LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Comparison of photon calibrator results with the official calibration.

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1 Introduction

One of the main motivations for implementing the LIGO photon calibrators is validation of the calibration of the interferometer error signals performed via calibration of the end test mass coil actuator transfer functions. The purpose of this document is to describe and present the results of recent efforts to compare calibration of the test mass coil actuator transfer functions using the photon calibrators with the coil actuator transfer function calibration results that are part of the "official" calibration procedure.

2 Calibration of the test mass coil actuators

The principal goal of the LIGO calibration group is to provide accurate amplitude and phase calibration of the interferometer *gravitational wave channels*, e.g. H1:LSC-DARM_ERR. Simply stated, this involves introducing an external differential length modulation and measuring the transfer function between this length modulation and the DARM_ERR signal. If the calibration of the actuator that generates the external length modulation is known, then DARM_ERR can in turn be calibrated in units of counts/meter of differential motion.

As the name implies, DARM_ERR, the "GW channel," is the error point of the interferometer differential arm-length feedback control loop. The actuators for this control loop are the end test mass coil actuators. The LIGO calibration group has utilized several independent techniques in efforts to attain the absolute calibration of the end test mass coil actuators. These techniques are described in detail in Ref. [1]. The technique currently preferred, sometimes referred to as the "free-swinging AS_Q" method, is described in Section 2.2, below.

2.1 Photon Calibrator calibration of the test mass coil actuators

Validation of the *official* calibration of the GW channels boils down to comparison of the independent calibration of the ETM coil actuation functions using the Pcals with the *official* ETM coil actuation transfer function calibrations. In practice, this is achieved by simultaneously[1] driving the coil actuator and the Pcal actuator for a particular test mass at closely separated frequencies, and comparing the transfer coefficients to the DARM_ERR signal. The force on the test mass induced by the Pcal can be calculated based on knowledge of the modulated laser power incident on the test mass. Knowledge of the mass of the test mass allows calculation of the expected test mass motion in response to the modulated force. The calculated Pcal-induced motion is compared with that induced via the coil actuators via the transfer coefficients to DARM_ERR and the *official* calibration of the coil actuation transfer functions. The comparison results for the three LIGO interferometers (six end test mass coil actuators) are presented in Section 3.

2.1.1 Pcal and coil transfer coefficients to DARM_ERR

We measure the Pcal and coil actuator transfer coefficients by driving the ifo:LSC-ETMi_CAL_EXC (for the photon calibrator) and ifo:LSC-ETMi_EXC (for the coil actuator) channels and measuring the transfer function to the ifo:LSC-DARM_ERR channel. Actually, the ifo:LSC-ETMi_CAL channel, the readback of the photodetector that monitors the photon calibrator output power, is used for the Pcal transfer function. The drive frequencies are separated by 1.5 Hz. This allows simultaneous, although slightly offset in frequency, measurement of the transfer functions which we expect to eliminate systematic errors that would result from variations in interferometer parameters such as the DARM loop gain or beam positions on the

[1] Or nearly simultaneously – before Feb. 2007 we staggered the transfer function measurements by one or two data points (frequencies). More recently, we drive simultaneously, but offset the drive frequencies by 1.5 Hz.

test masses. The systematic discrepancy resulting from the 1.5 Hz offset in the Pcal and coil drive frequencies can be estimated using the V2 interferometer model. The results are shown in Figure 1.

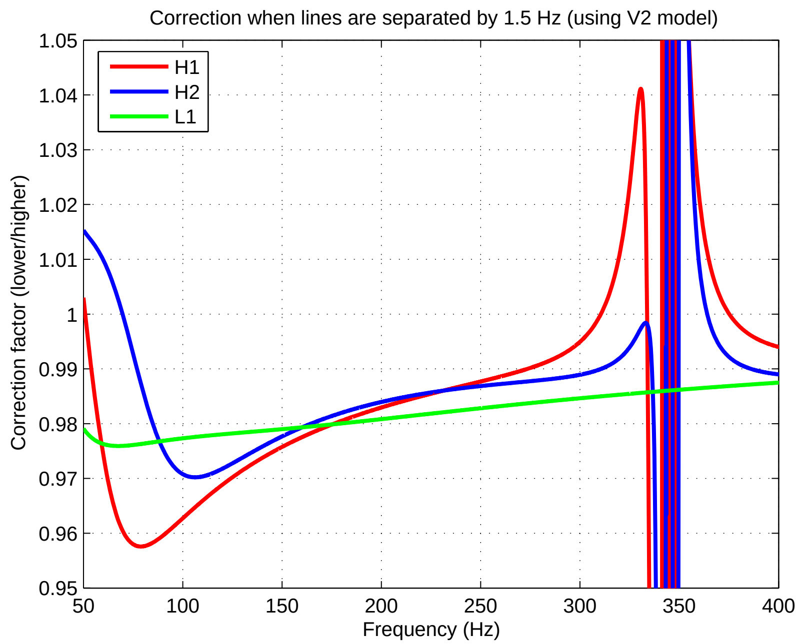

Figure 1 Correction to DARM_ERR for a 1.5 Hz separation between the coil and Pcal drive frequencies. Estimated using V2 interferometer model.

Because the interferometer response function varies significantly over the 1.5 Hz frequency separation (by about 2% for L1, the sweep is repeated with the coil and Pcal drive frequencies swapped and the results are averaged (see Section 3). The measured Pcal and ETM coil to DARM_ERR transfer functions are shown below. These measurements were made with sequential sweeps The results of more recent L1 measurements made with simultaneous sweeps on Mar. 16, 2007 are shown in Section 3.

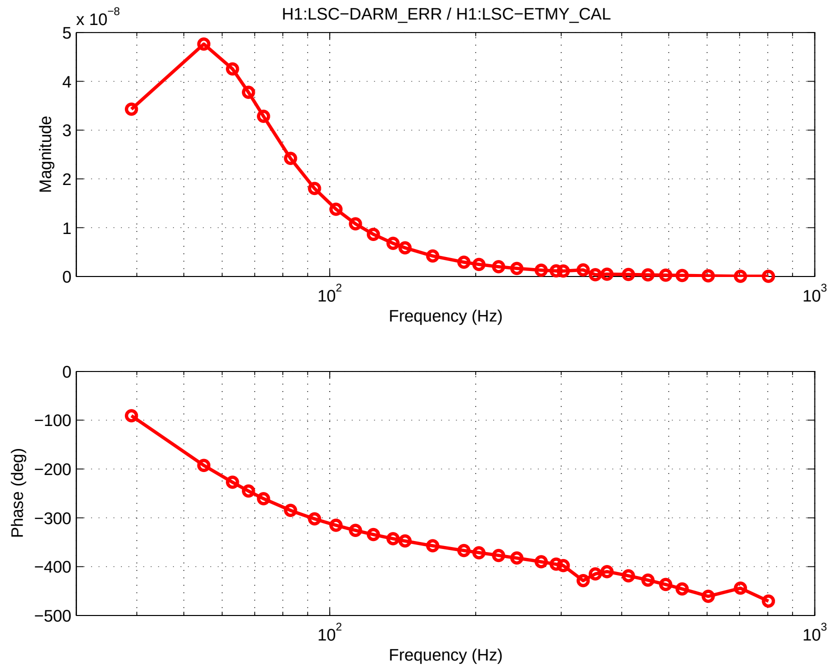

Figure 2 H1 Yarm Pcal to DARM_ERR transfer function measured on 9/28/06/.

