

Conceptual Design of a Double Pendulum for the Output Modecleaner

Norna A Robertson

For OMC team (D Coyne, C Echols, P Fritschel, V Mandic, J Romie, C Torrie, S Waldman)

Updated 13th December 2006 – version T060257-02-R. Supercedes previous versions (T060257-00-R and T060257-01-R). More of the design details are now addressed.

Further update on 21st December 2006 - version **T060257-03-R**

Appendix E added: contains details of suspension model incorporating two sets of blades – at the top and at the upper mass.

1. Introduction

The Suspensions group (SUS) has been asked to consider a design of suspension for the output modecleaner (OMC) which will hang in a HAM chamber. This modecleaner is likely to be installed during the enhanced LIGO upgrade after the S5 run, and it should also satisfy requirements for use in Advanced LIGO. The suspension requirements are at present given on the OMC wiki page at

http://lhocds.ligo-wa.caltech.edu:8000/advligo/OMC_Suspension_Specs

Although a single pendulum with blades included for vertical isolation might satisfy the requirements, we decided to investigate a double pendulum for the following functional reasons.

- 1) Damping can be applied at the upper mass and its design, including positioning of the OSEMs (and/or eddy current damping) for local control can proceed without detailed knowledge of the layout of the mode cleaner bench with associated optics.
- 2) Static adjustments for pitch, yaw and roll (as needed) can be incorporated at that stage.
- 3) Addition/subtraction of mass to flatten blades, if needed, can be done at that stage

In addition uncertainty in the required isolation for the OMC argues for the additional isolation provided by the double pendulum.

The current concept for the OMC bench is to rigidly mount the cavity optics and associated photodetectors to a baseplate. The baseplate could be made of fused silica with silicate bonding used for attaching the optics in a manner similar to the LISA optical bench design. Alternatively an aluminum baseplate with mirrors glued or otherwise attached could be used. The current plan is to use aluminium for Enhanced LIGO. See section 4 for further discussion. For the purposes of the suspension conceptual design we will assume the baseplate is fused silica of dimension 450 mm x 150 mm x 40 mm (mass

= 5.94 kg) and the mounted optics would take the total mass to 6 kg. For aluminium, the bench would have the same outer dimensions with lightweighting to achieve the same mass and similar moments of inertia as for silica.

It was noted that using an upper mass of 3 kg, together with the OMC bench mass of 6 kg would give a total suspended mass of 9 kg – which is the same as in the current Advanced LIGO input modecleaner triple pendulum design, whose three stages are each 3 kg (see T010103-04, Advanced LIGO Suspension System Conceptual Design - pg. 33 lists the (Input) Mode Cleaner Triple Pendulum suspension parameters using naming convention as in T040072-01. See also G040402 for summary of IMC design). Thus several elements of the input modecleaner design, including the top blades and the top mass might be used or adapted for the output modecleaner, saving on design effort. We have thus proceeded with a design along these lines – see figure 1 (ref D060104 – need to update with newer version when available).

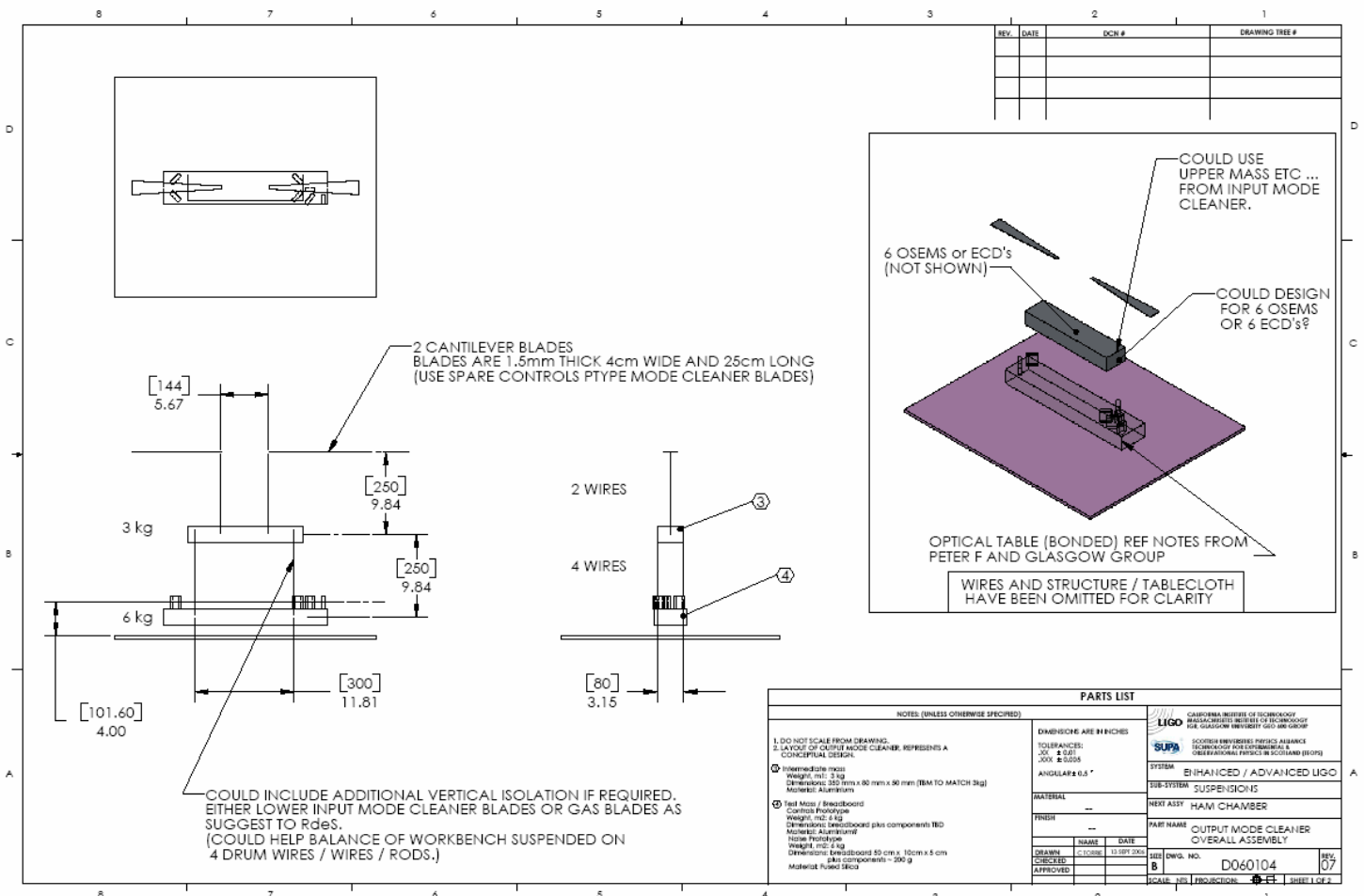


Figure 1. Outline conceptual design for output modecleaner D060104 rev 7

A more detailed concept has been put together as shown in Figure 2 (left). Here we see a modified input modecleaner top mass suspended from two blades, suspending an optical bench where the optics are attached on the lower side. By turning the bench upside down we avoid any interference of optical beams with the wire suspensions. A further design step is shown in figure 2 (right) – where the lower wires are in fact two loops going around “ears” attached to the side of the bench. Details of attachment method are TBD. This design has the advantage that installation and removal of the bench becomes straightforward. The positioning of the break-off points at the bench are chosen to be ~ 20% in from the ends of the bench to minimise sag.

