw2		
16	2.68339	roll1	roll0		
17	2.98807	pitch1			
18	2.99766	yaw1	yaw0		
19	3.2743	roll1	roll0		
20	3.39024	x0	x1		
21	3.6949	z1	z0		
22	5.09589	roll0			
23	8.83825	z2	z3		
24	11.667	roll2	roll3		