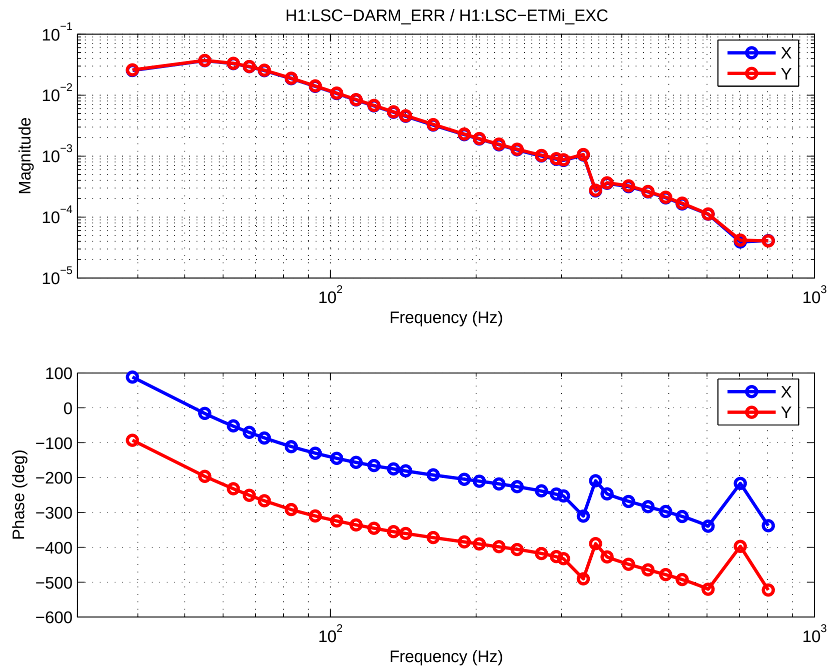

Figure 3 H1 Xarm and Yarm Coil to DARM_ERR transfer functions measured on 9/28/06/.

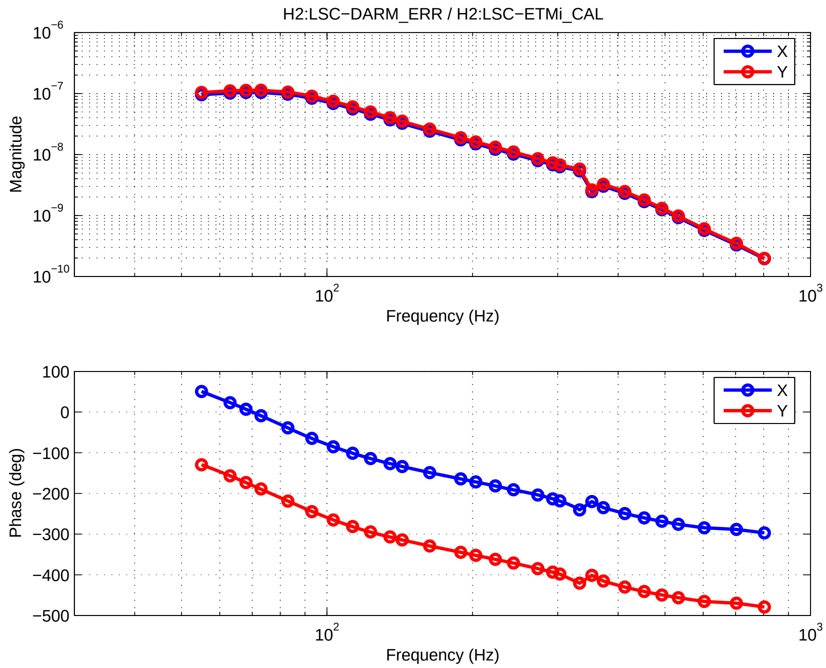

Figure 4 H2 Xarm and Yarm Pcal to DARM_ERR transfer function measured on 11/8/06.

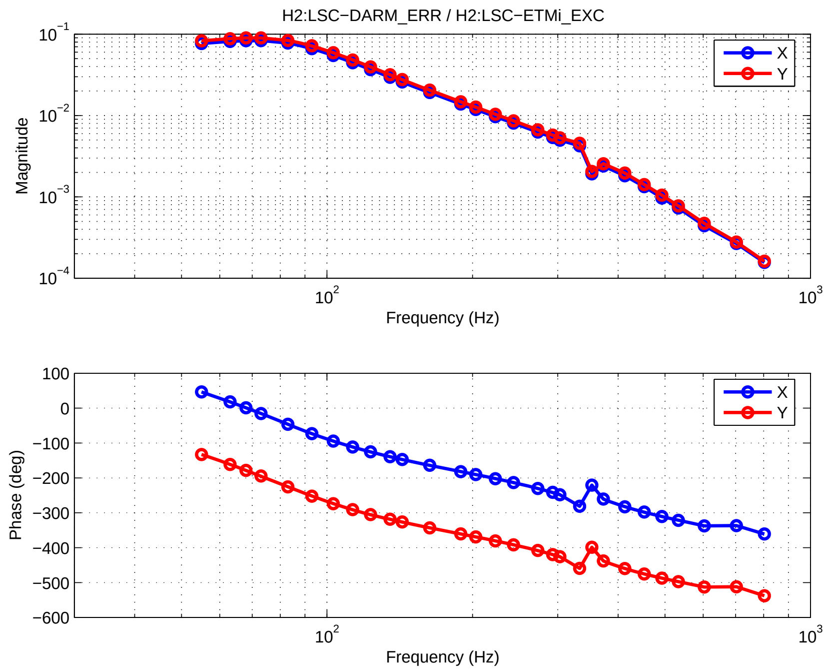

Figure 5 H2 Xarm and Yarm Coil to DARM_ERR transfer function measured on 11/8/06.

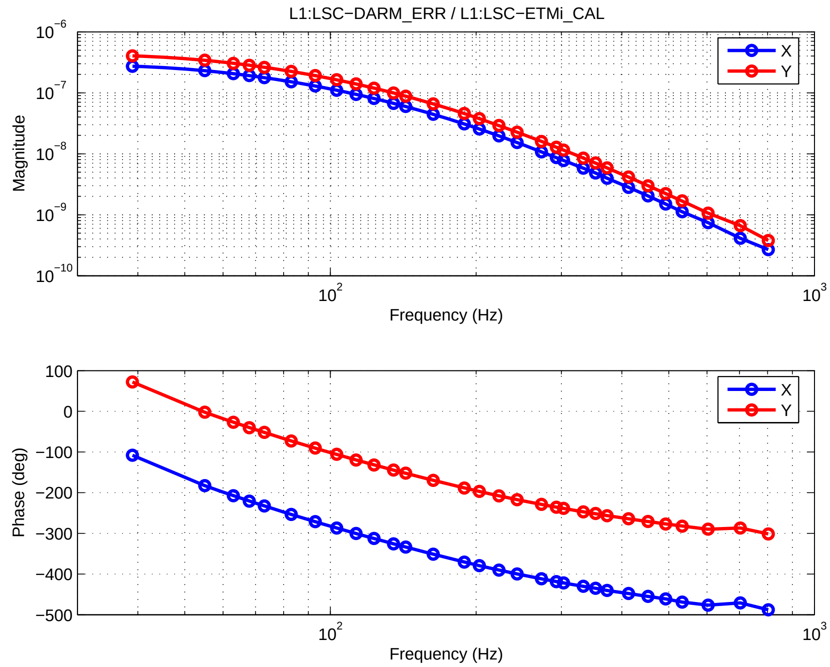

Figure 6 L1 Xarm and Yarm Pcal to DARM_ERR transfer function measured on 11/16/06.