Starting from this conceptual design we have put together a set of parameters and investigated the isolation and local control in six degrees of freedom. The results are presented in the following sections. A detailed list of the parameters is given in appendix A along with diagrams to explain the nomenclature. In Appendix B we give details of number and positions of damping actuators with gains and lever arms.

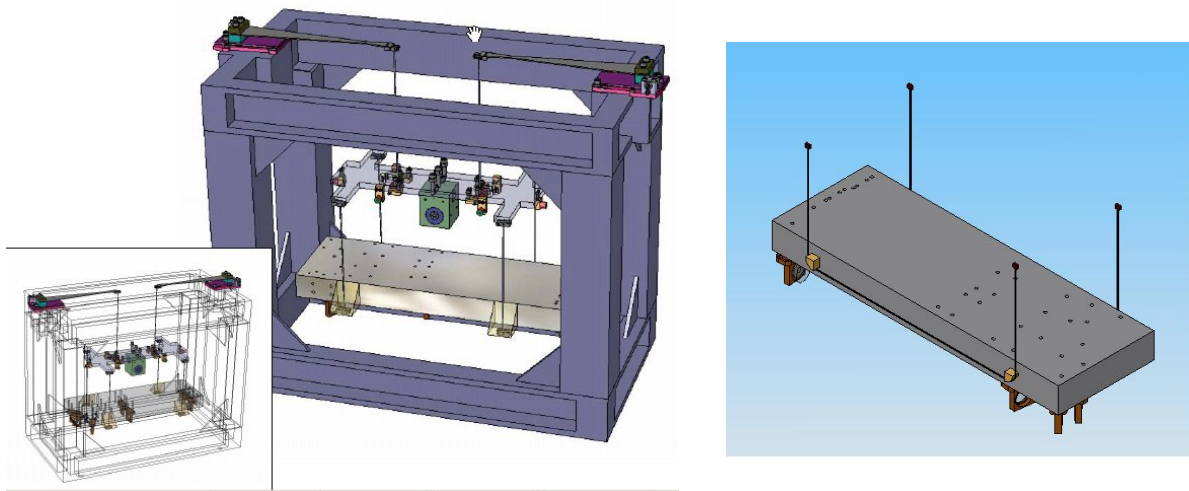


Figure 2 On the left is an early rendering of the OMC suspension mounted in its support structure. Features such as the “tablecloth” supporting the OSEMS and /or eddy current units, earthquake stops for upper and lower masses and blade guards are not shown. Shown on the right is the proposed method of suspending the table using two wire loops.

2. Design Details and Results

A MATLAB/Simulink symmetric double pendulum model has been developed by C Cueva (Stanford summer student) similar to those used for triple and quadruple pendulums. The results were checked with that of a standard MATLAB triple pendulum model with a parameter list set to simulate a double pendulum. Using the double pendulum model and the parameters listed in appendix A we obtained the following results. Note that these values are given only as preliminary and are not the final numbers. At present the mass and moments of inertia of the top mass of the *input* modecleaner has

been assumed for the top mass of the OMC. These parameters will be updated as the design of the OMC proceeds. However the mode frequencies are not expected to vary significantly from what is given below.

2.1 Normal mode frequencies (Hz)

```
longpitch1: [5.9928e-001 7.4506e-001 2.2926e+000]  
longpitch2: 7.4044e+001  
yaw: [5.2867e-001 2.9894e+000]  
transroll1: [7.4033e-001 8.4786e-001 2.2926e+000]  
transroll2: 4.2950e+001  
vertical: [1.3399e+000 3.0864e+001]
```

The longitudinal and transverse modes are indicated in italics.

The requirements document asks for the low frequency rigid body frequencies to lie approximately in the range 0.8 to 2 Hz. The above low frequencies cover a slightly larger range: 0.53 to 3 Hz. We believe this is acceptable. We note that there are three high frequency modes (pitch roll and vertical), all associated with the stretching of the lower set of wires. These are in the range 31 to 74 Hz for the current parameters. Damping of such modes is addressed in section 2.2.

2.2 Isolation and Damping

Transfer functions for longitudinal and vertical directions are shown in figures 3 and 4. The isolation at 10 Hz is indicated on the plots.

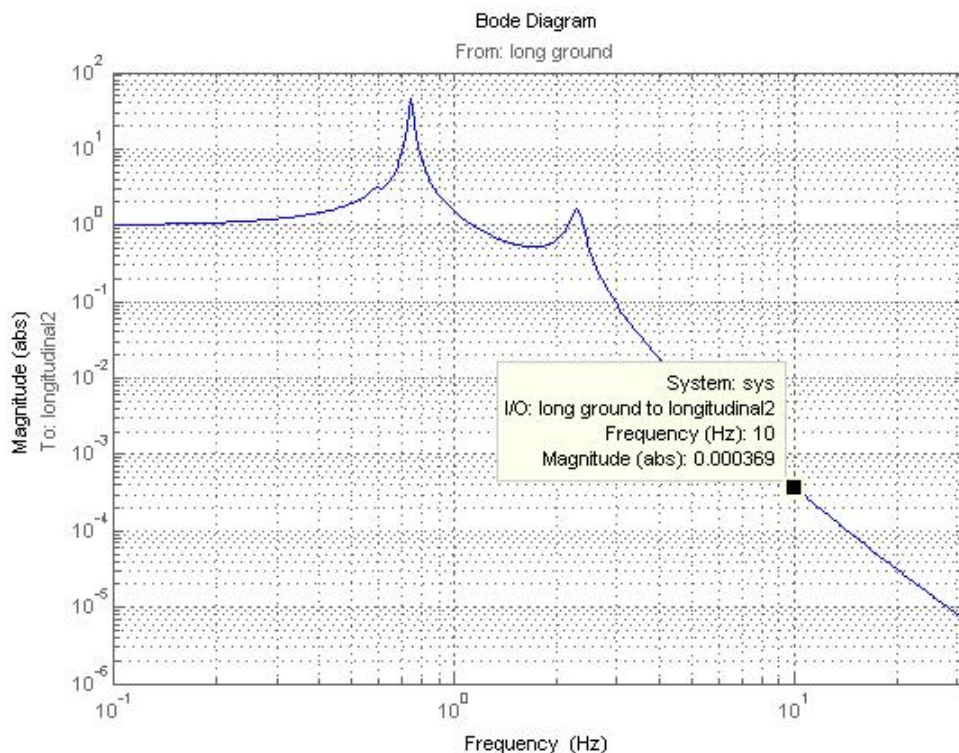


Figure 3 Longitudinal transfer function with eddy current damping and settling time = 13.1 secs.

