Minimum Inductance Calculation

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R. Abbott, Caltech

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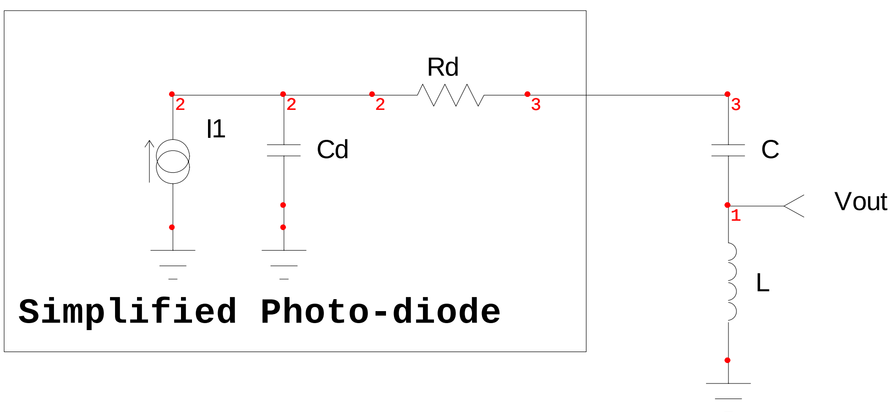
1. A derivation is presented for the minimum inductor value, consistent with a lower boundary signal to noise ratio, for a series resonant RFPD read-out circuit.
   1.1. Figure 1 shows a simplified schematic of a series read-out topology

Figure 1



The photodetector equivalent parameters are in the boxed region. The output voltage, V_{out}, represents the input to a low-noise RF amplifier.

C_d is the junction capacitance of the photo-diode. The photo-diode is assumed to be reverse-biased such that the internal diode can be neglected.

R_d is the series resistance of the photo-diode. Any series resistance in the external resonant components can be considered to be added to R_d in this model.

1.2. Assuming L and C are driven at their series resonant frequency ω, the magnitude of the current flowing through R_d can be shown to equal:

$$I_{Rd} = I_1 \cdot \sqrt{\frac{1}{1+(\omega \cdot R_d \cdot C_d)^2}}$$

1.3. The component of I1 attributable to shot-noise is given by:

$$I_{shot} = \sqrt{2 \cdot e \cdot I_{dc}}$$

Where I_{dc} is the photo-current flowing in the detector and e is is the electron charge (1.6 e -19 coulombs)

1.4. The output voltage due to shot-noise is given by:

$$Vout_{shot} = \omega \cdot L \cdot \sqrt{\frac{2 \cdot e \cdot I_{dc}}{1 + (\omega \cdot R_d \cdot C_d)^2}}$$

1.5. If $Vout_{shot}$ is to be a factor of N greater than the electronic noise E_n of the RF amplifier stage, a lower limit on L is given by:

$$L \geq \frac{N \cdot E_n}{\omega \cdot \sqrt{\frac{2 \cdot e \cdot I_{dc}}{1 + (\omega \cdot R_d \cdot C_d)^2}}}$$

1.6. Example calculation:

N = 5
En = 1nV/√Hz
$\omega = 2 \cdot \pi \cdot 100$ MHz
I_{dc} = 50mA
R_d = 10 Ω
C_d = 100 pF

Calculated L ≥ 74 nH