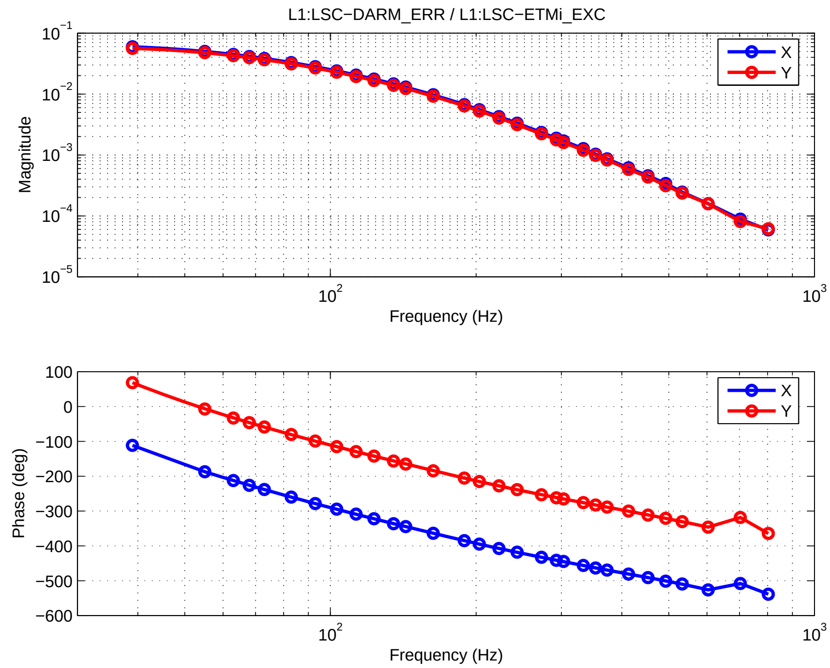

Figure 7 L1 Xarm and Yarm Coil to DARM_ERR transfer function measured on 11/16/06.

2.1.2 Calculation of Pcal-induced force

The force induced by a Pcal is given by:

$$F(t) = \frac{2\cos(\theta)}{c} P(t)$$

where θ is the Pcal beam angle of incidence on the ETM HR surface (from the vacuum side) and P(t) is the power reflected from the ETM (we neglect that the small amount of incident power not reflected, about 0.03%, effectively assuming it is transmitted, not absorbed).

The Pcal angle of incidence is determined from the ISC layout drawing (LIGO-D970220-C-D). The relevant distances from the center of the Pcal window on the vacuum equipment spool to the center of the ETM are given in Table 1. The calculated angle of incidence is 9.6 deg.

Parameter	***value***
Transverse horizontal distance	0.96 m
Transverse vertical distance	0.049 m
Longitudinal distance from input surface of Pcal viewport to ETM surface	5.723 m
Transverse distance in plane of incidence	0.96 m
Angle of incidence	9.6 Deg.

Table 1 Physical dimensions for calculation of the Pcal angle of incidence.

The fraction of Pcal output power reflected from the ETM is calculated using the calibration of the Pcal photodetector, the vacuum window transmission, and the test mass reflectivity. Determination of these parameters is described in detail in Ref. [2]. The values used for these calculations are listed in Table 2.

Parameter	***H1X***	***H1Y***	***H2X***	***H2Y***	***L1X***	***L1Y***
Photodetector calibration (mW/count)		0.04560	0.02233 0.02222[2]	0.04843	0.088[3]	0.1218
Viewport reflectivity (%)	7.1	1.1	1.1	0.8	7.0	7.7
ETM reflectivity (%)	99.97	99.97	99.97	99.97	99.97	99.97

Table 2 Pcal PD calibrations and viewport and ETM reflectivities.

Using the Pcal incidence angle of 9.7 deg. and the parameters in Table 2, we can calibrate the PCal PD readback channels in newtons/count.

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[2] The H2 Xarm Pcal is in a two-beam configuration. These are the calibrations of the photodetector for the upper and lower beams.

[3] The LLO Pcal PD calibrations are described in detail in an LLO elog entry by R. Savage on Mar. 26, 2007.

Location	*H1X*	*H1Y*	*H2X*	*H2Y*	*L1X*	*L1Y*
Calibration (N/count)		2.79e-13	2.90e-13	3.16e-13	5.003-13	6.92e-13

Table 3 Calculated calibration of PCal force actuation coefficients.

2.1.3 Calculation of Pcal-induced motion

The measurements made with the Pcals span the frequency range from 50 Hz to 2 kHz. Because the ETM pendulum resonance frequencies are below 1 Hz, in this frequency range the suspended optics are essentially free, so their response to the photon calibrator (or coil actuator) forces is given by

$$x(\omega) = -\frac{F(\omega)}{M\omega^2} = \frac{2\cos(\theta)}{Mc\omega^2} P(\omega) .$$

The mass of the ETMs is calculated based on the mechanical fabrication drawings and the published density of the Suprasil 312 test mass material, 2.201 g/cm^3 [2]. This procedure was cross-checked once by measurement of the weight of the H1 ITMX optic that was removed from the interferometer in 2005. For this optic (SPETM01), the calculated mass is 10.227 kg. The measured mass was 10.214 kg, within 0.13% of the calculated value. The mass values utilized in these calculations are given in Table 4.

	H1 ETMX	*H1 ETMY*	*H2 ETMX*	*H2 ETMY*	*L1 ETMX*	*L1 ETMY*
Mass	10.346 kg	10.388 kg	10.372 kg	10.363 kg	10.353 kg	10.365 kg

Table 4 Calculated masses of ETMs.

2.1.4 Correction for rotation-induced motion caused by beam offsets

If the Pcal beam is not centered on the ETM, it will induce rotational motion of the ETM and if the interferometer beam is not centered on the ETM, the rotational motion will be sensed by the interferometer as length modulation. This motion will either increase or decrease the Pcal-induced length modulation, depending on the relative sign of the beam displacements.

When beam offsets are taken into account, the test mass displacement formula, given in Section 2.1.3 becomes

$$x(\omega) = -\frac{2\cos(\theta)}{Mc\omega^2}\left(1 + \frac{abM}{I}\right)P(\omega)$$

where a and b are the photon calibrator and main interferometer beam offsets from center, respectively, and I is the moment of inertia of the ETM about its center of mass [2].