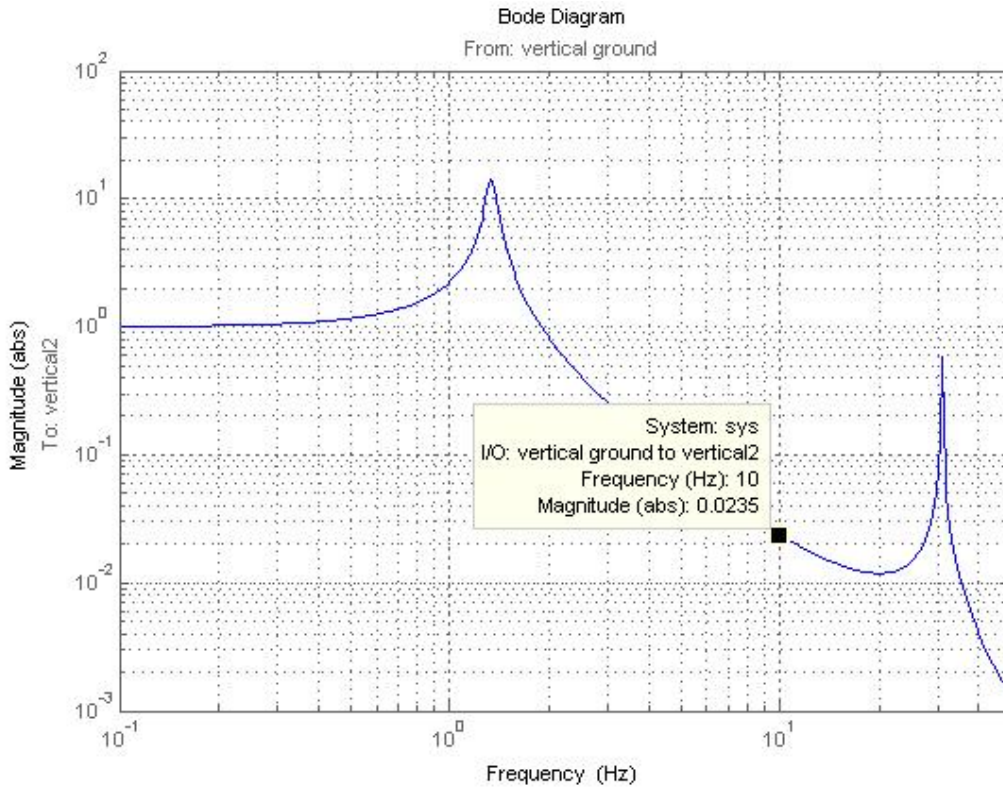


Figure 4 Vertical transfer function with eddy current damping and settling time = 3.2 secs.

Damping was modeled initially assuming a simple eddy current damping (ECD) law to check that low frequency modes were suitably coupled and that damping of similar magnitude in different directions could be applied. Active damping for all degrees of freedom using OSEMS similar to those being developed by the University of Birmingham for the Adv. LIGO quadruple and triple pendulums are the baseline design. This allows for damping and alignment control. The active damping loops are still to be fully modeled. Filtering in the feedback loop to avoid instability at the highest vertical, pitch and roll modes will be required. However this should present no problems. The OSEM or ECD positions and relative gains used in the current modeling are given in Appendix B. Positions and leverarms have been assumed to be the same as for the input modecleaner. With the gains indicated in Appendix B, the settling times (to 1/e) in all directions are in the range 3 to 17 seconds. The positions and relative gains can be adjusted if required as the design develops.

As can be seen in the vertical transfer function, the high frequency vertical mode is only weakly coupled to the top mass where the damping is applied and hence is not well damped. The same is true for the higher pitch and roll modes. We investigated how much eddy current damping (ECD) is needed to damp those modes say to a Q of 10. The amount is much larger than easily applied, and also such large damping overdamps the lower modes reducing isolation. Thus we do not advocate trying to damp these modes to this degree. If they prove problematic, three possible methods to damp to a low Q are: a) employ tuned-mass dampers; b) introduce blades into the top mass from which to suspend the optical bench, (i.e. go back to a design of top mass which is closer to the input mode cleaner top mass), and hence lower the higher frequency modes and increasing coupling; and c) use of ECDs to lightly damp the high frequency modes. Option c) is further investigated below and has been chosen for inclusion in the preliminary design.

We consider the use of ECD to damp the high frequency modes to a more modest level of $Q \sim 100$. Using for example 4 groups of two 1cm diam. by 1 cm thick magnets placed on the top face of the top mass, the high freq. vertical, roll and pitch modes could all be damped to Qs of order 100. See for example figure 5 showing transfer functions for vertical motion. Here the blue curve has simple active damping (no aggressive roll off) to give lower mode a Q of ~ 7 . The green curve is done using ECD as above i.e. 4 x 2 magnets ($b \sim 13\text{kg/s}$), giving a Q of the upper mode of 73. The isolation at 10 Hz is virtually the same. It doesn't fall off so steeply above the upper resonance but there is likely enough isolation by then. Note that this amount of ECD would be sufficient to damp the lower mode adequately.

