The three-detector null-stream combination with filtered data

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I. INTRODUCTION

The expression of the null-stream for a network of three ground-based interferometric detectors of gravitational waves, coherently observing a signal emitted by a coalescing binary system, is derived. The resulting statistics is based on a specific liner combination of the Fourier transforms of the filtered data, and it is characterized by its ability to suppress the filtered signal when the parameters of the filter coincide with those of the gravitational wave chirp. The resulting "filtered" null-stream combination can be used as veto against non-Gaussian events triggered by noise [1, 2].

II. THE DETECTOR RESPONSE TO A CHIRP-SIGNAL

Let m_1, m_2 be the masses of the two stars spiraling around each other, and let r be the distance separating the center of mass of this system from Earth. In the Newtonian approximation and under the assumption of circular orbit, the wave's two independent amplitudes can be written in the following form [3]

$$h_+(t;t_c,\xi,r,\delta_c) \;=\; \frac{2N}{r} \; a^{-1/4}(t;t_c,\xi) \; \frac{1+\cos^2\iota}{2} \; \cos[\chi(t;t_c,\xi)+\delta_c] \;, \tag{1}$$

$$h_\times(t;t_c,\xi,r,\delta_c) \;=\; \frac{2N}{r} \; a^{-1/4}(t;t_c,\xi) \; \cos\iota \; \sin[\chi(t;t_c,\xi)+\delta_c] \;, \tag{2}$$

where $(t_c, \xi, r, \delta_c, \iota)$ are the time to coalescence with respect to the arrival time, the time spent by the signal within a given detector's bandwidth, the distance to the source, the phase of the signal at time $t = t_c$, and the inclination of the plane of the binary with respect to the Earth line of sight respectively; N, ξ, a, and χ are functions of these parameters as well as the masses of the inspiraling stars, and they have the following analytic forms

$$N = \left[\frac{2\ G^{5/3}\ \mu\ (\pi\ M f_s)^{2/3}}{c^4}\right] \ , \tag{3}$$

$$\xi = 34.5\ \left(\frac{\mu M^{2/3}}{\mu_\odot\ M_\odot^{2/3}}\right)^{-1}\ \left(\frac{f_s}{40\ \text{Hz}}\right)^{-8/3}\ \text{sec.}\ , \tag{4}$$

$$a(t;t_c,\xi) = \frac{t_c - t}{\xi}\ , \tag{5}$$

$$\chi(t;t_c,\xi) = -\frac{16}{5}\ \pi\ f_s\ \xi\ a^{5/8}(t;t_c,\xi)\ . \tag{6}$$

In equation (3) M and μ are the total mass and the reduced mass of the system respectively, and f_s is the seismic cut-off frequency of an observing detector.

As a consequence of the particular time dependence of the wave's two polarization components, it is convenient to introduce the following complex, normalized waveform

$$S(t;t_c,\xi) \equiv \frac{a^{-1/4}(t;\xi)}{g\ \xi}\ e^{i\ \chi(t;\xi)}\ , \tag{7}$$

where the normalizing factor g is chosen in such as way that

$$\langle S, S\rangle = 2\ . \tag{8}$$

The angle brackets in Eq. (8) denote the following complex scalar product

$$\langle a, b\rangle \equiv 2\ Re\ \int_0^{+\infty} \frac{\widetilde{a}^*(f)\ \widetilde{b}(f)}{P_\Lambda(f)}\ df\ , \tag{9}$$

where a and b are two arbitrary complex functions, the "tilde" symbol denotes the Fourier transform operation, and P_Λ is the two-sided power spectral density of a random process $\Lambda(t)$.

In the stationary-phase approximation, the Fourier transform of the normalized waveform $S(t)$ can be written in the following form [3]

$$\widetilde{S}(f;t_c,\xi) = \frac{2}{g}\ \sqrt{\frac{2}{3\ f_s}}\ \left(\frac{f}{f_s}\right)^{-7/6}\ e^{i\Psi(f;t_c,\xi)}\ , \tag{10}$$

where the phase Ψ is a polynomial function of the Fourier frequency f [3].

From the above considerations it follows that the Fourier transform of the detector response to a chirping signal can be written as

$$\widetilde{d}(f) = 2\kappa\ Re[E^*\ \widetilde{S}(f)] + \widetilde{\Lambda}(f)\ , \tag{11}$$

where $\kappa = g\sqrt{\xi}N/r$, and the complex function E is equal to

$$E = F_+\ \left(\frac{1+\cos^2\iota}{2}\right) + i\ F_\times \cos\iota \tag{12}$$

III. THE CORRELATION VECTOR

In order to perform coherent searches for chirping waveforms one first construct the following complex correlation vector with the network data, $\vec{d}(t)$ [3]

$$C_i^*(\tau_{1i}) \equiv \langle S(t), d_i(t + \tau_{1i}) \rangle \ , \tag{13}$$

where τ_{1i} is the time-delay between detector 1 and detector i. In other words, the data streams are time-delayed with respect to a pre-chosen fiducial detector 1, and then cross-correlated with the chirp filter (Eq. 7). Although each filter depends on the frequency cut-off of the detector's data it is applied to, without loss of generality (and without loss of signal-to-noise ratio), we will choose such a frequency to be the lowest within the network. This way the same filter will be applied to the entire data vector.

If we now take the Fourier transform of the correlation vector C_i^*, computed at the point in the parameters space coinciding with that of the signal present in the data, we may notice that the following linear combination

$$\sum_{j=1}^{3} K_j(\theta_s, \phi_s) P_{\Lambda_j}(f) \widetilde{C}_j^*(f) \ , \ K_j = \epsilon_{jlm} \ F_{+l} \ F_{\times m} \ , \ l, m = 1, 2, 3 \ , \tag{14}$$

identically cancels the filtered signal. From this consideration it is then easy to derive the following χ^2-statistics for the null-stream of the filtered data

$$L^2(\theta, \phi, t_c, \xi) = \int_0^{+\infty} \frac{|\sum_{j=1}^{3} K_j(\theta, \phi) P_{\Lambda_j}(f) \widetilde{C}_j^*(f)|^2}{\sum_{j=1}^{3} K_j^2(\theta, \phi) P_{\Lambda_j}(f) |\widetilde{S}(f)|^2} \ df \tag{15}$$

The above χ^2-statistics can be used as a veto against noise-triggered fluctuations affecting the filtered data in a way similar to that derived in [1]. A generalization of this algorithm for 3-detectors to networks of N-detectors should be derivable by following a procedure similar to that discussed in [1].

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[1] S. Chatterji, A. Lazzarini, L. Stein, P. Sutton, A. Searle, M. Tinto, *Phys. Rev. D*, In press

[2] A. Searle *et al.*, In preparation.

[3] A. Pai, S. Dhurandhar, S. Bose, *Phys. Rev. D*, **64**, 042004 (2001)