For the H2 and L1 interferometers, the photon calibrator beams impinge on the ETMs within approximately 1 cm of the center of the optic. This is determined using Spiricon frame grabber or spool camera images of the spots on the test masses. The OSEM spots provide fiducials and parallax is taken into account [2]. For H1, the end station baffle supports obstruct the view of the center of the ETMs from any position on the Pcal viewports. Thus the H1 Xarm Pcal beam impinges on the ETM approximately 4 cm left of center and for the Yarm the Pcal beam is approximately 1.4 cm left of center. Still, measurements of the interferometer spot positions indicate that even for H1 the Pcal motion correction due to rotation is less than one percent so we neglect rotational effects in the results presented in this document.

2.2 *Official* calibration of the end test mass coil actuators via the *free-swinging AS_Q method*

The LIGO calibration team typically presents their test mass coil actuator transfer functions in the form of a "DC" actuator coefficient, in meters/count of drive, coupled with a pendulum resonance frequency and a Q value for the resonance. While this allows, at least in theory, calculation of the actuation coefficient for all frequencies from 0 Hz to several kHz, the actual measurements that determine the length response are made at frequencies between approximately 20 Hz and 4 kHz. If we restrict our investigation of the comparison between the photon calibrator results and those of the *official* calibration to this range of frequencies, i.e. we only consider frequencies above about 10 Hz, then there is no reason to "propagate" the *official* calibration to DC which requires knowledge of the parameters of the pendulum resonance. One of the most appealing features of the "free-swinging" calibration method is that it only depends on the wavelength of the laser light and a series of transfer function measurements. In this sense, it is a true length calibration that does not require knowledge of the mass of the test mass or the characteristics of the suspension pendulum[4].

However, as we will describe below, the actual procedure followed to determine the *official* calibration results in a transfer function, measured between 40 Hz and 4 kHz, giving the ETM coil actuation function in units of meters/count of drive to the ETM coils. The plots below were generated by E. Goetz on 1/23/07. They are included to give examples of what the data products from the *official* calibration procedure look like.

2.2.1 Michelson free-swinging AS_Q calibration

The first step in the *official* calibration procedure uses the wavelength of the laser light to calibrate the peak-to-valley variations in the AS_Q signal when the interferometer is aligned in the Michelson configuration but the MICH servo is not locked. In th$_{is}$ free-swinging mode, seismic-induced motions of the input test masses and beamsplitter result in differential length variations that span the full range of resonance conditions (bright to dark fringe). The full peak-to-valley (or bright-to-dark) variations in the AS_Q signal correspond to a differential length change of one fourth of a wavelength of the laser light [3]. Thus $AS\_Q_{p\text{-}v}$ divided by $\lambda/4$ is the calibration of AS_Q in counts/meter in the Michelson configuration. The free-swinging AS_Q variations for H2 are shown in Figure 8.

[4] This is a fundamental difference between this method and the calibration of the coil actuators via the photon calibrators. The Pcals utilize calibrated forces applied to the test masses to predict the resulting motions, which of course depends on the mass of the test mass.

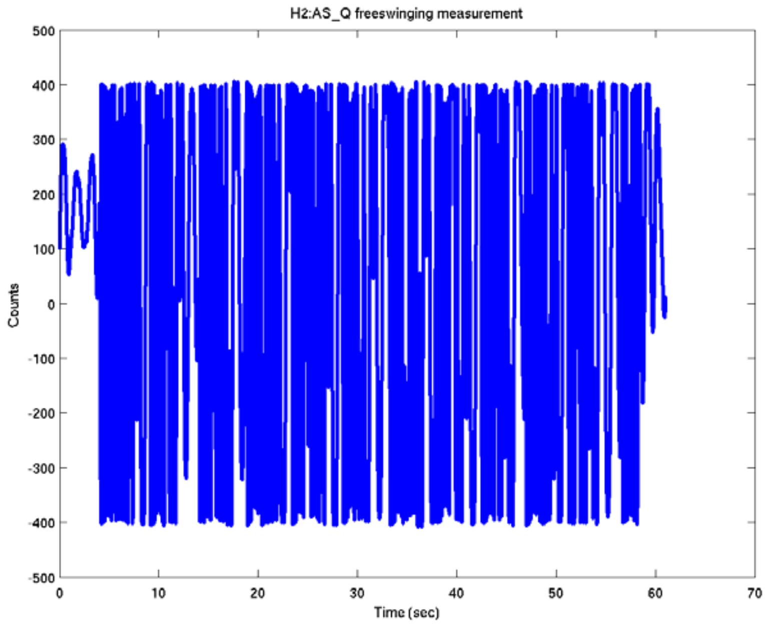

Figure 8 AS_Q variations with the H2 interferometer aligned in the Michelson configuration, but with the MICH servo unlocked. Measured on Dec. 8, 2006.

The p-v AS_Q variations are ?? counts which results in an AS_Q calibration of ?? m/count.

2.2.2 Calibration of the ITM coil actuation transfer function.

The next step in the *official* coil calibration is calibration of the ITM coil actuator transfer functions in the Michelson configuration. This is accomplished by locking the interferometer in the Michelson configuration, measuring the ITM coil to AS_Q transfer function (Figure 9), measuring the open-loop gain of the MICH servo (Figure 10), then dividing the ITM coil to AS_Q transfer function by the open loop gain. The result is shown in figure?? where it has been multiplied by the measurement frequency squared to flatten the plot.

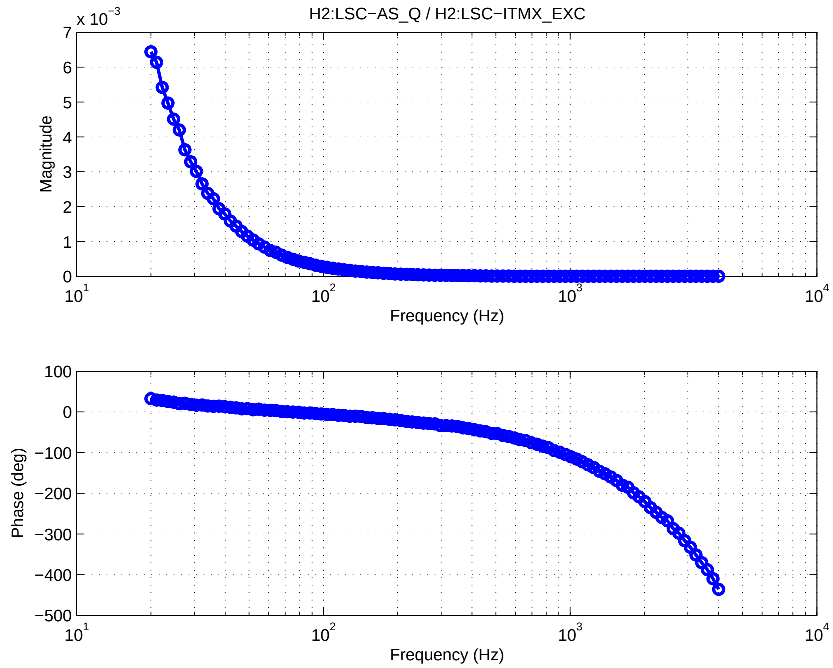

Figure 9 H2 ITMX Coil to AS_Q transfer function measured in the locked Michelson configuration on 1/23/07.