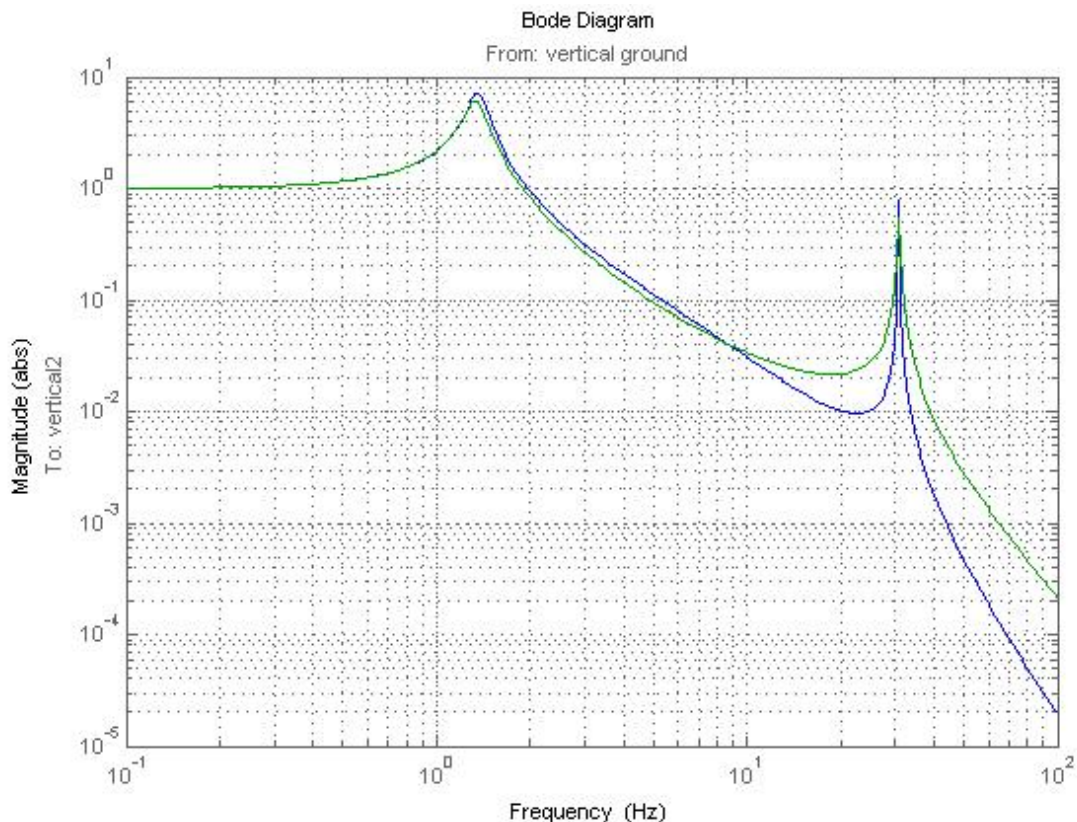
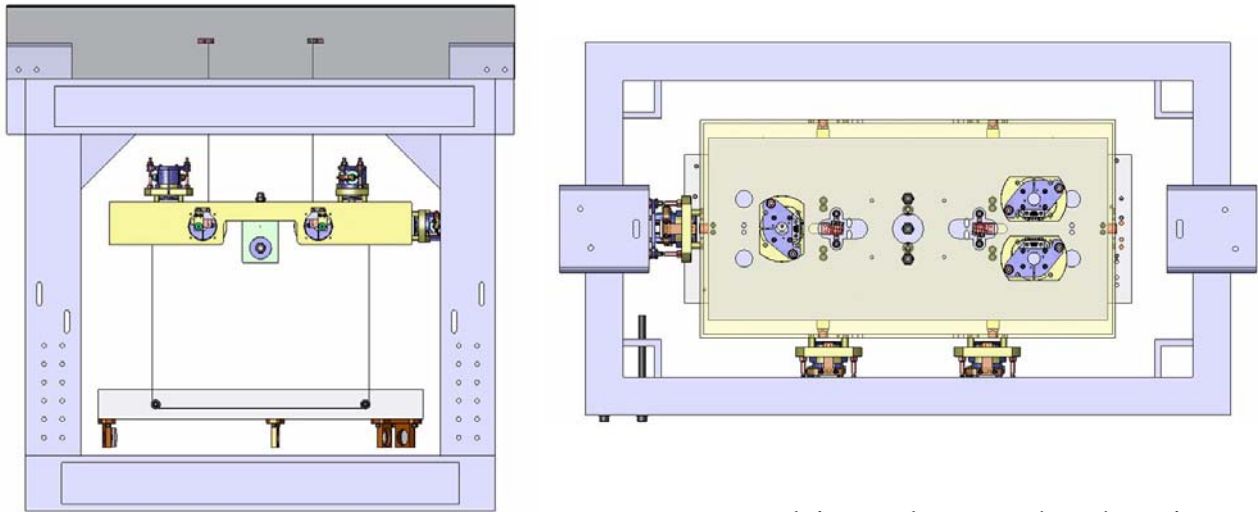


Figure 5 vertical transfer function comparing active damping (blue) and eddy current damping (green) where the ECD is sufficient to reduce the high frequency peak to a Q of ~ 70 .

We have checked that the magnets can safely be put on the top mass (rather than on the support structure) without introducing excess noise due to coupling with ambient magnetic fields. See Appendix C.

The amount of pitch and roll damping achieved by the same number of magnets depends on the lever arms assumed. Figure 6 shows a possible layout of the OSEMs on the top



mass, and it can be seen that there is space to put ECD units. The actual positioning of the OSEMS and the ECD units is still TBD and the representation of the OSEM positions in figure 6 should not be taken as finalized, and are not the same as those assumed in Appendix B.

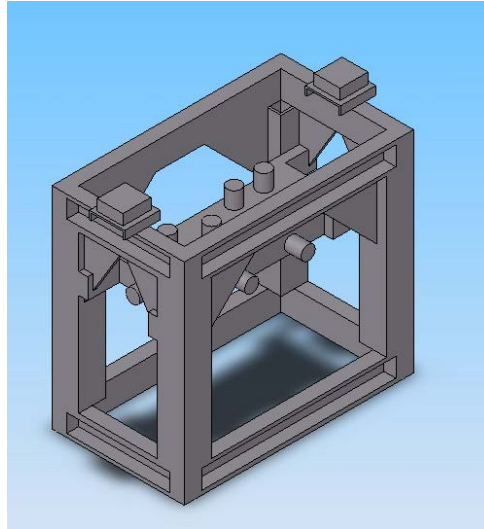
Figure 6. Diagrams showing the tablecloth (in yellow) and possible placement of OSEMS (still to be finalized).

The effect of ECD on the high frequency pitch and roll modes is still to be fully analysed. However as an example – a pitch leverarm of 3 cm with 4 units of 2 magnets as above would appear to damp the high frequency pitch mode at 74 Hz to a Q of ~ 140 , and with a roll leverarm of 6 cm the roll mode at 43 Hz has a Q of ~ 180 (these numbers and the effect on isolation in these directions are TBC).

2.3 Support structure.

The support structure has been slightly modified from that shown in figure 2 to give a height such that a double pendulum 25 cm plus 25 cm in length can be incorporated. It is 10 cm taller, slightly wider to give more space round the optical bench and with thicker walls for the box section to stiffen. (see figure 7). The first resonance of this structure is \sim

170 Hz from FEA suggests that an meet the resonance to be



analysis (TBC) which actual structure would requirement for this greater than 150 Hz.

Figure 7 Revised FEA model of structure

2.4 Earthquake stops.

Earthquake stop design is TBD. We advocate a design which incorporates fluorel (aka viton) to provide some impact damping.

2.5 Design of tablecloth

The tablecloth is the metal support for the OSEMS which sits over and around the top mass and is attached to the support structure. The design is based on of the IMC tablecloth design – suitably enlarged. See figure 6 for a conceptual rendering. Not shown is the mounting of the tablecloth to the support structure.