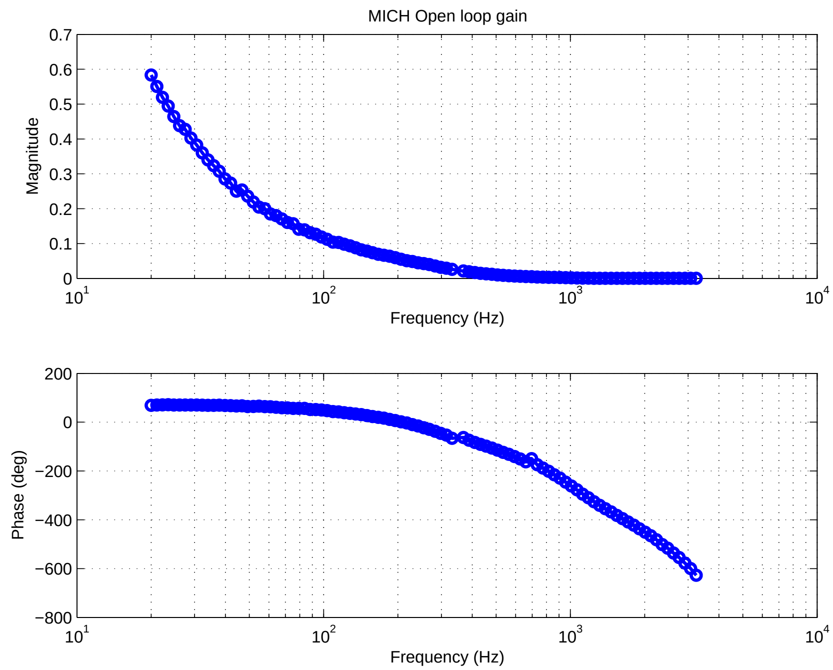

Figure 10 H2 MICH open-loop transfer function (measured with the loop closed) measured on Dec. 8, 2006.

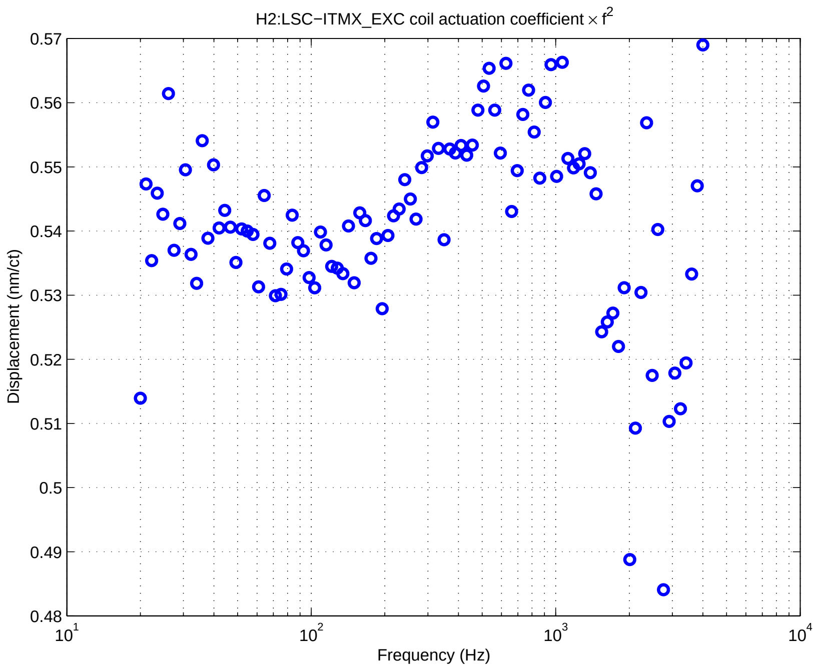

Figure 11 H2 ITMX coil actuator transfer function calibrated by dividing by the MICH closed loop transfer function, then multiplying by the AS_Q calibration. The result is multiplied by the measurement frequency squared to flatten the plot.

2.2.3 Calibration of the ETM coil actuator transfer function

The final step in the ETM coil calibration is transferring the ITM coil actuation calibration to the ETM coil actuator in single arm lock configuration. This is accomplished by making transfer functions between the ITM and ETM coil actuator drives and AS_I. The ratio of these two transfer functions gives the ratio of the ETM to ITM actuation coefficients. The H1 ETMX/ITMX transfer function is shown in Figure 12.

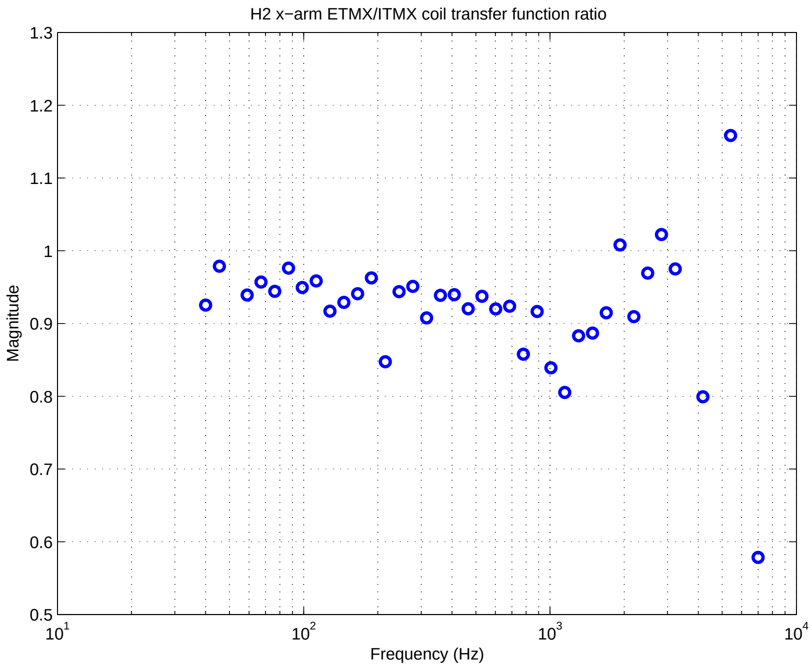

Figure 12 H2 ETMX/ITMX coil actuator transfer function ratio measured on Dec. 8, 2006.

The ITM coil actuator transfer function (in m/count) given in Figure 11 is multiplied by the ratio of the actuation transfer coefficients given in Figure 12 to yield the ETM coil actuation transfer function (also in m/count) given in Figure 13.

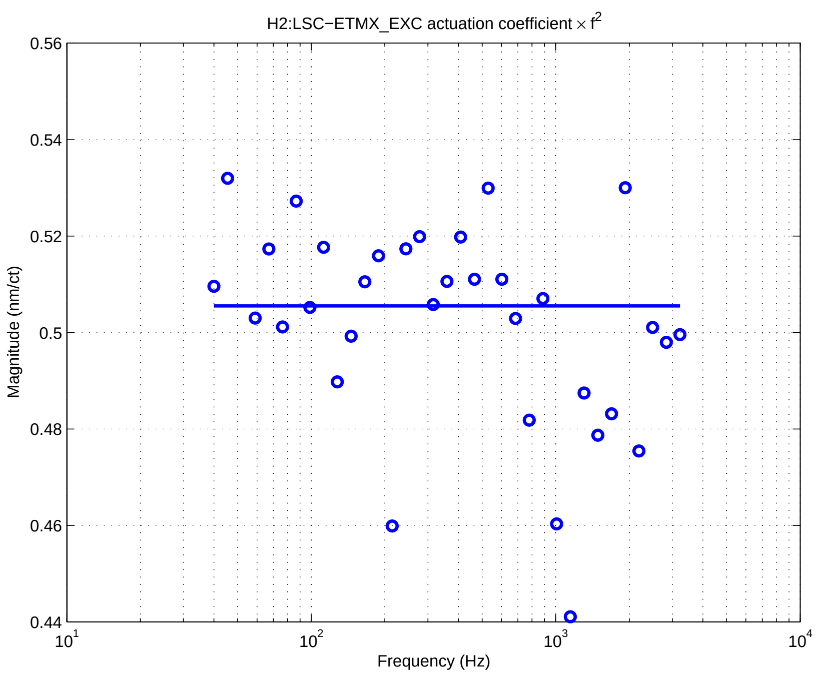

Figure 13 H1 ETMX coil actuator transfer function measured on Dec. 8, 2006

The S5-V2 ETM coil actuation coefficients, derived following the procedure described above, but averaged over several measurements made during the S5 run, are given in Table 5.

	H1X	*H1Y*	*H2X*	*H2Y*	*L1X*	*L1Y*
V2 Official calibration	0.812 nm/ct	0.831 nm/ct	0.860 nm/ct	0.896 nm/ct	0.434 nm/ct	0.418 nm/ct
Pendulum frequency	0.761 Hz	0.767 Hz	0.749 Hz	0.764 Hz	0.766 Hz	0.756 Hz

Table 5 S5-V2 ETM coil actuation coefficients.

3 Comparison between Pcal and *official* calibration of the coil actuators

We typically compare the Pcal calibration of the coil actuators with the single "DC" value reported in the *official* calibration. However, to get a feeling for the scatter in the Pcal and *official* measurements a a function of frequency, we present the full data for the Pcal and Ocal measurements for H1Y, H2X and H2Y, below.

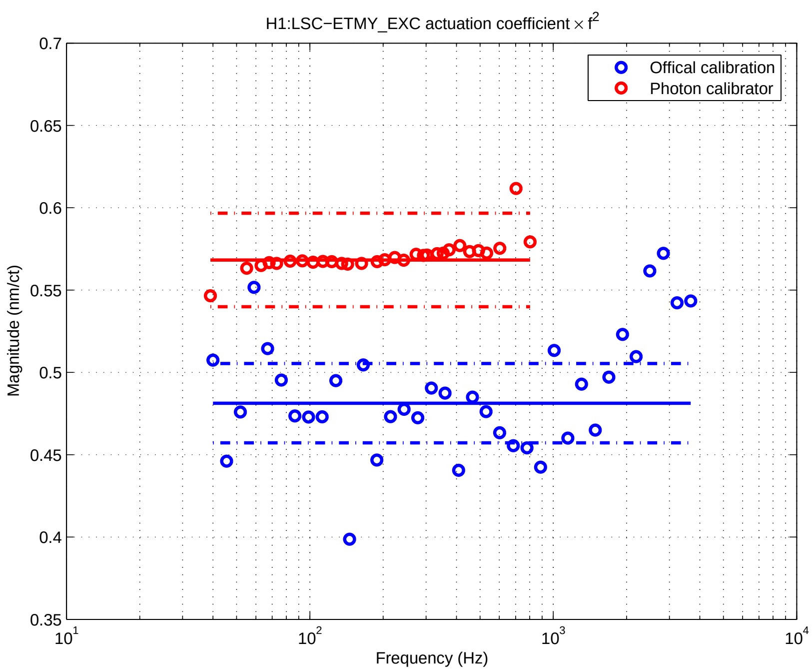

Figure 14 H1 ETMY coil actuation coefficients determined by the official calibration procedure (blue) and by the photon calibrator (red). The transfer functions have been multiplied by the square of the measurement frequency to flatten the plots.

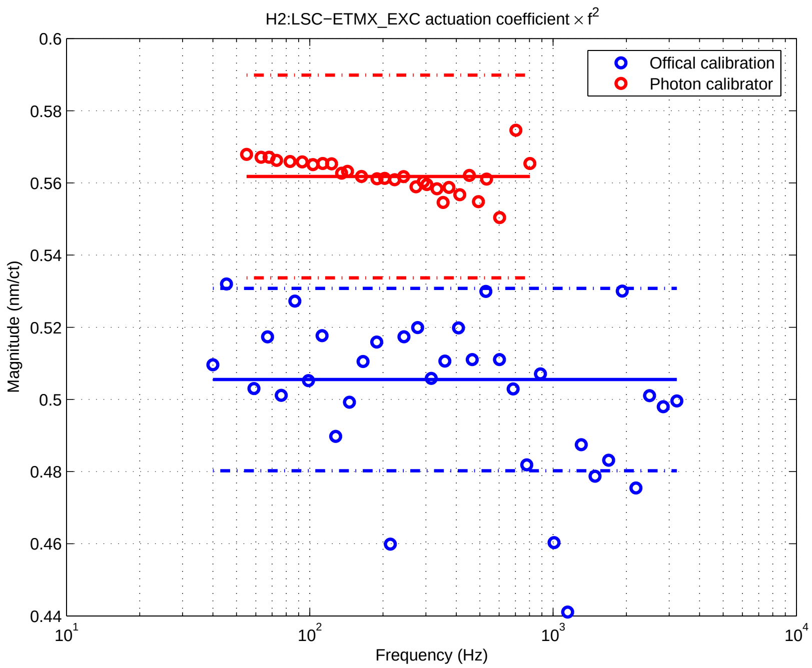

Figure 15 H2 ETMX coil actuation coefficients determined by the *official* calibration procedure (blue) and by the photon calibrator (red). The transfer functions have been multiplied by the square of the measurement frequency to flatten the plots.

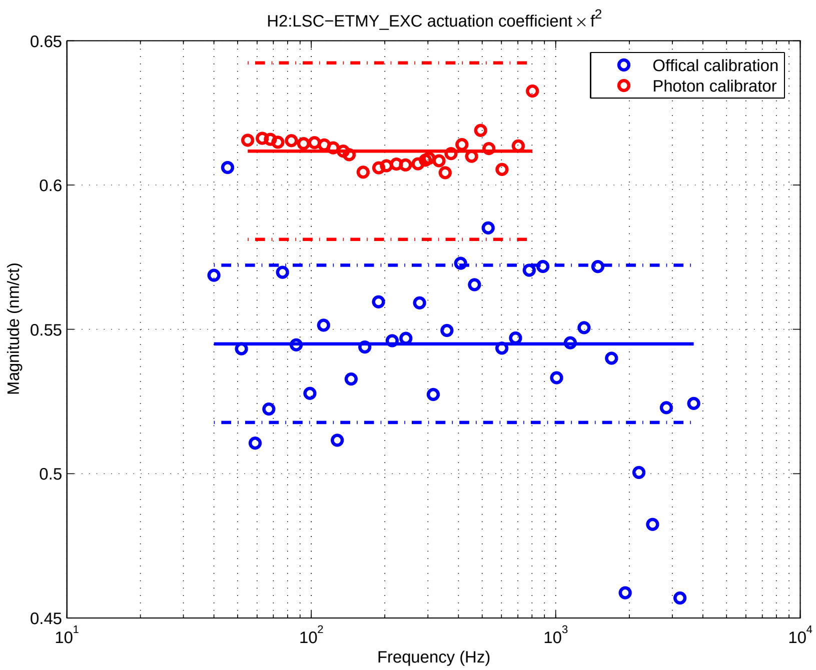

Figure 16 H2 ETMY coil actuation coefficients determined by the official calibration procedure (blue) and by the photon calibrator (red). The transfer functions have been multiplied by the square of the measurement frequency to flatten the plots.