2.6 Routing of wires up the suspension.

It was suggested that some of the wiring from the optics bench might be routed directly to the support structure rather than up the suspension. Based on some preliminary cable stiffness measurements it seems that the cabling may compromise the OMC double suspension isolation at frequencies above ~15 Hz. Further stiffness measurements may change the numbers and conclusions. Details are posted on the Wiki page:

http://lhocds.ligo-wa.caltech.edu:8000/advligo/OMC_Suspension

The current baseline for wiring is given in Appendix D, where it is assumed all cables will run up the suspension chain with anchoring at each mass.

3. Responsibilities of SUS and ISC

The design and construction of the OMC is a joint effort of the SUS and ISC groups. ISC provides the final “optic” (by which we mean the optical bench and the associated optics mounted on it). SUS provides the suspension, support structure, electronics, and installation tooling. SUS will also provide a dummy “optic” made of aluminium, for initial assembly and tests of the suspension.

4. Change from Enhanced LIGO to Advanced LIGO

The OMC team is designing the Enhanced LIGO OMC to fit the needs of Enhanced and Advanced LIGO. However, there are a number of things that should be re-addressed after testing, installation and integration of the OMC into a LIGO interferometer. These topics include:

- a) The material of the optics bench. The OMC prototype will have a metal bench with metal optics holders. If better performance is required (for example, stiffer bench with more thermal stability) a fused silica bench could be used with silicate bonded optics holders.
- b) Vertical isolation. If more isolation is found to be required, a second stage of blades could be incorporated at the top mass. Such blades also give the advantage of easier damping of the highest rigid body modes (as mentioned in 2.2)
- c) Relationship between the OMC and the rest of the interferometer. Currently, there are no inputs from other systems, so we will roll-off the feedback to the actuators at a frequency consistent with the local damping requirements. Before production starts, we should review whether the OMC stands alone or if input signals from other systems may be applied.

5. Conclusion

We have presented a conceptual design for a double pendulum suspension of an output modecleaner. The normal mode frequencies all lie in a range of ~ 0.5 to 3 Hz apart from the highest vertical pitch and roll modes. The low frequency modes can all be adequately damped by applying forces at the top mass. The isolation at 10 Hz in longitudinal is $\sim 4 \times 10^{-4}$ and in vertical $\sim 2 \times 10^{-2}$ (which assuming 0.1% coupling gives a vertical to longitudinal isolation of $\sim 2 \times 10^{-5}$). The design makes use of existing designs from the input modecleaner, including aspects of the mechanical parts (blades, top mass, mechanical alignment) and electronics design (OSEMS for damping and DC alignment).

Appendix A.

Parameters for Output Mode Cleaner (SI). See also figures 7 and 8

pend =

```
pendulum_model: 'double_pendulum'  
  m1: 3.1250e+000  
  l1x: 2.3800e-002  
  l1y: 2.4000e-003  
  l1z: 2.3800e-002  
  m2: 6  
  ix: 1.5000e-001  
  iy: 4.5000e-001  
  iz: 4.0000e-002  
  l2x: 1.0205e-001  
  l2y: 1.2050e-002  
  l2z: 1.1250e-001  
  l1: 2.5000e-001  
  l2: 2.5000e-001  
  nw1: 2  
  nw2: 4  
  r1: 1.5000e-004  
  r2: 8.5000e-005  
  Y1: 2.1200e+011  
  Y2: 2.1200e+011  
  ufc1: 2.3000e+000  
  ufc2: 0  
  d0: 1.0000e-003  
  d1: 1.0000e-003  
  d2: 1.0000e-003  
  su: 0  
  si: 7.5000e-002  
  n0: 7.2000e-002  
  n1: 7.2000e-002  
  n2: 1.3500e-001  
  n3: 1.3500e-001
```

ufc1 = uncoupled resonant frequency of the top mass on the top blades
ufc2 = 0 corresponds to no lower blades.

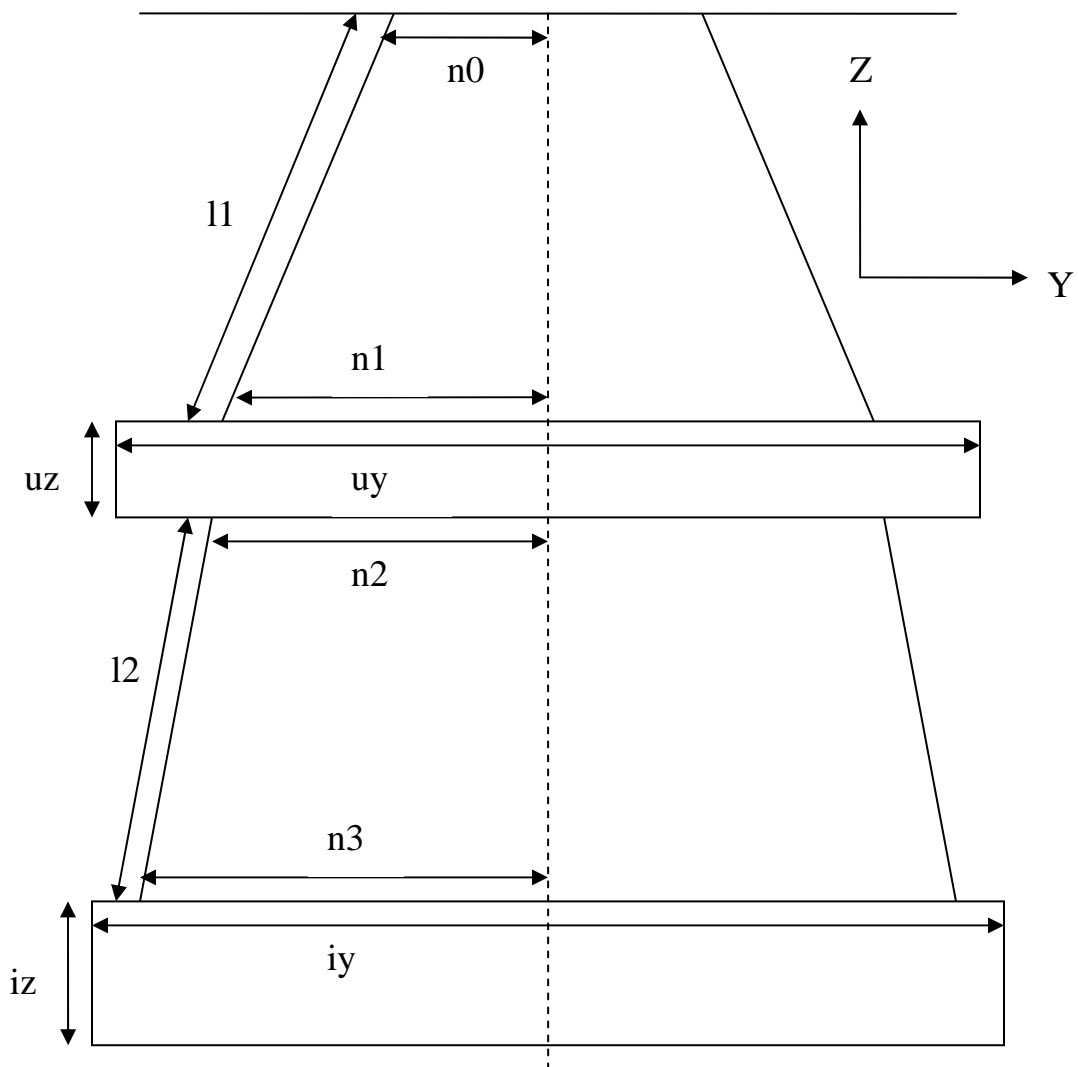


Figure 7. Parameters for a double pendulum (face on view)