The Pcal data compared with the single S5 V2 official calibration values for H1Y, H2X and H2Y is shown below.

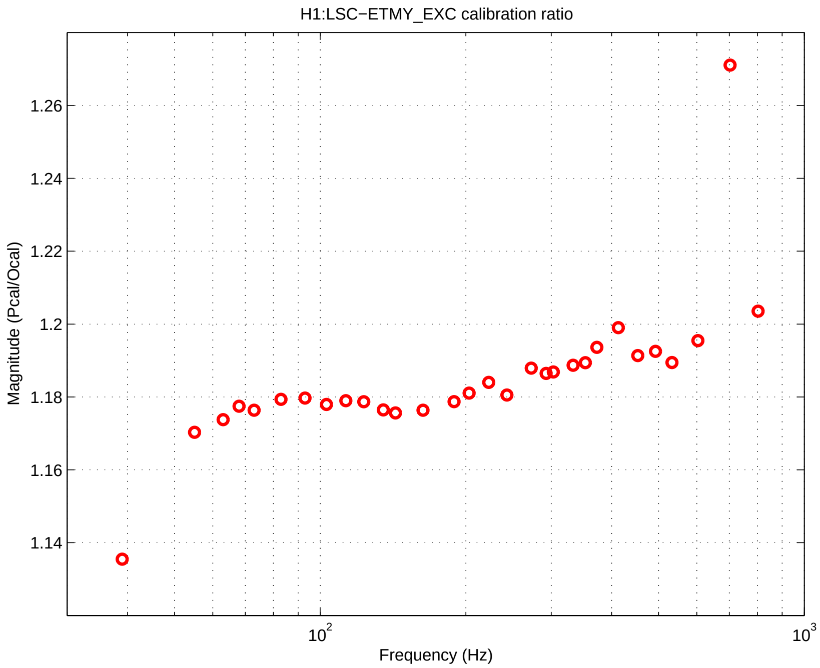

Figure 17 H1 Yarm ratio of Pcal calibration of ETM coil actuation to the *official* value.

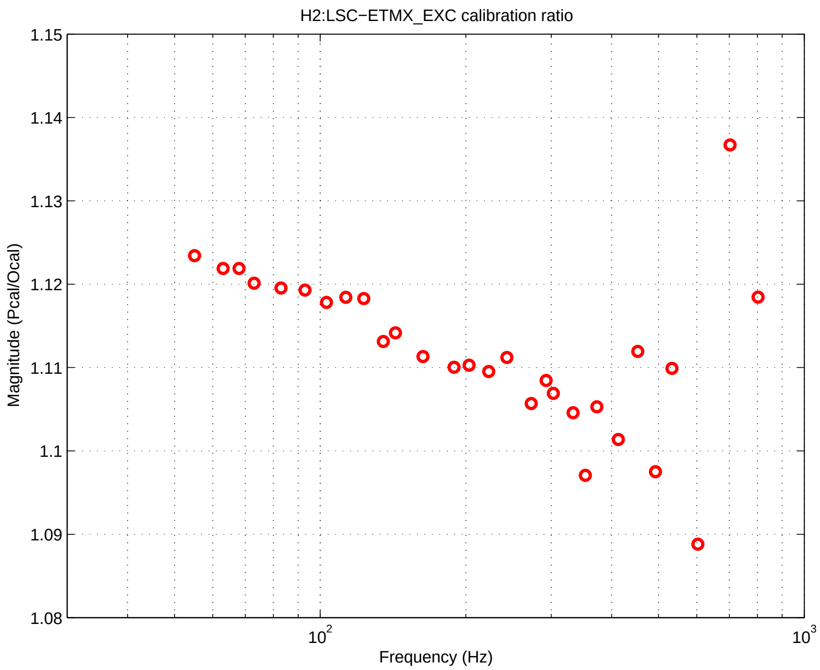

Figure 18 H2 Xarm ratio of Pcal calibration of ETM coil actuation to the official value.

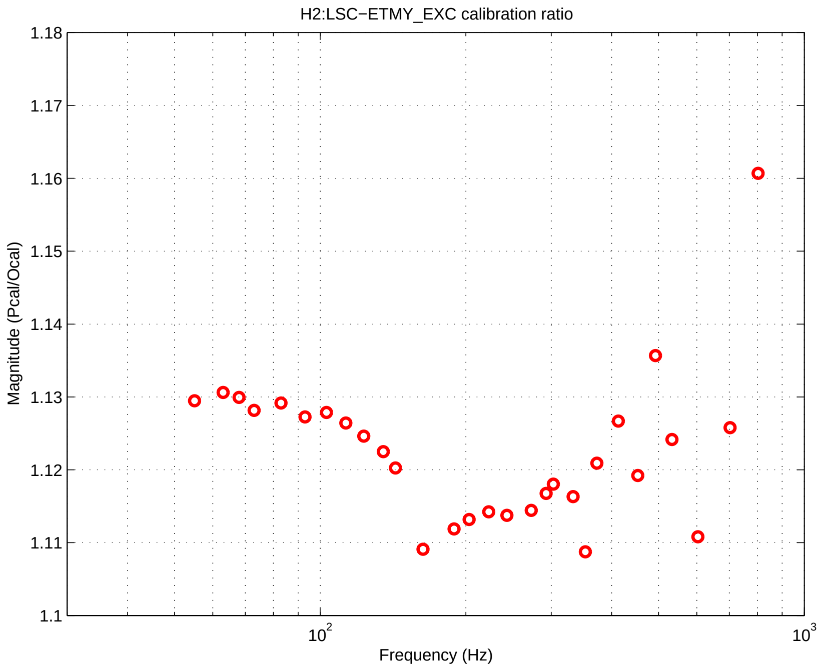

Figure 19 H2 Yarm ratio of Pcal calibration of ETM coil actuation to the official value.

For L1, we made two simultaneous sweeps for each ETM, one with the coil drive frequency y1.5 Hz below the Pcal drive frequency (labeled *Coil lo* in the plots below) and one with the coil drive frequency 1.5 Hz above the Pcal drive frequency ((labeled *Coil High* in the plots below). The mean of the two measurements at each frequency is used to estimate the Pcal/Ocal ratio.