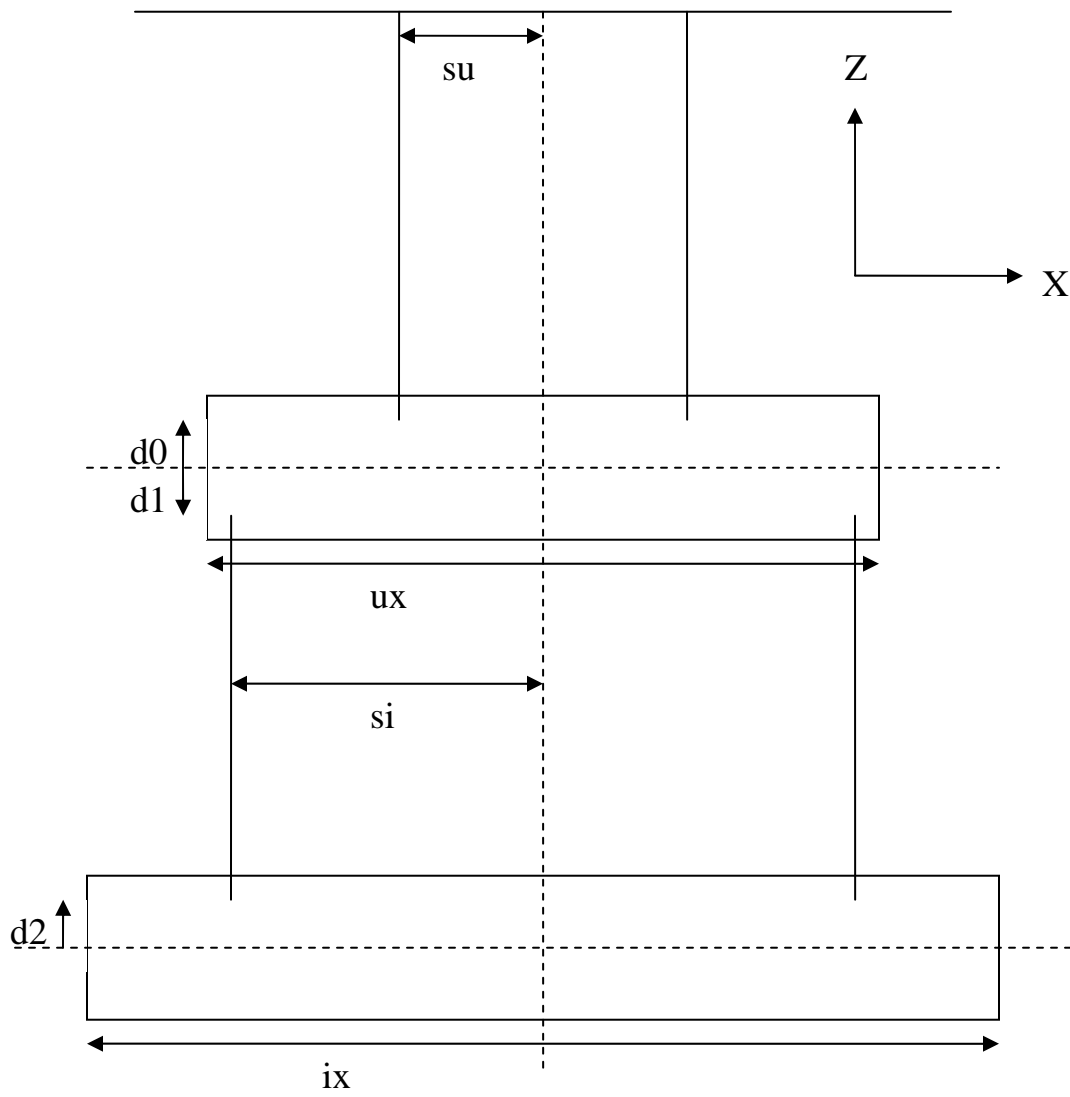


Figure 8. Parameters for a double pendulum (side view)

Appendix B Actuator Gains and Leverarms

Layout assumed same as input modecleaner (this could easily be modified).

3 actuators on top for vertical roll and pitch
2 actuators on side for longitudinal and yaw
1 actuator on end for transverse

Eddy current damping function veldamp = zpk([0],-1000,30000)

Gain parameters:

% gain = 0.06

Gain triangle = (leverarm)² * (no. of coils) * gain

%gainztrl = gain; % vertical, z, pitch, rt, roll rl (coils on top of upper mass)
%gaint = gain.*2; % transverse, t (coil on one end of upper mass)
%gainlrz = gain; % longitudinal, l, yaw, rz (coils on long rear side of upper mass)

%long = (1)² * 2 * gainlrz = 0.12
%pitch = (0.03)² * 2 * gainztrl = 1.08e-4
%vert = (1)² * 3 * gainztrl = 0.18
%yaw = (0.08)² * 2 * gainlrz = 7.68e-4
%trans = (1)² * 1 * gaint = 0.12
%roll = (0.06)² * 3 * gainztrl = 6.48e-4

Appendix C Magnetic Coupling

Copy of e-mail sent out by NAR on 7 Dec 2006

Colleagues

I have done back-of-envelope calculation to check the effect of having the ECD magnets on the top mass rather than on the support structure in respect of external mag field coupling. I consider below coupling in vertical direction.

1) Assume 8 magnets each 10mm by 10 mm

Magnetic dipole strength of each is 0.5 Am² (ref T050105 K Strain)

Assume no cancellation due to arranging with opposing poles - so total strength = 8 x 0.5 = 4 Am² (This is conservative - we would have pairs of magnets adjacent to each other)

2) Average coupling factor at 10 Hz = 10 N/T/(Am²) (reference T050271 P Fritschel)

3) Magnetic field noise at 10 Hz = 10^{-11} T/rt Hz (reference T050271 P Fritschel)

4) Force to displacement vertical transfer function at 10 Hz from top mass to bottom mass, assuming ECD of strength 13 kg/s (as provided by 1/2 of a 4 by 4 ECD array - i.e. 8 magnets) = 3.2×10^{-5} m/N (from MATLAB model)

1) x 2) x3) x4) gives vertical displacement noise due to magnetic coupling of $4 \times 10 \times 10^{-11} \times 3.2 \times 10^{-5} = 1.3 \times 10^{-14}$ m/rt Hz

Compare this to direct seismic noise coupling.

5) Single stage platform noise at 10 Hz = 4×10^{-11} m/rt Hz (taken from M060062 - HAM single stage report - where this number is horizontal requirement and I assume vertical same as horizontal. I checked with Brian Lantz that the design should more than meet this number).

6) TF of OMC suspension in vertical at 10 Hz, assuming ECD as above, = 3.4×10^{-2}

Thus residual vertical motion = 5) x 6) = 1.4×10^{-12} m/rt Hz

Conclusion: the vertical motion due to magnetic coupling is 2 orders of magnitude smaller than the vertical residual seismic motion and so negligible. Thus we can attach magnets to the top mass OK.

Appendix D Routing of wires

From: Rich Abbott <abbott@ligo.caltech.edu>

Subject: Re: OMC

Cc: Norna A Robertson <nornar@stanford.edu>,

Calum Torrie <c.torrie@physics.gla.ac.uk>, echols_c@ligo.caltech.edu,

jay@ligo.caltech.edu, Mandic Vuk <mandic_v@ligo.caltech.edu>,

Sam Waldman <waldman_s@ligo.caltech.edu>,

Janeen Romie <janeen@ligo-la.caltech.edu>, Rana <rana@its.caltech.edu>

X-Spam-Score: undef - Sender Whitelisted (abbott@ligo.caltech.edu: Mail from user authenticated via SMTP AUTH allowed always)

X-Canit-Stats-ID: 5440012 - c79ad7eebd68

X-Scanned-By: CanIt (www . roaringpenguin . com) on 131.215.115.19

Hi Dennis,

I've been thinking about what you wrote below, and I think it would be good to be crystal clear as to the cabling I am currently envisioning for the OMC. I had told Janeen that my baseline plan would be to run all cables up the suspension chain anchoring at each mass. This is the more conservative approach, and is fine with me as far as the electrical issues are concerned.

At present, I envision a minimum of two separate cables bundles going back up the suspension chain from the OMC base-plate. The need for two cables is due to a desire to

keep low-noise analog signals separate from other less stringently controlled wires. I am not exploring ribbons as Dennis mentioned they have their own problems. My plan is to use individual wires, perhaps twisted in some cases, but contained within an overall shield.

Cable bundle #1 would contain the following number of wires of the given description contained within an overall braided shield:

(2 wires) Supply and Return for OMC Length piezo actuator, likely to be twisted as the OMC piezo has a kHz dither on it as well as the high voltage length signal

(2 wires) Supply and Return for the OMC thermal length actuator. I am envisioning using a higher voltage so I can use the same tiny wire as the rest of the wires in the bundle.

(2 wires) Supply and Return for the OMC thermal length actuator temperature read-back. Eventually, this might not be needed, but for prototype testing I think it's prudent

(5 wires) QPD 1 for position information

(5 wires) QPD 2 for position information

Total of 16 wires

Cable bundle #2 would contain the following number of wires of the given description contained within an overall braided shield:

(2 wires) For the differential DC readout signal from DC readout diode 1

(2 wires) For the differential DC readout signal from DC readout diode 2

(4 wires) For the DC power used by the DC readout diode pre-amplifier (Positive voltage supply and return, and Negative voltage supply and return). This function represents a high current connection at ~200mA

(2 wires) For the variable bias voltage of DC readout diode 1. This function represents a high current connection at ~100mA

(2 wires) For the variable bias voltage of DC readout diode 2. This function represents a high current connection at ~100mA

Total of 12 wires

As for the gauge of wire to use, I haven't made an exact choice yet, but I don't see why I can't use the same stuff that has been used before. Some data was taken by Ben on Cooner wire ampacity in a vacuum environment, but the data doesn't indicate the wire size. The data does suggest that whatever wire he was measuring will be fine at 200mA.

Rich

Appendix E Addition of second set of blades

See discussion in section 2.2 page 6 above with regards to how to handle high frequency, high Q modes in vertical roll and pitch. The following describes option b).

Below is copy of e-mail sent by NAR to OMC team on 18 Dec 2006.

I have taken a look at what happens when blades are added at the upper mass in the OMC suspension. The only thing i changed in the parameter set as given in the current conceptual design doc was the so-called ufc2 - which relates to the blade spring constant (it is the frequency of the bottom mass on the blades at the upper mass). I put in the appropriate value assuming we use blades of same design as in the IMC. Assuming I did this correctly (will check again) I get the following mode frequencies

longpitch1: [5.9674e-001 7.4491e-001 2.2926e+000]
longpitch2: 9.4400e+000
yaw: [5.2867e-001 2.9894e+000]
transroll1: [7.4009e-001 8.0846e-001 2.2926e+000]
transroll2: 5.7092e+000
vertical: [1.1999e+000 4.3613e+000]

The higher vert mode is now 4.4 Hz, the higher roll is 5.7 Hz and the higher pitch is 9.4 Hz. It is that pitch mode that makes things not straightforward. If we want to damp that mode actively we need feedback extending beyond that frequency, and if we do that we will have to worry about sensor noise possibly compromising the isolation.

For those used to seeing lower frequencies in triple and quad models you may ask why that frequency is so high if we have blades which give soft vertical modes. It is because the spacing of the wires in the pitch direction is large since we have chosen to attach the wires on the side of the bench. Full width is 15 cm. And the frequency in pitch scales with that spacing. For the roll mode the spacing is also wide - but the moment of inertia in the roll sense is significantly larger and more than compensates, even though freq goes as square root of I. (*should read "inverse square root"*)

Quick look at sensor noise. Assuming the "adapted geo active" control, with the lowpass frequency increased from 9 Hz to 18 Hz for stability and using gain to give a settling time of around 10 secs for long and for pitch (gain box in simulink model set at 0.5 for long and pitch) the TF in longitudinal from sensor to bottom mass = 1.2×10^{-4} at 10 Hz. Multiply this by 10^{-10} m/rt Hz sensor noise gives 1.2×10^{-14} m/rt Hz. Compare this to residual seismic. From Brian's single stage HAM info the residual long noise at the suspension point of the OMC will be around 1.4×10^{-11} m/rt Hz (though actual requirement is i believe 4×10^{-11} m/rt Hz). The longitudinal TF of the OMC with the above gains etc is 4.3×10^{-4} . So residual long. motion is 6.0×10^{-15} m/rt Hz. Thus

sensor noise dominates by factor of 2.

Actual 10 Hz sensor noise may be a bit better than this - I'll check* (the 10^{-10} m/rt hz number is for 1 hz). However this is food for thought and i welcome any thoughts, comments etc

Norna

* (checked – number is correct)

In follow up discussion on this it was suggested that this is a case where the modal damping approach might work very nicely, allowing us to provide significant damping for the 9 Hz pitch mode without compromising the longitudinal passive isolation.