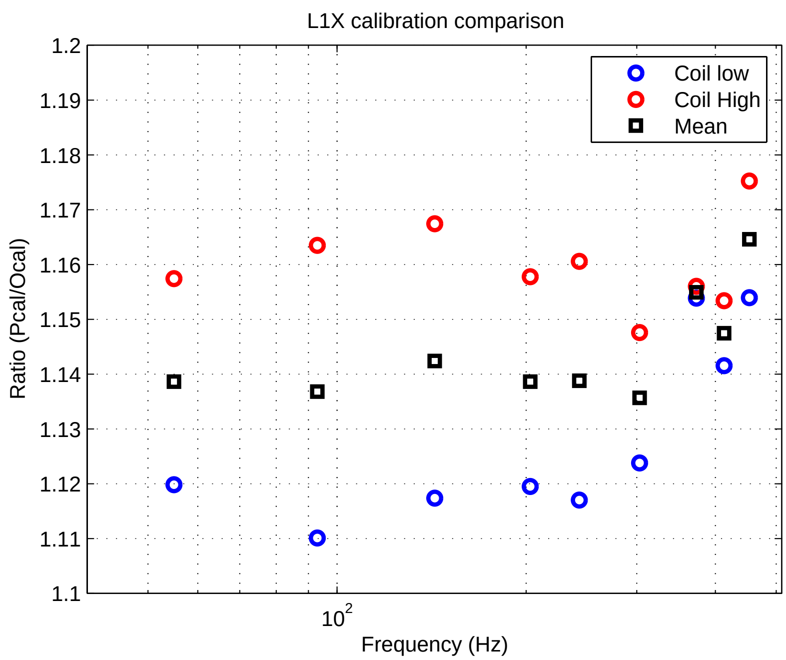

Figure 20 L1 Xarm ratio of Pcal calibration of ETM coil actuation to the official value.

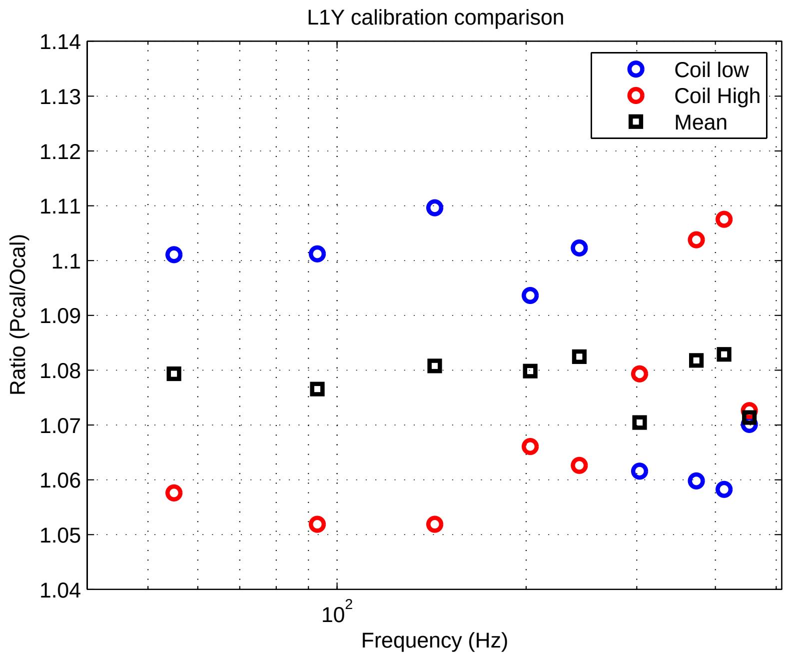

Figure 21 L1 Yarm ratio of Pcal calibration of ETM coil actuation to the official value.

To report a single number for each ETM, we take the mean of the Pcal/Ocal results, considering only the data between 50 Hz and 400 Hz because we have found the Pcal results to be excessively noisy outside this band. The results are given in Table 6, below. The ratios are calculated using the Matlab script in Appendix 1 with the S5 V2 values given in Table 5 for the official calibration.

	H1 Xarm	**H1 Yarm**	**H2 Xarm**	**H2 Yarm**	**L1 Xarm**	**L1 Yarm**
Pcal/Ocal ratio	N/A	1.16	1.16	1.17	1.14	1.08
Date measured		Sept. 28, 06	Nov. 8, 06	Nov. 8, 06	Mar. 16, 07	Mar. 16, 07

Table 6 Ratio of ETM coil calibration factors (m/ct) measured using the photon calibrators to those measured using the official calibration technique (mean of values over the frequency range from 50 to 400 Hz).

If we take the mean of the values reported in Table 6 for the five ETMs (H1Xarm data is not reported because we have not performed Pcal and coil swept sine measurements for that ETM) we get 1.14 with a standard deviation of 0.03. Thus, our measurement indicate a systematic discrepancy between the photon calibrators and the official calibration technique of about 15% with the photon calibrators indicating a larger actuation coefficient for the coils. The photon calibrator results would reduce the interferometer sensitivity by about 15% thus reducing, for instance, the binary inspiral range by this factor.

We have not addressed the error budget for the photon calibrators in this document, but this topic is discussed in detail in Ref. [2].

4 References

1. Calibration of the LIGO detectors for S4, A. Dietz et al., LIGO-T050262-01-D

2. Status of the LIGO photon calibrators: February 2007, E. Goetz, P. Kalums, and R. Savage, LIGO-T070026-00-W.
3. Calculation of the AS_Q signal for a Michelson interferometer, E. Goetz and M. Rakhmanov, LIGO-T0700??-00-W.

5 Appendix 1

Matlab script used to calculate Pcal to Ocal ratios.

File: pcalCompare.m

```
% R. Savage 070327
% Pcal calibration of coil actuators and
% comparison with official calibration
%
%
clear all
% vector order [H1x H1y H2x H2y L1x L1y]
%
c = 299792458 %speed of light in m/sec;
%
% Pcal over coil actuation ratio (ETM*_CAL/ETM*_EXC)
% determined from simultaneous or sequential sweeps
% H1x ratio was never measured, so value of 1 inserted
p2c = [1 7.837e5 7.935e5 7.897e5 2.209e5 1.423e5];
%
% PD calibration (W/ct)
pd = [4.407e-5 4.56e-5 4.455e-005 4.843e-5 8.8e-5 1.218e-4];
%
% Masses of test masses
M = [10.346 10.388 10.372 10.363 10.353 10.365];
%
% Vacuum window reflectivities (%)
Rw =[7.1 1.1 1.1 0.8 7.3 7.4];
Tw = 1-Rw./100; %window transmission
%
% Test mass reflectivities
Rtm = 0.9997;
%
% Pcal incidence angles (deg)
A = 9.6;
%
%%%%%%%%%% Pcal calibration of coil actuator %%
pcal1 = p2c.*pd.*Tw.*Rtm.*2*cosd(A)./(M*c);
%
%%%%%%%%%% Official calibration %%%%%%%%%%%%%%%%
% "official" coil calibration (m/ct) from S5 V2
Ocal = [0.812e-9 0.831e-9 0.860e-9 0.896e-9 0.434e-9 0.418e-9];
%
% Pendulum frequencies for official calibration
pend =[0.761 0.767 0.749 0.764 0.766 0.756];
%
% propagation of official calibration DC coeff. to 1 Hz
Ocal1 = Ocal.*(4*pi^2*pend.^2); % m/ct at 100 Hz
%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
rat = pcal1./Ocal1  %ratio of Pcal to Official calibration of coil actuators

% rat =  0.0000    1.1593    1.1638    1.1660    1.1385    1.0796
% mean (2:6) = 1.1414
% std/mean =0.0317
```