Another point to note is that the moments of inertia (MOI) used in the model which gave the frequencies quoted above are likely to be underestimates since

- a) the upper mass was modeled as being the same in mass and MOIs as the current input modecleaner mass, and for the OMC it will be wider in the “pitch” direction and hence have larger relevant MOI,
- b) the optical bench is modeled simply as a rectangular box shape. Additional mass (optics, diodes preamp) will tend to be near the edges, again raising MOI.

Larger MOIs will lower the higher pitch frequency, easing the sensor noise problem.

Examples of the longitudinal and pitch transfer functions and impulse responses with active damping and gains as described above are given in figures 9 and 10.

The parameters used are as in Appendix A with one change: $ufc2 = 2.3$.

This number is derived as follows.

$ufcn$ = uncoupled frequency of nth stage = frequency of that stage which would be observed for a set of blades in a particular stage supporting only the mass directly below (the use of this definition is historical).

For the input modecleaner $ufc2 = 3.22$ (ref calculation of the lower blade frequency by CIT 01/09/02), and mass supported in that stage is 2.98 kg.

Thus summed spring constant of the 4 blades, k , = $m \cdot (2 \cdot \pi \cdot f)^2 = 1.22 \times 10^3$

For the OMC the mass supported is 6 kg = M say.

Thus $ufc2 = \sqrt{k/M} \cdot 1 / (2 \cdot \pi) = 2.3$

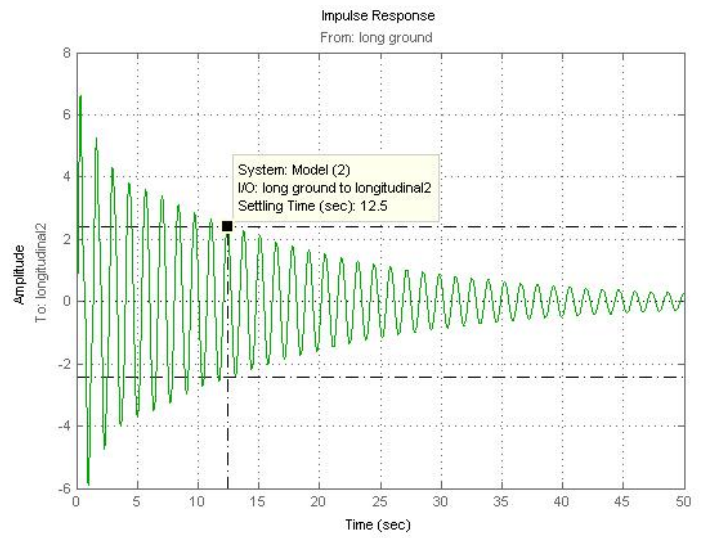
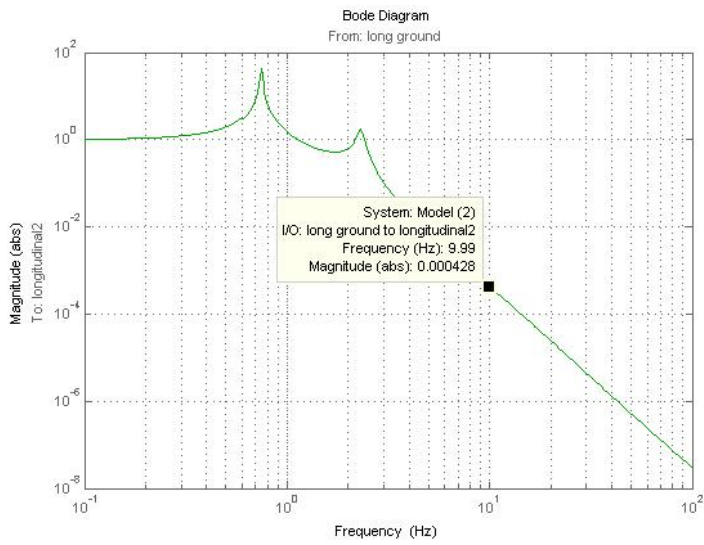


Figure 9. Longitudinal transfer function (TF) and impulse response with active damping. TF at 10 Hz is indicated on left. Time to decay to $1/e$ is indicated on right.

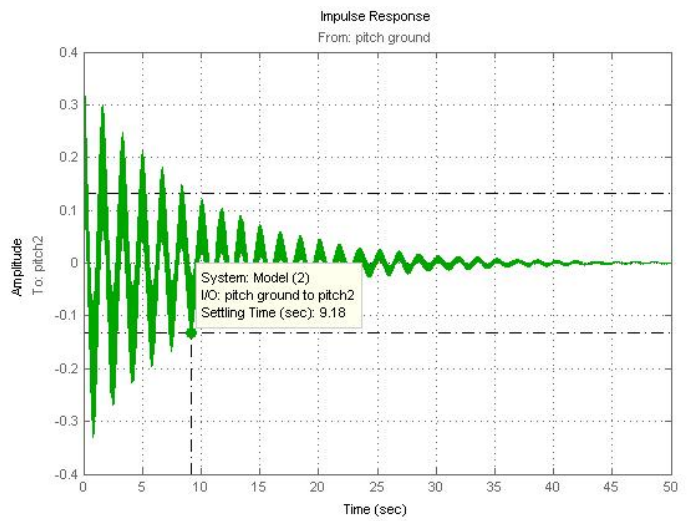
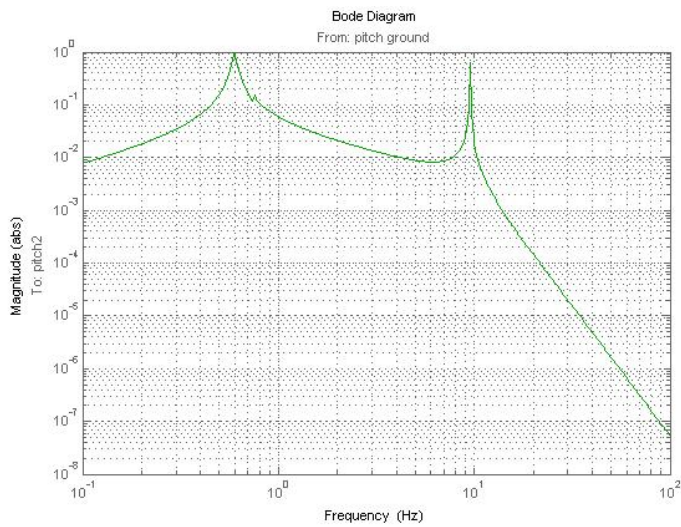


Figure 10. Pitch transfer function (TF) and impulse response with active damping. TF at 10 Hz is indicated on left. Time to decay to $1/e$ is indicated on right.